Introduce to Supersymmetry

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Introduction to supersymmetry

General feature of supersymmetry is that, we have all super in this theory: superfield, supercharge, super-algebra and so on. Another word supersymmetry is really a super theory (must read the best of other theory). Its first motivation for interesting of this theory. Its funny, but what have this theory to our world, which we observe? Is it theory which describe our world and physics in it or only funny toy-model? Now we can't answer exactly but some thing suggest us that answer is first:

1. we have on 100% predicable supersymmetric quantum mechanics. For heavy atomic nucleus which have even number of nucleons and which is bosons, we can easy equate Schrodinger equation and easy solve it. But for nucleus with odd number of nucleons (fermionic nucleus) all this operation is not such easy. But we can equate for bosons nucleons and than very easy by supersymmetric transformation obtain equation for fermionic nucleus. And we can test every kind of nucleons. And our prediction which we obtain by supersymmetric transformation is exactly identical with which we obtain in experiment.

2. supersymmetry can help us with solvation problem of divergence in quantum field theory.

3. supersymmetry is natural conclusion from string theory. And without supersymmetry string theory doesn't exist. Bosons and fermions particle is different modes of vibration same object call string. And by changing modes we can from boson's particle obtain fermion's particle. And in string theory this transformation is very natural.

4. supersymmetry should be solve problem of dark matter. How we know, universe behave like have more mass than we can observe. We have possibility to approximate how much matter in black hole and in interstellar space, but its not enough anymore. And Standard Model doesn't predict any another matter. But supersymmetric extension of Standard Model do possibility to existent stability supersymetric partner of "normal" particle. Which call WIMP(Weak Interaction Massive Particle).

How supersymmetry work?

How we know that exist contain Lorentz symmetry, two discrete Lorentz symmetry - time and space conjugation, and we know about existence charge conjugating symmetry. Time conjugation symmetry, for example, tell us that after a time reversal transformation

$$T : t \to -t$$

we will have exactly a same physics. Contain Lorentz transformation tell us that translation, rotation and boosts doesn't matter for physics. How we know bosons describe by Klein-Gordon equation, but fermions only by Dirac equation. It's a very different equation at first sight. But supersymmetry tell us, that we must have a same physics for bosons and for fermions.

Before we will begin look for operator which should be transforms bosons to fermions and back, and condition of this operator, I should be say a few word about Coleman-Manduli's "no-go" theorem. Before was discover supersymmetry, Coleman and Manduli proofed theorem which tell us that "we can't build Lie algebra grater than which consist from generator of angular momentum M, momentum P and boost N, and for massless particle can be exist two more generator - dilatation D and conformal transformation H". But Coleman and Manduli didn't think about anti commutation relation. If we enter in the Lie algebra anti commutation relation we should be build without building new mathematical tools, consistence theory.

So, we are beginning. How I say above object which will be look for must be operator, probably it's clear why. Second condition, which we would, our object (we will call it 'supercharge') is spinor. It's condition of parity in field theory. Spinor is object which transform by following properties [1]:

$$\psi\prime_\alpha = N_\alpha{}^\beta \psi_\beta \quad (1)$$

$$\overline{\chi}\prime_{\dot\alpha} = (N^*)_{\dot\alpha}{}^{\dot\beta} \overline{\chi}_{\dot\beta} \quad (2)$$

where
N - is representation of $SL(2,\mathbb{C})$ group,
N^* - complexes conjugate representation of representation N

By the Jacobi identity and the method which i show below [2] we can extend Poincare Lie algebra to Superpoincare Lie algebra:

$$[P^\mu, P^\nu] = a \tag{3}$$
$$[M^{\mu\nu}, P^\rho] = i(\eta^{\nu\rho}P^\mu - \eta^{\mu\rho}P^\nu) \tag{4}$$
$$[M^{\mu\nu}, M^{\rho\sigma}] = i(M^{\mu\sigma}\eta^{\nu\rho} + M^{\nu\rho}\eta^{\mu\sigma} - M^{\mu\rho}\eta^{\nu\sigma} - M^{\nu\sigma}\eta^{\mu\rho}) \tag{5}$$
$$[Q_\alpha, P^\mu] = 0 \tag{6}$$
$$[Q_\alpha, M^{\mu\nu}] = (\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta \tag{7}$$
$$\{Q_\alpha, Q^\beta\} = 0 \tag{8}$$
$$\{Q_\alpha, \overline{Q}_{\dot\beta}\} = 2(\sigma^\mu)_{\alpha\dot\beta}P_\mu \tag{9}$$

for example will divine relation for commutator $[Q_\alpha, M^{\mu\nu}]$. How I say above our supercharge Q is spinor and must be transform how spinor

$$Q\prime_\alpha = (e^{-\frac{i}{2}\omega_{\mu\nu}\sigma^{\mu\nu}})_\alpha{}^\beta Q_\beta \approx (\mathbb{I} - \frac{i}{2}\omega_{\mu\nu}\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta \tag{10}$$

but Q also is operator, and must be transform like operator:

$$Q\prime_\alpha = U^\dagger Q_\alpha \tag{11}$$

where

$$U = (e^{-\frac{i}{2}\omega_{\mu\nu}M^{\mu\nu}}) \approx (\mathbb{I} - \frac{i}{2}\omega_{\mu\nu}M^{\mu\nu}) \tag{12}$$

if we compare this two terms we should be obtain following equation

$$Q_\alpha - \frac{i}{2}\omega_{\mu\nu}(\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta = Q_\alpha - \frac{i}{2}\omega_{\mu\nu}(Q_\alpha M^{\mu\nu} - M^{\mu\nu}Q_\alpha) + \mathcal{O}(\omega^2) \tag{13}$$

after vanishing alike terms we have our commutation relation.

$$[Q_\alpha, M^{\mu\nu}] = (\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta \tag{14}$$

More, how take other commutation relation we can find in Flip Tanedo's notes "Supersymmetry and Extra Dimensions" [2]

How acts Supercharge?

We have spinor field which describe fermions particles, but bosons are describing by scalar, pseudoscalar, pseudovector and vector field. And we must build consistent's theory where all particle bind in group by supersymmetric transformation. One of possibility build consistent theory choose following condition[3]:

$$Q_\alpha S = \psi_\alpha \tag{15}$$
$$Q_\alpha P = \gamma_5 \psi_\alpha \tag{16}$$
$$Q_\alpha \psi_\beta = -(\gamma^\mu)_{\alpha\beta}\partial_\mu S + (\gamma^\mu\gamma_5)_{\alpha\beta}\partial_\mu P \tag{17}$$

where:
S - scalar field
P - pseudoscalar field
ψ_a - spinorial field

it's one of possible way. But we can use super-Poincare transformation on superfield and obtain in direct way how our field must transform [2].

$$U_{SP} = exp(i(\omega_{\mu\nu}M^{\mu\nu} + a_\mu P^\mu + \theta^\alpha Q_\alpha + \overline{\theta}_{\dot{\alpha}}\overline{Q}^{\dot{\alpha}})) \tag{18}$$

where θ and $\overline{\theta}$ is Grassmann parameters.

U_{SP} is Lie group representation.

And our superfield have all possible 'normal' field components and look like

$$S(x^\mu, \theta^\alpha, \overline{\theta}_{\dot{\alpha}}) = \varphi(x) + \theta\psi(x) + \overline{\theta}\overline{\chi}(x) + \theta\theta M(x) + \overline{\theta\theta}N(x) + (\theta\sigma^\mu\overline{\theta})V_\mu + \theta\theta\overline{\theta}\overline{\lambda}(x) + \overline{\theta\theta}\theta\rho(x) + \theta\theta\overline{\theta\theta}D(x) \tag{19}$$

$\varphi(x)$, $M(x)$, $N(x)$, $D(x)$ - scalar fields
$V_\mu(x)$ - vector field
$\psi(x)$, $\overline{\chi}$, $\overline{\lambda}(x)$, $\rho(x)$ - spinorial field

if we acts our transformation U_{SP} on the our superfield $S(x^\mu, \theta^\alpha, \overline{\theta}_{\dot{\alpha}})$ we should have all possible transformation. How it do explicitly we can find in notes of Flip Tanedo [2]

Where we can use supersymmetry?

Supersymmetry is not some special theory of 'normal' field theory. We can use supersymmetrization for all quantum theory. How was tell, we have grate results in supersymetric quantum mechanics. Supercharge acts to non relativistic state change it like this:

$$Q|B\rangle \sim |F\rangle \tag{20}$$

$$Q|F\rangle \sim |B\rangle \tag{21}$$

and, for example, we can build supersymmetric Yang-Mills theory. One can check that following lagrangian is invariant not only gauge transformation but under supersymmetric transformation too [3]

$$\mathcal{L}_{SYM} = -\frac{1}{4} Tr F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \overline{\Psi} \not{D} \Psi \tag{22}$$

References

[1] J.Lopuszanski ”Rachunek spinorow”, Warsz.(1985)

[2] F.Tanedo ”Supersymmetry and extra dimension”(2009)

[3] J.M.Figueroa-O’Farrill ”BUSSTEPP Lectures on Supersymmetry”(2001)

[4] S.Weinberg ”Quantum Theory of Fields. III Supersymmetry.”(2000)