Symmetry Breaking in Quantum Systems

Maciej Bobrowski

University of Wrocław

Wrocław, April 20, 2010

Outline

1. Symmetry in physics
2. Spontaneous Symmetry Breaking in classical field theory
3. The Linear Sigma Model
4. Goldstone's Theorem

Symmetry in physics

Symmetry it is invariance under the influence of some transforms. With respect to the theory of invariance property of symmetry is apparent, as is Emma Noether showed, the existence of conservation laws.

Thanks to the conservation laws, even without precise knowledge of the dynamics of physical processes we can say much about the possible behavior of the physical system, in particular, we exclude certain events. For example, from the invariance symmetry of shifts in the space appear the conservation of momentum, so states of different value than the initial momentum is excluded.

Symmetry in physics

Types of symmetry:

1. Kinematic
   - Continuous - functions of continuous parameters: e.g. rotation
   - Discrete - functions of discrete parameters: e.g. reflected in time and space
2. Dynamic e.g. Kepler's move in the field of potential $V \sim -\frac{1}{r}$

Symmetry in physics:

- translation in time T,
- mirror image P,
- charge-coupled C.

Consider Lagrangian with negative parameter $-\mu^2$

$$L = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4 \tag{1}$$

This Lagrangian has a discrete symmetry. It is invariant under the operation $\phi \to -\phi$.

The corresponding Hamiltonian is

$$H = \int d^3x[\frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4] \tag{2}$$

The minimum-energy classical configuration is a uniform field $\phi(x) = \phi_0$, with ϕ_0 chosen to minimize the potential

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4 \tag{3}$$

This potential has two minima given by

$$\phi_0 = \pm v = \pm\sqrt{\frac{6}{\lambda}}\mu \tag{4}$$

The constant v is called the vacuum expectation value of ϕ. To interpret this theory, suppose that the system is near one of the minima (say the positive one). Then it is convenient to define

$$\phi(x) = v + \sigma(x) \tag{5}$$

and rewrite L in terms of $\sigma(x)$. Plugging (3) into (1), we find that the term linear in σ vanishes (as it must, since the minimum of the potential is at $\sigma = 0$).

Dropping the constant term as well, we obtain the Lagrangian

$$L = \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}(2\mu^2)\sigma^2 - \sqrt{\frac{\lambda}{6}}\mu\sigma^3 - \frac{\lambda}{4!}\sigma^4 \qquad (6)$$

The symmetry $\phi \to -\phi$ is no longer apparent; its only manifestation is in the relations among the three coefficients in (4), which depend in a special way on only two parameters.

The Linear Sigma Model

A more interesting theory arises when the broken symmetry is continuous, rather than discrete. The Lagrangian of the linear sigma model involves a set of N real scalar field $\phi^i(x)$:

$$L = \frac{1}{2}(\partial_\mu \phi^i)^2 + \frac{1}{2}\mu^2(\phi^i)^2 - \frac{\lambda}{4!}[(\phi^i)^2]^2 \tag{7}$$

with an implicit sum over i in each factor ϕ^i.

The Lagrangian (5) is invariant under the symmetry

$$\phi^i \to R^{ij}\phi^j \tag{8}$$

for any $N \times N$ orthogonal matrix R. The group of transformations (6) is just the rotation group in N dimensions, also called the N-dimensional orthogonal group O(N). Again the lowest-energy classical configuration is a constant field ϕ_0^i, whose value is chosen to minimize the potential

$$V(\phi^i) = -\frac{1}{2}\mu^2(\phi^i)^2 + \frac{\lambda}{4}[(\phi^i)^2]^2 \tag{9}$$

This potential is minimized for any ϕ_0^i that satisfies

$$(\phi_0^i)^2 = \frac{\mu^2}{\lambda} \tag{10}$$

$$\phi_0^i = (0, 0, ..., 0, v) where v = \frac{\mu}{\sqrt{\lambda}} \tag{11}$$

We can now define a set of shifted fields by writing

$$\phi^i(x) = (\pi^k(x), v + \sigma(x)), k = 1, ..., N - 1 \tag{12}$$

It is now straightforward to rewrite the Lagrangian (5)in terms of the π and σ fields.

The result is

$$L = \frac{1}{2}(\partial_\mu \pi^k)^2 + \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}(2\mu^2)\sigma)^2 - \sqrt{\lambda}\mu\sigma^3 - \\ - \sqrt{\lambda}\mu(\pi^k)^2\sigma - \frac{\lambda}{4}\sigma^4 - \frac{\lambda}{2}(\pi^k)^2\sigma^2 - \frac{\lambda}{4}[(\pi^k)^2]^2 \qquad (13)$$

We obtain a massive σ field just as in (4), and also a set of N-1 massless π fields. The massive σ field describes oscillations of ϕ^i in the radial direction. The massless π fields describe oscillations of ϕ^i in the tangential directions.

Goldstone's Theorem

The appearance of massless particles when a continuous symmetry is spontaneously broken is a general result, known as Goldstone's theorem. Goldstone's theorem states that for every spontaneously broken continuous symmetry, the theory must contain a massless particle. The massless fields that arise through spontaneous symmetry breaking are called Goldstone bosons.

Bibliography

- D. H. Perkins "Introduction to high energy physics" Cambridge University Press, 4 edition, 2000,
- M. E. Peskin and D. V. Schroeder "Introduction to quantum field theory" Addison-Wesley Publishing Company, 1995,
- F. Strocchi "Symmetry Breaking" Springer, Berlin Heidelberg, 2 edition, 2008,
- A. Zee "Quantum field theory in a nutshell" Princeton University Press, 2003.