

Analiza matematyczna

Zajęcia nr 3

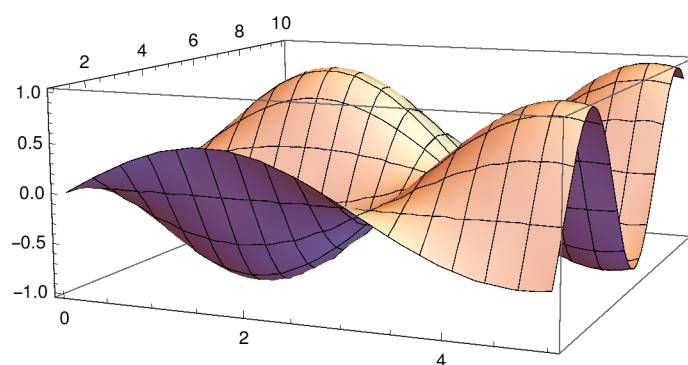
Wykres 3D

? Plot3D

`Plot3D[f, {x, xmin, xmax}, {y, ymin, ymax}]` generates a three-dimensional plot of f as a function of x and y .

`Plot3D[{f1, f2, ...}, {x, xmin, xmax}, {y, ymin, ymax}]` plots several functions. >>

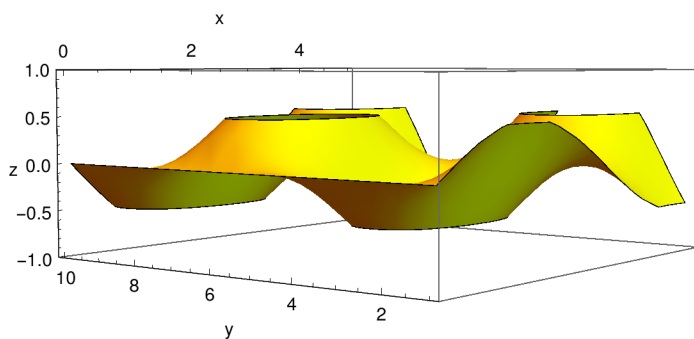
`Plot3D[Sin[x] Cos[y], {x, 0, 5}, {y, 1, 10}]`



```

Plot3D[Sin[x] Cos[y], {x, 0, 5}, {y, 1, 10},
  Mesh → False,
  PlotStyle → Yellow,
  AxesLabel → {"x", "y", "z"},
  PlotRange → {-1, 1},
  AxesLabel → {"x", "y", "z"},
  RegionFunction → Function[{x, y, z}, Abs[z] < 0.5]]

```



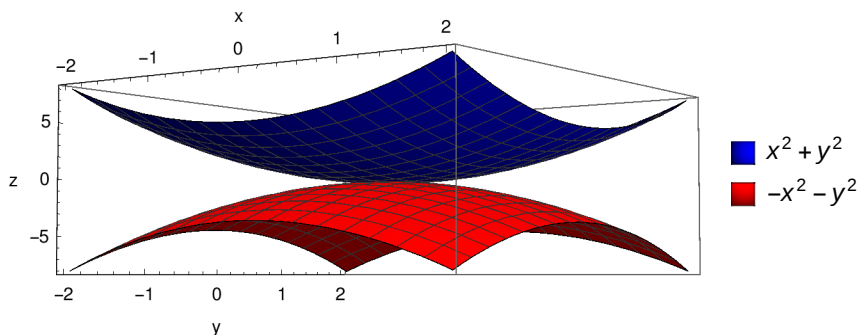
? RegionFunction

RegionFunction is an option for plotting functions that specifies the region to include in the plot drawn. >>

```

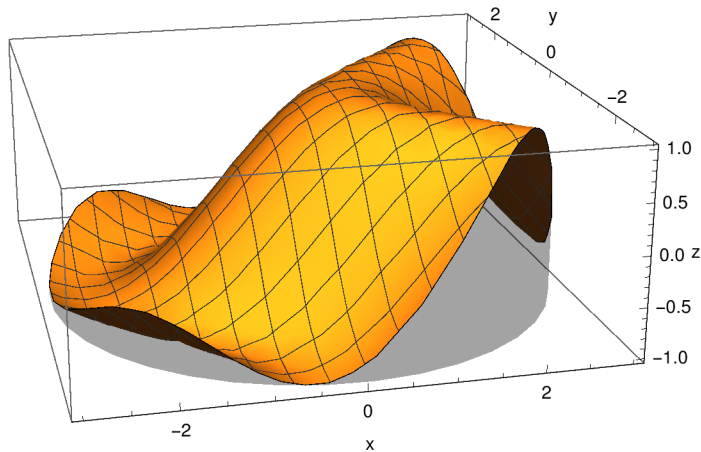
Plot3D[{x^2 + y^2, -x^2 - y^2}, {x, -2, 2}, {y, -2, 2},
  PlotStyle → {Blue, Red},
  AxesLabel → {"x", "y", "z"},
  PlotLegends → "Expressions"]

```

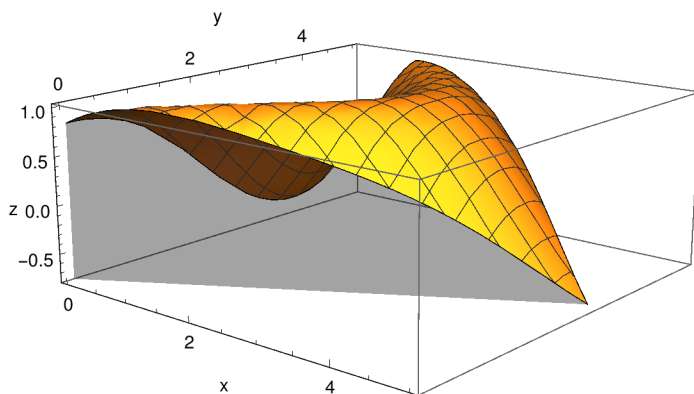


∈ \[Element] (bez spacji)

```
Plot3D[Sin[x + Cos[y]], {x, y} ∈ Disk[{0, 0}, 3],
  Filling → Bottom, AxesLabel → {"x", "y", "z"}]
```



```
Plot3D[Sin[x + Cos[y]], {x, y} ∈ Polygon[{{0, 0}, {1, 5}, {5, 3}}],
  Filling → Bottom, AxesLabel → {"x", "y", "z"}]
```

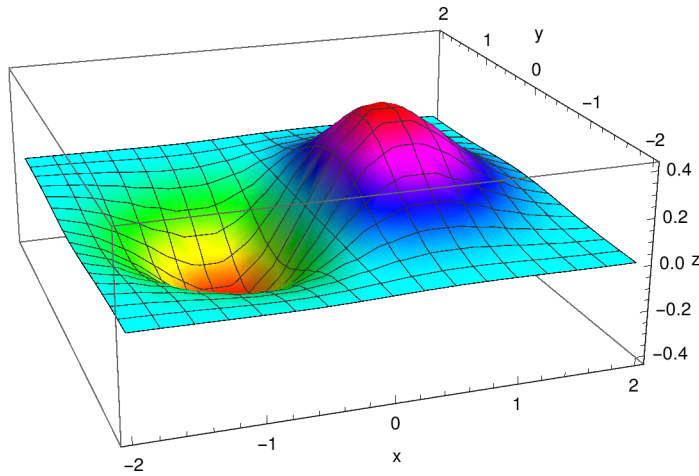


Hue - odcien wykresu

? Hue

Hue[h] is a graphics directive which specifies that objects which follow are to be displayed, in a color corresponding to hue h .
 Hue[h, s, b] specifies colors in terms of hue, saturation, and brightness.
 Hue[h, s, b, a] specifies opacity a . >>

```
Plot3D[x / Exp[x^2 + y^2], {x, -2, 2}, {y, -2, 2},
  AxesLabel -> {"x", "y", "z"},
  ColorFunction -> Function[{x, y, z}, Hue[z]]
]
```



Zadanie

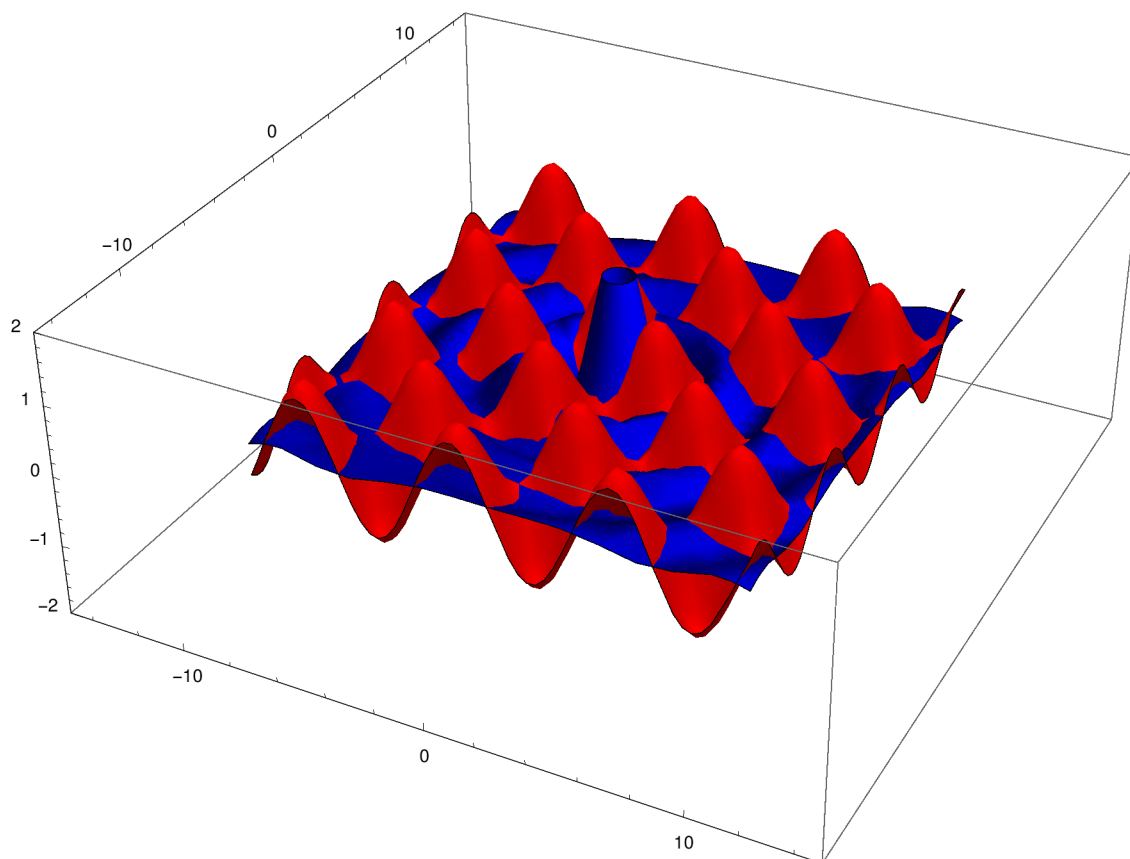
A) zdefiniować funkcje:

$$F(x, y) = \frac{\sin(2\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}}$$

$$G(x, y) = \sin(x) \cos(y)$$

B) Narysować je na jednym wykresie 3D

- dziedzina $x \in (-10, 10)$, $y \in (-10, 10)$
- zakres osi x: $(-15, 15)$, zakres osi y: $(-15, 15)$, zakres osi z: $(-2, 2)$ (PlotRange)
- tak aby wykresy miały różne kolory (PlotStyle)
- usunąć siatkę (Mesh->False)
- dodać parametr RegionFunction z warunkiem $|z| < 1.5$



RegionPlot3D

Rysuje bryłę w obszarze, dla którego spełniony jest warunek.

? RegionPlot3D

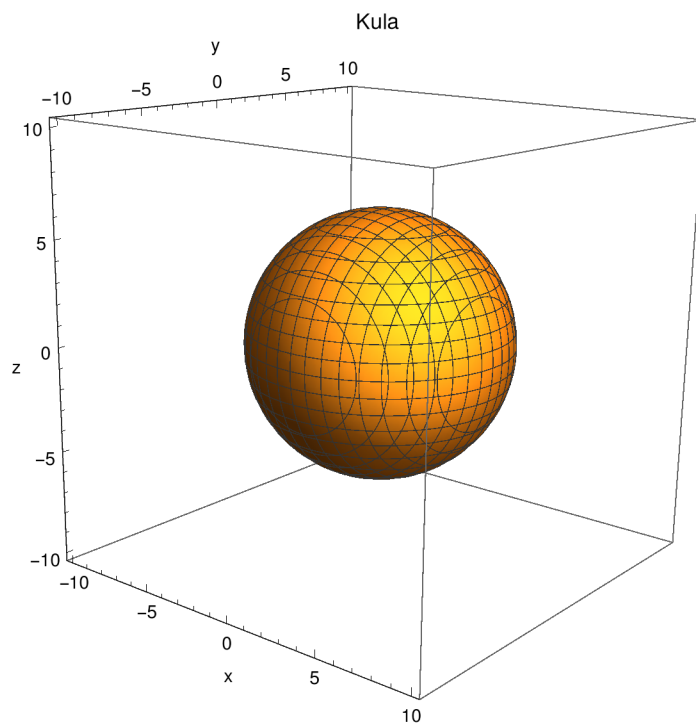
`RegionPlot3D[pred, {x, xmin, xmax}, {y, ymin, ymax}, {z, zmin, zmax}]`

makes a plot showing the three-dimensional region in which *pred* is True. >>

```

RegionPlot3D[x^2 + y^2 + z^2 < 40, {x, -10, 10}, {y, -10, 10}, {z, -10, 10},
  AxesLabel -> {"x", "y", "z"},
  PlotLabel -> "Kula",
  PlotPoints -> 100]

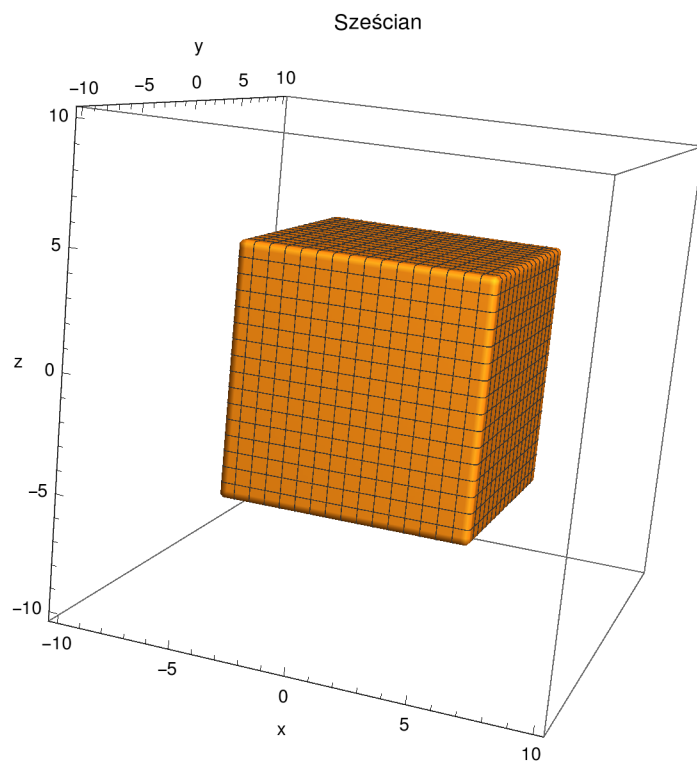
```



? PlotPoints

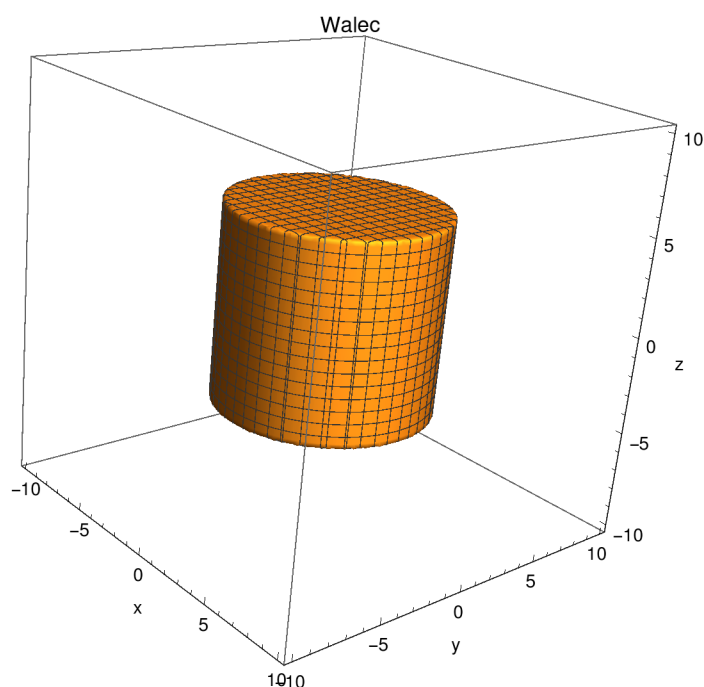
PlotPoints is an option for plotting functions that specifies how many initial sample points to use. >>

```
RegionPlot3D[x^2 < 30 && y^2 < 30 && z^2 < 30,
  {x, -10, 10}, {y, -10, 10}, {z, -10, 10},
  AxesLabel -> {"x", "y", "z"},
  PlotLabel -> "Sześcián",
  PlotPoints -> 100]
```



Zadanie

Narysować walec przy użyciu RegionPlot3D



ParametricPlot3D

Krzywa w 3D sparametryzowana przez 1 zmienną lub płaszczyzna w 3D sparametryzowana przez 2 zmienne

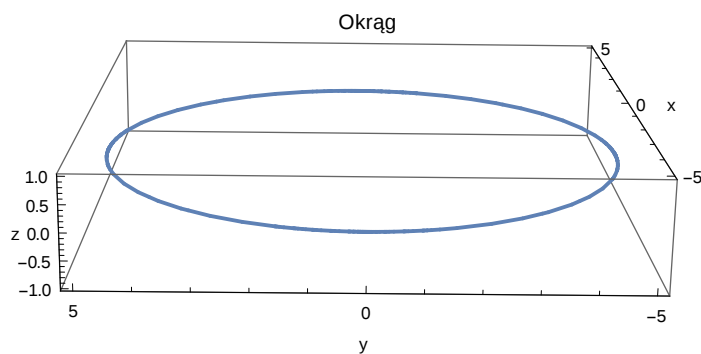
? ParametricPlot3D

`ParametricPlot3D[{fx, fy, fz}, {u, umin, umax}]` produces a three-dimensional space curve parametrized by a variable u which runs from u_{min} to u_{max} .

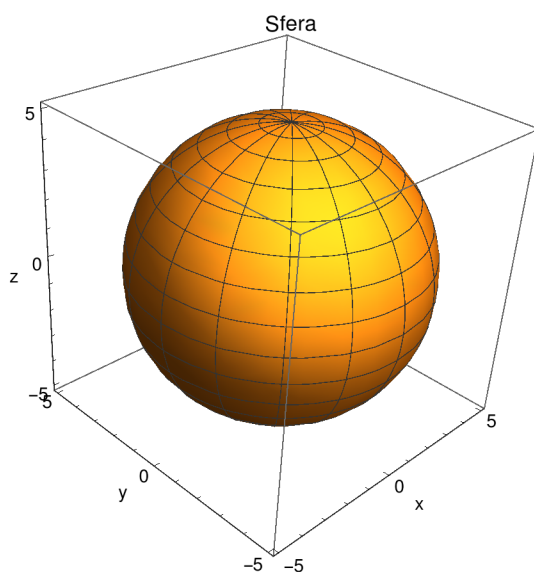
`ParametricPlot3D[{fx, fy, fz}, {u, umin, umax}, {v, vmin, vmax}]` produces a three-dimensional surface parametrized by u and v .

`ParametricPlot3D[{fx, fy, fz}, {gx, gy, gz} ...] ...]` plots several objects together. >>


```
ParametricPlot3D[{5 Cos[ $\phi$ ] , 5 Sin[ $\phi$ ] , 0}, { $\phi$ , 0, 2 Pi},
  AxesLabel -> {"x" , "y", "z"},
  PlotLabel -> "Okrąg"]
```

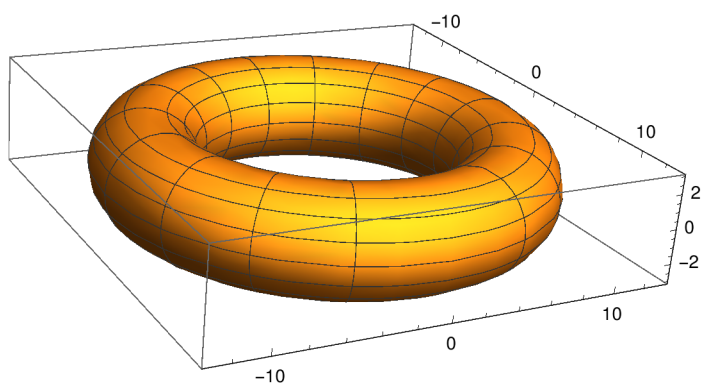


```
ParametricPlot3D[
  {5 Cos[ $\phi$ ] Sin[ $\theta$ ], 5 Sin[ $\phi$ ] Sin[ $\theta$ ], 5 Cos[ $\theta$ ]}, { $\phi$ , 0, 2 Pi}, { $\theta$ , 0, Pi},
  AxesLabel -> {"x" , "y", "z"},
  PlotLabel -> "Sfera"]
```



Zadanie

Narysować torus przy użyciu ParametricPlot3D



Granice

? Limit

Limit[*expr*, $x \rightarrow x_0$] finds the limiting value of *expr* when x approaches x_0 . >>

Limit[Sin[x] / x, x → Infinity]

0

Założenie:

Limit[x^a, x → Infinity, Assumptions → a < 0]

0

Limit[x^a, x → Infinity, Assumptions → a == 0]

1

Limit[x^a, x → Infinity, Assumptions → a > 0]

∞

Granice prawostronne i lewostronne

lewostronne:

Limit[1 / x, x → 0, Direction → 1]

$-\infty$

prawostronne:

Limit[1 / x, x → 0, Direction → -1]

∞

Zadanie

policzyc granice

a) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$

b) $\lim_{n \rightarrow \infty} \frac{\sin(n!)}{\sqrt{n}}$

c) $\lim_{n \rightarrow \infty} \sqrt{1+n} - \sqrt{n}$

d) $\lim_{n \rightarrow \infty} \frac{1}{n^q}$ dla $q > 0$

e) $\lim_{n \rightarrow \infty} \frac{(2n+1)! - (2n-1)!}{(2n)! n!}$

f) $\lim_{n \rightarrow \infty} \frac{c^n}{n^k}$ dla $c > 1$

g) $\lim_{x \rightarrow \infty} \sin(x)$

h) $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$