General Reference Frame

Aneta Wojnar

University of Wrocław

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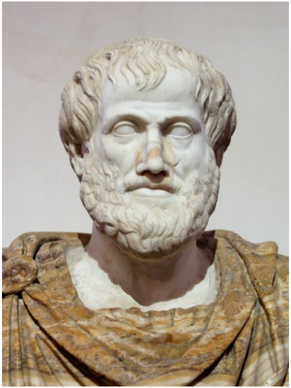
Outline

Aristotle



Aristotle (384 BC – 322 BC)

Galileo Galilei

Galileo Galilei (1564 - 1642)

Albert Einstein

Albert Einstein (1879 - 1955)

Outline

A few short definitions

Elementary event

It is a physical phenomenon in a small area in space lasting very shortly. Features of this event are not important.

Spacetime

It is a set of all elementary events. By a world line of material point we will understand a history of this point in spacetime.

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Definition of Aristotle's spacetime E

- Space and time are absolute quantities.
- There exists one distinguished frame that is always at rest.

Aristotele's spacetime

Spacetime E one can present as a Cartesian product:

$$E = T \times S = \{(t, x);\ t \in T, x \in S\}$$

where $T = E^1$ is a time axis and $S = E^3$ a three-dimensional position space (here, E^n means n-dimensional Euclidean space).

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Inertial frames

Assumptions of galilei spacetime

- Rejection of existence of reference frame distinguished by nature,
- Time is still absolute (fixed).

Distinguished class of reference frames

A particle not subject to any force moves with constant velocity.

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Inertial frames

Connection between two inertial frames

$\vec{x} = (x^1, x^2, x^3)$ variable t indicates time.

For two frames θ and θ', which coincided at $t = 0$ and later one of them, e.g. θ', was moving with constant velocity $\vec{u}$, we have:

$$\vec{x}' = \vec{x} - \vec{u}t \text{ where } t' = t.$$

Inertial frames as an equivalence class

Relation of being inertial is:

- reflexive,
- symmetric,
- transitive.

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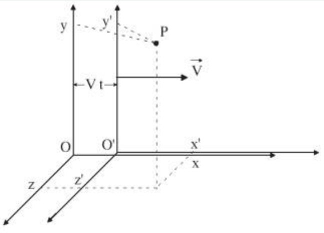
Summary

- The laws of nature are the same in all of inertial frames (Galilean relativity),
- There exist preferential reference systems for which, given initial inertial frame, all the other inertial systems are in uniform rectilinear motion with respect to it (inertia).



Outline

Generalization of Galilean principle

Einstein's principle of relativity

1. Physical laws have the same form in all inertial frames.
2. The speed of light is finite and equal in all inertial frames.

Lorentz transformation

Connection between two inertial frames when one of them is moving with constant velocity $\vec{u}$ along $\hat{x}$ axis is:

$$\vec{x}' = \frac{\vec{x} - \vec{v}t}{\sqrt{1 - \frac{v^2}{c^2}}}, \qquad t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}, \qquad y' = y, \qquad z' = z.$$

$$\Downarrow$$

$$\vec{u}'_x = \frac{\vec{u}_x - \vec{v}}{1 - \frac{\vec{u}_x \vec{v}}{c^2}}$$

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The principle of equivalence

At every point in an arbitrary gravitational field we can choose a locally inertial frame in which the laws of physics take the same form as in special relativity.

- $x'^0 = f((x^\mu)), \quad x'^j = f^j((x^k)); \, (j, k = 1, 2, 3)$

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Definition and theorem

General reference frame F

Let V be a differentiable manifold of dimension $N+1$ ($N \geq 1$) and let $\mathscr{A}$ be the atlas that defines the manifold structure of V. For any open set $U \subset V$, define the subset $\mathscr{A}_U \equiv \{\chi \in \mathscr{A}; Dom(\chi) \supset U\}$ of the atlas $\mathscr{A}$. Let the open set U be such that $\mathscr{A}_U$ is non-empty and, for any two charts $\chi, \chi' \in \mathscr{A}_U$, set

$\chi \mathscr{R}_U \chi'$ iff $[\forall \mathbf{X} \in \chi(U), F^0 = x^0$ and $\frac{\partial F^j}{\partial x^0}(\mathbf{X}) = 0 \quad (j = 1, ..., N)]$

where $F \equiv \chi' \circ \chi^{-1}$ is the transition map, whose domain $\chi(Dom(\chi) \cap Dom(\chi'))$ contains $\chi(U)$. Then $\mathscr{R}_U$ is an equivalence relation on $\mathscr{A}_U$. The equivalence classes for this relation are called **reference frames** (over the domain U).

Proof

We have to show:

- Relation $\mathscr{R}_U$ is reflexive,
- Relation $\mathscr{R}_U$ is symmetric,
- Relation $\mathscr{R}_U$ is transitive.

For Further Reading I

- Mayeul Arminjon and Frank Reifler.
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