Introduce to Supersymmetry

Alaksiej Kacanovic

Uniwersytet Wroclawski

May 24, 2010

What is supersymmetry it?

Supersymmetry is expanded symmetry

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We hope find a some operator which will be transform fermions to bosons and bosons to fermions

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In supersymmetry field theory we can have separately bosons and fermions fields or composition of that fields which call superfield

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Let try find super Lie algebra

What about of Coleman-Manduli "no-go" theorem?

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Let try find super Lie algebra

What about of Coleman-Manduli "no-go" theorem?

Super Lie algebra include anti-commutation relation

Few important information

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Spinor is object which transform by $SL(2,\mathbb{C})$ representation

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What is spinor it?

Spinor is object which transform by SL(2,$\mathbb{C}$) representation

- $\psi\prime_\alpha = N_\alpha{}^\beta \psi_\beta$
- $\overline{\chi}\prime_{\dot{\alpha}} = (N^*)_{\dot{\alpha}}{}^{\dot{\beta}} \overline{\chi}_{\dot{\beta}}$

Extended Lie algebra

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- $\{Q_\alpha, Q^\beta\} = 0$
- $\{Q_\alpha, \overline{Q}_{\dot\beta}\} = 2(\sigma^\mu)_{\alpha\dot\beta} P_\mu$

One example

How show that extended try?

One example

How show that extended try?

for example $[Q_\alpha, M^{\mu\nu}] = (\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta$

Supercharge transform like spinor

$$Q\prime_{\alpha} = \left(e^{-\frac{i}{2}\omega_{\mu\nu}\sigma^{\mu\nu}}\right)_{\alpha}{}^{\beta} Q_{\beta} \approx \left(\mathbb{I} - \frac{i}{2}\omega_{\mu\nu}\sigma^{\mu\nu}\right)_{\alpha}{}^{\beta} Q_{\beta}$$

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Supercharge transform like operator

$$Q\prime_\alpha = U^\dagger Q_\alpha U$$

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If we compare two $Q\prime_\alpha$

$$Q_\alpha - \frac{i}{2}\omega_{\mu\nu}(\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta = Q_\alpha - \frac{i}{2}\omega_{\mu\nu}(Q_\alpha M^{\mu\nu} - M^{\mu\nu}Q_\alpha) + \mathcal{O}(\omega^2)$$

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After vanishing we have

$$[Q_\alpha, M^{\mu\nu}] = (\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta$$

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$$[Q_\alpha, M^{\mu\nu}] = (\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta$$

... and for conjugate representation this we can show same

$$[\overline{Q}^{\dot\alpha}, M^{\mu\nu}] = (\overline{\sigma}^{\mu\nu})^{\dot\alpha}{}_{\dot\beta}\overline{Q}^{\dot\beta}$$

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How supercharge acts on field?

Few definition

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- $Q_\alpha S = \psi_\alpha$
- $Q_\alpha P = \gamma_5 \psi_\alpha$
- $Q_\alpha \psi_\beta = -(\gamma^\mu)_{\alpha\beta} \partial_\mu S + (\gamma^\mu \gamma_5)_{\alpha\beta} \partial_\mu P$

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What about superfields?

We must have super-Poincare transformation

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$$U_{SP} = exp(i(\omega_{\mu\nu}M^{\mu\nu} + a_\mu P^\mu + \theta^\alpha Q_\alpha + \overline{\theta}_{\dot{\alpha}}\overline{Q}^{\dot{\alpha}}))$$

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How look like that superfield?

$S(x^\mu, \theta^\alpha, \overline{\theta}_{\dot{\alpha}}) = \varphi(x) + \theta\psi(x) + \overline{\theta}\overline{\chi}(x) + \theta\theta M(x) + \overline{\theta\theta} N(x) + (\theta\sigma^\mu\overline{\theta}) V_\mu + \theta\theta\overline{\theta\lambda}(x) + \overline{\theta\theta}\theta\rho(x) + \theta\theta\overline{\theta\theta} D(x)$

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where

- $\varphi(x)$, $M(x)$, $N(x)$, $D(x)$ - scalar fields
- $V_\mu(x)$ - vector field
- $\psi(x)$, $\overline{\chi}$, $\overline{\lambda}(x)$, $\rho(x)$ - spinorial field

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Where we can use supersymmetry?

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anywhere

Example 1. Yang-Mills

We can modify Yang-Mills Lagrangian for supersymmetry invariants

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$\mathcal{L}_{SYM} = -\frac{1}{4} TrF_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\overline{\Psi}\not{D}\Psi$

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$Q|B\rangle \sim |F\rangle$

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Is it supersymmetry try?

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We have corroboration supersymmetry in quantum mechanics

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What about Standard Model?

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Is it supersymmetry try?

We have corroboration supersymmetry in quantum mechanics

What about Standard Model?

we don't know...

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Why supersymmetry is good?

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- Grand unification

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- Grand unification
- String theory

Why supersymmetry is good?

- Grand unification
- String theory
- Divergence

Why supersymmetry is good?

- Grand unification
- String theory
- Divergence
- Dark matter

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Thank you for attention!

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