CHIRAL CONDENSATE AND THE STRUCTURE OF HADRONS*

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(Received February 14, 2014)

A model of hadron masses based on the quark structure of hadrons combined with effects of chiral dynamics is used to calculate $2+1$ flavour chiral condensate in the hadron resonance gas framework. Results are discussed in the context of recent lattice QCD data. Improvements of the dynamical models of hadron structure will be suggested with the aim to estimate strange sigma term of the nucleon.

DOI:10.5506/APhysPolBSupp.7.225
PACS numbers: 12.38.Lg, 11.25.Tq, 25.75.–q, 12.38.Gc

1. Introduction

Spontaneous chiral symmetry breaking is, apart from colour confinement, the most important physical aspect of strong interactions. Virtual quarks and antiquarks of opposite chirality are attracted to each other due to the strong interactions and destabilize the trivial vacuum state. A condensate is formed which gives rise to a nonvanishing expectation value of the bilinear fermionic operator $\bar{\psi}\psi$.

Once the temperature is raised, chiral symmetry will be affected and eventually restored in the deconfined phase of QCD. In order to find the temperature dependence of the chiral condensate in the hadronic medium, we use the framework of the hadron resonance gas (HRG) with the partition function $Z_{\rm HRG}$. From the generic definition follows in this case

$$\langle\bar{q}q\rangle = -\frac{\partial}{\partial m_0} T \ln Z_{\rm HRG} = \langle\bar{q}q\rangle_0 + \sum_{\rm H} \frac{\partial m_{\rm H}}{\partial m_q} n_{\rm H}(T)\,, \tag{1}$$

* Progress report presented by J. Jankowski at the XXXI Max Born Symposium and HIC for FAIR Workshop "Three Days of Critical Behaviour in Hot and Dense QCD", Wrocław, Poland, June 14–16, 2013.

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where the scalar densities for hadrons have been introduced as

$$n_{\rm H}(T) = \frac{d_{\rm H}}{2\pi^2} \int\limits_0^\infty dk k^2 \frac{m_{\rm H}}{\sqrt{m_{\rm H}^2 + k^2}} \frac{1}{e^{\beta\sqrt{m_{\rm H}^2+k^2}} \pm 1} \tag{2}$$

with the inverse temperature $\beta = 1/T$ and the hadron degeneracy $d_{\rm H}$. The sum over hadronic states takes into account all light and strange hadrons up to a mass $m_{\rm max} \sim 2$ GeV.

In this note, we explore the effect of the quark substructure of hadrons on the melting of the condensate for temperatures below the QCD critical temperature. The results are discussed in the context of recent first principle QCD simulations on the lattice.

2. Constituent quark picture

Some prior knowledge about the structure of hadrons is required to evaluate (1). The constituent quark picture (CQP) adopted recently [1] assumes that baryon and meson masses scale as

$$m_{\rm B} = (3 - N_{\rm s})M_{\rm q} + N_{\rm s}M_{\rm s} + \kappa_{\rm B}\,, \tag{3}$$

$$m_{\rm M} = (2 - N_{\rm s})M_{\rm q} + N_{\rm s}M_{\rm s} + \kappa_{\rm M}\,. \tag{4}$$

The quark masses in these mass formulae are the dynamical (constituent) ones and are denoted by $M_{\rm q}$ for the light quarks and by $M_{\rm s}$ for the strange quark. The parameter $N_{\rm s}$ measures the strangeness content of the hadron and the quantities $\kappa_{\rm B}$, $\kappa_{\rm M}$ denote the state dependent binding energies, assumed to be independent on the current quark masses. For the open strange hadrons $N_{\rm s}$ is simply the number of strange (anti)quarks in the hadron. For hidden strange mesons — such as the η or the h_1 — it is modified by the square modulus of the coefficient of the $\bar{s}s$ contribution to the meson wave function. There are two possible flavour assignments related to the flavour singlet and flavour octet structure of the hidden strange mesons. The strangess counting parameter is $N_{\rm s}^{(0)} = 2/3$ for the singlet and $N_{\rm s}^{(8)} = 4/3$ for the octet.

Two further simplifying assumptions are made: excited states are assumed to have the same flavour structure as their respective ground states, and any possible mixing between octet and singlet states is neglected. This approach obviously neglects virtual quark loops and thus for example the strange nucleon sigma term is strictly zero.

2.1. Dynamical quark mass

The dynamical quark mass is determined from the quark gap equation which in the momentum representation for a fixed quark flavour reads [2]

$$S_{\rm f}^{-1}(p) = i\gamma p + m_{\rm f} + \int \frac{d^4q}{(2\pi)^4} g^2 D_{\rho\sigma}(p-q)\gamma_\rho \frac{\lambda^a}{2} S_{\rm f}(q) \Gamma^a_\sigma(p;q)\,. \qquad (5)$$

We take Euclidean field theory with $\{\gamma_\rho, \gamma_\sigma\} = 2\delta_{\rho\sigma}$ and $\gamma_\rho^\dagger = \gamma_\rho$, $\gamma p = \gamma_\mu p_\mu$. Here, $D_{\rho\sigma}(p)$ is the renormalized dressed-gluon propagator and $\Gamma^a_\sigma(p;q)$ is the renormalized dressed-quark–gluon vertex for which similar, nonperturbative Schwinger–Dyson equations exist. In general, the analysis of these equations in the high-momentum limit, where due the asymptotic freedom of QCD perturbative methods apply, does reveal that in solving Eq. (5) regularization and renormalization procedures are required. Nevertheless, additional assumptions and nonperturbative techniques are required to obtain the nontrivial, chiral symmetry breaking and confining solutions which characterize the low-momentum sector of QCD. To gain further insights into the properties of this nonperturbative domain and to obtain nontrivial, semiquantitative solutions of the quark mass gap equations it is legitimate to assume a model gluon propagator and simplified vertex functions, but still obeying symmetry requirements of QCD such as chiral symmetry.

One of the possibilities is to adopt the local limit of a heavy-gluon propagator and the rainbow-ladder ansatz for the vertex function

$$g^2 D_{\rho\sigma}(p-q) = \frac{4\pi\alpha_{\rm IR}}{m_G^2}\delta_{\rho\sigma}\,, \qquad \Gamma^a_\rho(p;q) = \frac{\lambda^a}{2}\Gamma_\rho(p;q) = \frac{\lambda^a}{2}\gamma_\rho\,, \qquad (6)$$

where $m_G = 0.8$ GeV is a dynamically generated gluon energy scale. The fitted parameter $\alpha_{\rm IR} = 0.9\pi$ is commensurate with contemporary estimates of the zero-momentum value of a running-coupling in QCD [3]. This is a nonrenormalizable four point quark vertex, similar to the NJL model [4], providing chiral symmetry breaking but so far not confining.

To go beyond that, one of the many possibilities is to parametrize confinement by introducing an infrared cut-off in the proper-time regularization scheme [5, 6] and in addition to use regularization scheme preserving chiral Ward–Takahashi identities [7]. The cut in the low energy part of the integrals, on intuitive grounds, excludes propagation of modes on long distances. Here, confinement is meant to be violation of the reflection positivity axiom as discussed in [8, 9]. Then the gap equation takes the following form

$$M_{\rm f} = m_{\rm f} + 4\alpha_{\rm IR} M_{\rm f}^3 / \left(3\pi m_G^2\right) \left[\Gamma\left(-1, \tau_{\rm UV}^2 M_{\rm f}^2\right) - \Gamma\left(-1, \tau_{\rm IR}^2 M_{\rm f}^2\right)\right]\,, \qquad (7)$$

where $\tau_{\rm IR} = 1/\Lambda_{\rm IR}$ and $\tau_{\rm UV} = 1/\Lambda_{\rm UV}$, with $\Lambda_{\rm IR} = 0.24$ GeV and $\Lambda_{\rm UV} = 0.905$ GeV [3, 7]. The resulting quark sigma terms are defined as

$$\sigma_{\rm f} = m_{\rm f}\left(dM_{\rm f}/dm_{\rm f}\right)\,. \qquad (8)$$

Numerical results are shown in Fig. 1. It is plainly seen that dynamical effects vanish between the charm and bottom quark masses, *i.e.* around ~ 2 GeV. This effect is very well known under the name of heavy quark symmetry [10]. Quark sigma terms at the physical values of current quark masses read $\sigma_{\rm q} = 7.65$ MeV for the up and down quarks and $\sigma_{\rm s} = 121.16$ MeV for the strange quark. The resulting nucleon sigma term is $\sigma_N = 22.95$ MeV.

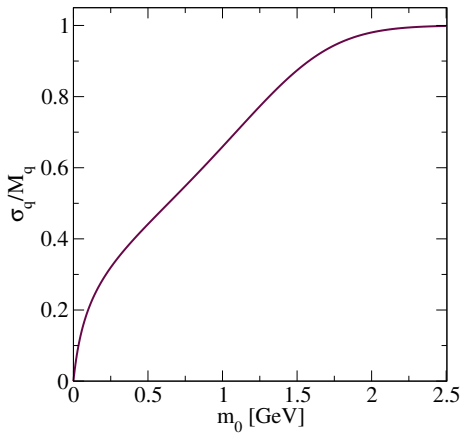

Fig. 1. Quark sigma term as a function of m_0 from Eq. (5).

An easy retrospection of the hadron mass formulas shows that the baryon octet Gell-Mann–Okubo relation $3M_\Lambda + M_\Sigma = 2(M_N + M_\Xi)$ is translated into a constraint on the state dependent contributions: $3\kappa_\Lambda + \kappa_\Sigma = 2(\kappa_N + \kappa_\Xi)$. Both equalities are satisfied with an accuracy of less than 5%.

3. HRG and the QCD lattice data

To compare with lattice QCD results [11, 12], one considers the quantity

$$\Delta_{\rm q,s}(T) = [\langle\bar{q}q\rangle - (m_{\rm q}/m_{\rm s})\langle\bar{s}s\rangle] \,/\, [\langle\bar{q}q\rangle_0 - (m_{\rm q}/m_{\rm s})\langle\bar{s}s\rangle_0] \ . \tag{9}$$

The reason to define this quantity on the lattice is purely technical: in this form it eliminates the quadratic singularity at nonzero values of quark mass $m_{\rm q}/a^2$ (where a is lattice spacing) and the ratio eliminates multiplicative ambiguities in the definition of condensates. Physically this quantity is sensitive to chiral symmetry restoration: it is normalized to unity in vacuum and vanishes with vanishing of the condensates as temperature grows.

The lattice results for the $\Delta_{\rm q,s}(T)$, taken from references [11, 12], are calculated for the lattices with different temporal extent with an extrapolation to the continuum limit. The results are calculated with dynamical quark flavours for (almost) physical values of the quark masses both for the

light and the strange quarks. The values of the quark masses are fixed by reproduction of experimental values of f_K/m_π and f_K/m_K.

Figure 2 shows a comparison of the lattice data to the HRG results with CQP mass formulas. It is evident that $\Delta_{\mathrm{q,s}}(T)$ varies rapidly in the temperature region around $T_{\mathrm{c}} \sim 155$ MeV. There is overall agreement with HRG results up to temperatures $T \sim 140$–150 MeV which is little below the pseudocritical temperature obtained from the lattice data [13].

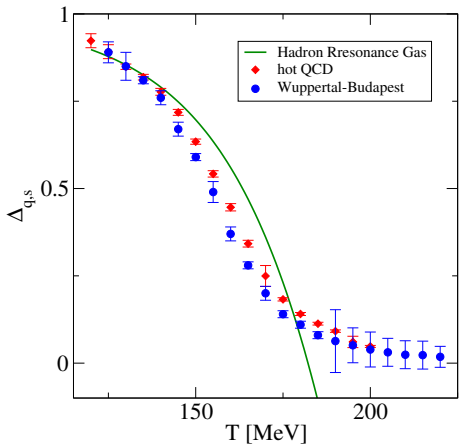

Fig. 2. Chiral condensate as a function of temperature calculated within the hadron resonance gas model (solid/green line), compared to the lattice QCD data of the Wuppertal–Budapest (dots) [11] and HotQCD (diamonds) [12] collaborations.

4. Quark interactions

To estimate effects of the interactions, we can use the QCD version of the Breit equation, based on taking into account leading order relativistic corrections to the Schrödinger equation [15]. In turn, the interaction Hamiltonian has an effective potential of the form $H^{\mathrm{I}} = H^{\mathrm{np}} + H^{\mathrm{SS}} + H^{\mathrm{C}}$ with the nonperturbative, spin–spin and Coulomb parts, respectively. Instead of considering all the details of hadron structure one can follow the logics of [14] to fit the relevant parameters with some hadron masses gaining an overall description of sigma terms.

Once we have a potential model of hadrons, virtual quark loop effects could be estimated as a Lamb shift of the proton mass. In this way, the polarization loop calculated with pQCD can be equipped with the dynamical quark mass M_{s} and the resulting screened potential V_{sc} can be obtained. In this way, the mass shift is given by $\Delta m_p = \int d^3r\psi^*(r)V_{\mathrm{sc}}(r)\psi(r)$ and can be related to the nucleon strange sigma term by the Hellman–Feynman theorem.

5. Summary

As shown in the above note, mild assumptions about dynamics of hadrons allow one to get a description of the chiral condensate at low temperatures in a fair agreement with the lattice data. Further improvements would include sigma terms for low lying states from chiral perturbation theory as those provide the dominant contributions to the hadron resonance gas thermodynamics.

The importance of the hadronic contribution to the melting of the condensate was appreciated in a model for the freeze-out stage of heavy ion collisions [16]. In this approach, freeze-out phenomena were assumed to be happening in the hadronic phase of QCD and were related to the Mott–Anderson localization of hadron wave functions driven by the universal chiral dynamics. This allows for a unified explanation of the freeze-out parameters observed in heavy ion collision experiments.

We acknowledge support by the Polish National Science Center (NCN) within the "Maestro" programme No. DEC-2011/02/A/ST2/00306 (D.B.), by a post-doctoral internship grant No. DEC-2013/08/S/ST2/00547 (J.J.) and by the Russian Fund for Basic Research grant No. 11-02-01538-a (D.B.).

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