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VACUUM PARTICLE-ANTIPARTICLE CREATION IN STRONG FIELDS AS A FIELD-INDUCED PHASE TRANSITION

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We study the special features of vacuum particle creation in an external classical field for two simple external field models in standard QED. Our investigation is based on a kinetic equation that is a nonperturbative consequence of the fundamental QED equations of motion. We identify the special features of system evolution that apply qualitatively also for other systems and are therefore rather general. The common basis for a description of these systems is formed by kinetic equations for vacuum particle creation belonging to the class of integro-differential equations of non-Markovian type with fastly oscillating kernel. This allows us to characterize the processes of this type as belonging to the class of field-induced phase transitions. Examples range from condensed matter physics to cosmology.

Keywords: electron-positron pair, strong field, vacuum creation, phase transition.

INTRODUCTION

In the present work we investigate the special features of the transition from an initial state of primordial vacuum oscillations to a final quantum field system of particles and antiparticles due to an external field (the dynamical Schwinger effect) as a field-induced phase transitions (FIPT). As an example, we consider a 3+1 dimensional QED system in the presence of a linearly polarized time-dependent electric field. This particular case allows a rather simple kinetic description in the framework of the quasiparticle representation. The corresponding kinetic equation (KE) has a specific structure: it is an integro-differential equation of non-Markovian type with fastly oscillating kernel describing the evolution of vacuum oscillations excited by the external field. The mathematical structure of this KE is preserved also for other systems with unstable vacuum. Therefore, one can expect that the investigated features of the FIPT can be recognized also in other quantum field systems of corresponding nature in the presence of a strong classical external field.

The basic KE is given in Section 2. Its numerical solutions are discussed in Section 3 for some simple models of the external electric field. The characteristic features in a description of the evolution of the particle-antiparticle plasma created from vacuum are summarized in Section 4.

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1. KINETIC EQUATION

According to the general theory of systems with unstable vacuum (e.g., see [1, 2]), two formulations of kinetic theory exist which are destined for the space-time description of vacuum exitations. These are the KE in the Wigner [3, 4, 5] and quasiparticle [6, 7, 8] representations. In [8, 9], the equivalence of these two approaches was demonstrated for the simplest external field models in QED. For these simple, spatially homogeneous field models both versions of the kinetic theory allow for a reduction of the complicated original equations to a rather simple system of ordinary differential equations (ODE) for three functions: the distribution function $f(\boldsymbol{p},t)$ of the quasiparticle excitations and two auxiliary functions for a description of the vacuum polarization. The corresponding KE [7] describing vacuum electron-positron plasma (EPP) creation in a homogeneous linearly polarized electric field $E(t) = -\dot{A}(t)$ with the vector potential (in the Hamilton gauge) $A^{\mu}(t) = (0,0,0,A(t))$ is

$$\dot{f}(\boldsymbol{p},t) = \frac{1}{2}\lambda(\boldsymbol{p},t)\int_{t_0}^{t} dt' \lambda(\boldsymbol{p},t')[1-2f(\boldsymbol{p},t')]\cos\theta(t,t'), \tag{1}$$

where

$$\lambda(\boldsymbol{p},t) = eE(t)\varepsilon_{\perp} / \varepsilon^2(\boldsymbol{p},t), \quad \theta(t,t') = 2\int_{t'}^{t} d\tau\varepsilon(\boldsymbol{p},\tau). \tag{2}$$

Here λ is the amplitude of the vacuum transitions, and θ is the high-frequency phase, describing the vacuum oscillations modulated by the external field. Furthermore, the quasienergy ε, the transverse energy $\varepsilon_{\perp}$, and the longitudinal quasi-momentum P are defined as

$$\varepsilon(\boldsymbol{p},t) = \sqrt{\varepsilon_{\perp}^2(\boldsymbol{p}) + P^2}, \ \varepsilon_{\perp} = \sqrt{m^2 + p_{\perp}^2}, \ P = p_{\parallel} - eA(t). \tag{3}$$

Here $p_{\perp} = |\boldsymbol{p}_{\perp}|$ is the modulus of the vector $\boldsymbol{p}_{\perp}$ perpendicular to the field vector and $p_{\parallel} = p_3$ is the momentum component parallel to the field.

The quasiparticle distribution function $f(\boldsymbol{p},t)$ is zero in the in-vacuum state where the external field strength is zero ($E_{\text{in}} = 0$ corresponds to $A_{\text{in}} = A(t_0)$), i.e., Eq. (1) is complemented by the initial condition $f(\boldsymbol{p},t_0) = f_{\text{in}} = 0$. It is also assumed that the electric field is switched off in the out-state (($E_{\text{out}} = E(t\to\infty) = 0$ and $A_{\text{out}} \neq A_{\text{in}}$). Thus, the in- and out-vacuum states are different.

The non-Markovian integro-differential equation (Eq. (1)) is equivalent to a system of three time-local ordinary differential equations

$$\dot{f} = \frac{1}{2}\lambda u, \quad \dot{u} = \lambda(1-2f) - 2\varepsilon v, \quad \dot{v} = 2\varepsilon u, \tag{4}$$

where $u(\boldsymbol{p},t)$ and $v(\boldsymbol{p},t)$ are auxiliary functions describing vacuum polarization effects. Dynamical system (4) has the following integral of motion: $(1-2f)^2 + u^2 + v^2 = 1$, compatible with the initial conditions $f_{\text{in}} = u_{\text{in}} = v_{\text{in}} = 0$. In the low-density approximation $2f \ll 1$, KE (1) has a closed formal solution in the form of a useful quadrature formula [10]

$$f(\boldsymbol{p},t)=\frac{1}{2}\int_{t_0}^{t}dt'\lambda(\boldsymbol{p},t')\int_{t_0}^{t'}dt''\lambda(\boldsymbol{p},t'')\cos\theta(t',t'')\,. \tag{5}$$

The total number density of pairs is defined as

$$n(t)=2\int\frac{d\boldsymbol{p}}{(2\pi)^3}f(\boldsymbol{p},t)\,, \tag{6}$$

where the factor 2 corresponds to the spin degree of freedom.

In the present work KE (1) is solved numerically for two relevant models of the electric field:

1. the Eckart-Sauter field with characteristic duration of action T

$$E(t)=E_0\cosh^{-2}(t/T),\quad A(t)=-TE_0\tanh(t/T)\,, \tag{7}$$

and

2. the Gaussian envelope model of the laser pulse [11]

$$E(t)=E_0\cos(\omega t)\,e^{-t^2/2\tau^2}\,, \tag{8}$$

$$A(t)=-\sqrt{\frac{\pi}{8}}E_0\tau\exp(-\sigma^2/2)\,\mathrm{erf}\left(\frac{t}{\sqrt{2}\tau}-i\frac{\sigma}{\sqrt{2}}\right)+\mathrm{c.c.}\,,$$

where $\sigma=\omega\tau$ is a dimensionless measure for the characteristic duration of the pulse τ connected with the number of periods of the carrier field. The Eckart-Sauter field (7) admits an exact solution of the problem [6, 8, 12]; it is a benchmark case.

In order to introduce the Keldysh parameter $\gamma=E_c\omega/E_0m$ for a discussion of field model (7), one can use the substitution $\omega\to1/T$. In the limiting case $\gamma\ll1$, the tunneling mechanism (with participation of an infinite number of photons) dominates, whereas for $\gamma\gg1$, pair creation is driven by the absorption of few photons.

The vacuum oscillations (Zitterbewegung) play a crucial role in the mechanism of vacuum EPP creation. The usual energy of vacuum oscillations $\varepsilon_0=\sqrt{m^2+\boldsymbol{p}^2}$ is transformed here to quasienergy (3) in the presence of the time-dependent electric field. The memory effect (non-Markovian character of the KE), the fastly oscillating factor with phase (2) and frequency 2ε (the dynamical energy gap) are essential elements in KE (1). This equation contains two characteristic time scales: a slow one associated with the time scale of the external field period, $2\pi/\omega$, and a fast one given by the Compton time $\tau_c=2\pi/m$. These scales are usually vastly different, $\omega\ll m$. The coupling of the dynamics related to these two scales leads to a very complicated structure of the distribution function, both in the first stage (generation of the quasiparticle EPP (QEPP)) and in the final stage (formation of the residual EPP (REPP)) [13].

2. FIELD-INDUCED PHASE TRANSITION

In the considered situation, the FIPT appears as rearrangement of the vacuum state under the action of a classical electromagnetic field. It leads to the t–noninvariant quasiparticle vacuum which corresponds to a non-stationary Hamiltonian of the system (the S. Coleman theorem [6, 14]). In this connection, the quasiparticle electron-positron pairs are the massive analog of the Goldstone bosons [6, 15]. Let us consider phenomena which accompany the FIPT.

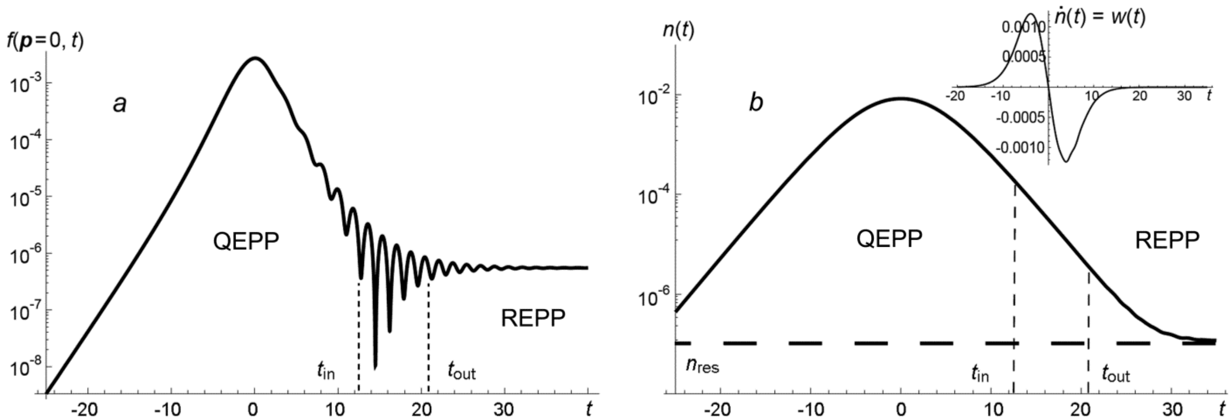

Fig. 1. Transition from the QEPP plasma to the final state for the Eckart–Sauter pulse type defined by Eq. (7) with $E_0 = 0.2\,E_c$ and $T = 8$. The labels t_{in} and t_{out} denote approximately the start and the end of the transient stage. (*a*) Evolution of the distribution function for the point $p_\perp = p_\parallel = 0$. (*b*) Evolution of the pair number density defined by Eq. (6) and the local pair production rate $w(t) = \dot{n}(t)$ (inset).

2.1. Transient stage

The typical picture of the EPP evolution under the action of smooth pulse (7) is shown in Fig. 1. The left panel (*a*) shows that the transient process of the fast EPP oscillations divides the evolution of the EPP into two domains, the QEPP and the REPP. After momentum integration, the fast oscillations of the transient process are smoothed out (see Fig. 1*b*). The inset in the panel shows the local production rate. The results of numerical solutions of KE (1) (or Eq. (4)) coincide with the exact solution [6, 12, 9]. In all figures, the time and frequency are scaled with the electron mass.

For qualitative orientation, one can introduce here the time interval of strong oscillations limited by point t_{in} of the start (that can be identified with the moment when the oscillations of the distribution function reach for the first time the level of the REPP) and the end t_{out} (corresponding to the moment when the mean level of oscillations approaches that of the REPP and the elongation of the oscillations is significantly reduced). This transient period of the Zitterbewegung separates the smoothed QEPP stage from the REPP stage.

Under similar conditions, strong oscillations are also observed in other physical models with massive constituents. For example, they appeared in the domain of the relativistic phase transition with dynamical mass generation (the inertial mechanism of particle creation) including the Higgs mechanism [16]. Their existence can be found also in the strong field dynamical models of strongly correlated systems (e.g., see [17]). Let us underline that the appearance of the transient region with strong oscillations takes place in the considered case of smooth impulse (7) without a carrier wave that would possess a high frequency component.

For a better understanding of this phenomenon, let us consider the mechanisms of particle creation acting in KE (1) or in its approximate solution (5). We will trace the evolution of the system in smooth field (7) for $t > 0$ which is accompanied by field strength depletion. If the electric field is rather strong, for $t < t_{\text{in}}$ the acceleration mechanism represented by the force factor $eE(t)$ in the numerator of amplitude (2) is dominant, whereas the fastly oscillating factor $\cos\theta(t,t')$ on the right hand side of KE (1) is smoothed out. The vicinity of the moment t_{in} of the start of the transient stage is characterized by weakening of the accelerating field action and by growth of the role of the fast oscillations with the frequency $2\varepsilon(\boldsymbol{p},t) \ge 2m$, in which one can neglect now the influence of a weak field so that the oscillation "beard" in the transient stage appears in Fig. 1. The subsequent field depletion accompanied by the growth of

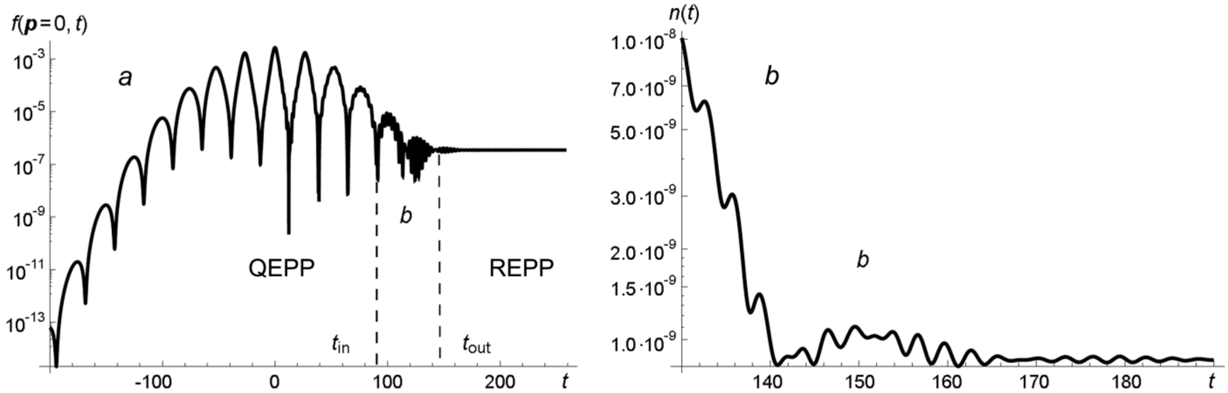

Fig. 2. Transition from QEPP to REPP in the case of a harmonic field with the Gaussian envelope defined by Eq. (8) with $\sigma = 5.0$. (*a*) Distribution function for the point $p_\perp = p_\parallel = 0$. (*b*) Density $n(t)$ described by Eq. (6) in the transition region *b*.

the vector potential (and the quasi-momentum $P(t)$ defined by Eq. (3)) in the denominator of amplitude (2) leads to asymptotic extinction of oscillations and approach of the final REPP state.

These features of the transient process are complicated in the case of a high-frequency periodic field with the Gaussian envelope defined by Eq. (8), $\omega \gg 1/\tau$. Typical patterns are presented in Fig. 2. In this case, the transient stage is separated into domains defined by the subcycle structure of the pulse. They are traced well also on the density curve of the EPP in the right panel of Fig. 2, showing the last domain of the stage *b*. Apparently, the effect of mutual amplification of EPP production as a result of nonlinear interaction of the fast and slow components [18, 19, 20] of field (8) is illustrated in the figure.

A new element now is the dependence on the carrier frequency ω. When comparing with the right panel of Fig. 1, we observe the modulation effect which becomes apparent also in the area of the "beard." The previously discussed picture of the transient regime is also observed at the end of each cycle of sub-pulses with half-period duration π/ω of external field (8), but it gets squeezed by the neighboring cycle. The general transient process arises at sufficient depletion of the envelope amplitude at $t > 0$.

The presence of a transient region of fast oscillations in the distribution function is characteristic for every field model. In this regard, the discussed phase transition under the action of a strong electric field is a universal effect for quantum field systems with energy gap. We remark that in the case of massless 2+1 dimensional QED (e.g., for graphene), the high-frequency transient region is absent, and the evolution of the particle-antiparticle plasma distribution function is smooth [21].

2.2. Strong nonequilibrium

The entire process of vacuum EPP creation is highly nonequilibrium, including the final out-state. In the first place, this conclusion follows from the exactly solvable models. The distribution functions of the out-state turn out to be the same for the constant field model $E(t) = E_0$ [22] and the Eckart–Sauter model described by Eq. (7) for $T \to \infty$ [2, 6]

$$f_{\text{out}}(\boldsymbol{p}) = \exp\left[-\frac{E_c}{E_0}\left(\frac{\varepsilon_\perp}{m}\right)^2\right]. \tag{9}$$

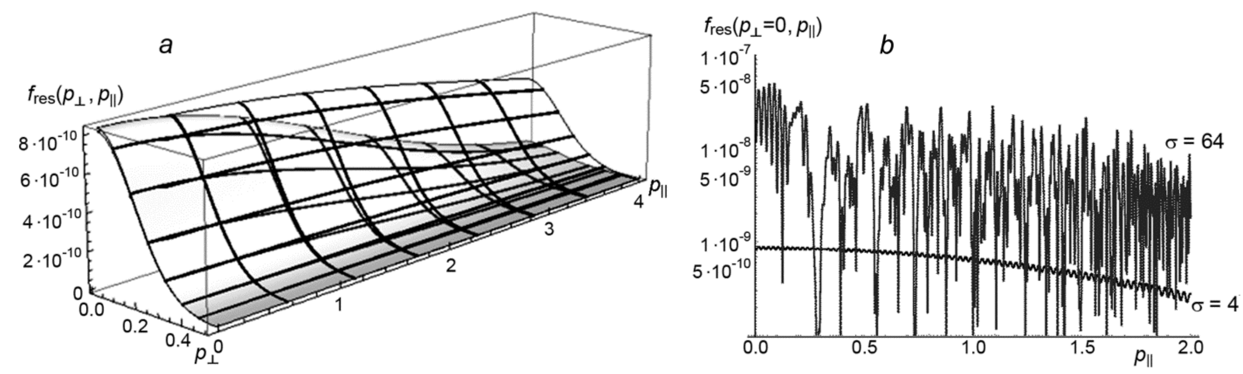

Fig. 3. (*a*) REPP distribution function $f(p_\perp, p_\|)$ for the Eckart–Sauter pulse defined by Eq. (7) with $T = 82.4$ (at the bottom) and $T = 164.8$ (at the top) at $E_0/E_c = 0.15$ in the momentum space for $0.0 \le p_\perp \le 0.5$ and $0.0 \le p_\| \le 4.0$. (*b*) REPP distribution functions $f(p_\perp = 0, p_\|)$ for a short pulse ($\sigma = 4$) and a long pulse ($\sigma = 64$) with the same amplitude of the field $E_0 = 0.15E_c$. The cyclic frequency ω of the oscillating field satisfies the condition $1/\omega = 82.4$ that corresponds to a wavelength of 0.1 nm.

This function is degenerate with respect to $p^3 = p_\|$ and therefore, is non-normalizable. This leads to the necessity to extend the definition of macroscopic observables of type (6). As a rule, the substitution

$$\int dp_\| \to eTE_0 \tag{10}$$

is introduced, which results in the well-known Schwinger formula [23] for the EPP production rate. The constant field model has recently been analyzed in detail in [24]. Strongly anisotropic nonequilibrium distribution (9) exists only in the presence of external field and is defined by its symmetry. Detailed consideration of the nonequilibrium feature of this distribution can be found in [25].

Asymptotic distribution (9) in the constant field model is a smooth function of the transverse energy $\varepsilon_\perp(p_\perp)$. In more realistic field models, the structure of the distribution function becomes very complicated. As an example, see the right panel of Fig. 3.

2.3. Non-monotonic entropy growth

The transition from the in-state to the out-state is accompanied by non-monotonic entropy growth. This phenomenon was noted and discussed long ego (for example, see [26, 22, 27]). For example, function (9) leads to the following entropy production rate:

$$\frac{S_{\text{out}}}{T} = \frac{m^4}{8\pi^2}\frac{E_0}{E_c}\left(1 + \frac{E_0}{\pi E_c}\right)\exp\left(-\pi\frac{E_c}{E_0}\right), \tag{11}$$

where the pulse duration is defined by relation (10). In Eq. (11) the definition of the information entropy has been used. The most complete investigation was implemented in [28] on the basis of KE (1). We note that KE (1) or system of ODE (4) is invariant with respect to time inversion, so that the entropy growth observed here, apparently, results from transforming the primordial vacuum fluctuations under the action of a strong external field in the statistical ensemble of the EPP with well-defined entropy.

SUMMARY

In this work we have considered the field-induced phase transition from primordial vacuum fluctuations to the final massive quantum field system of particle-antiparticle pairs under the action of a strong external field. This phenomenon possesses the following characteristic features:

1. presence of three stages of evolution: quasiparticle, transient, and final;
2. presence of fast oscillations in the transient stage;
3. strong nonequilibrium character, including the out-state;
4. non-monotonic entropy growth.

Apparently, these features are rather universal and are characteristic on the qualitative level for physical systems of different natures.

On the formal level, this universality appears because the corresponding KEs belong to the united class of integro-differential equations of non-Markovian type with fastly oscillating kernel. Examples of this kind are, e.g., KEs for description of the vacuum creation of scalar bosons and fermions in the FRW space-time [6] and the nonperturbative KE for description of the carrier excitations in graphene [21]. In the present work, we have restricted ourselves to the consideration of the domain of the tunneling mechanism of particle creation, $\gamma \ll 1$. We plan to consider the few-photon domain of particle creation ($\gamma \gg 1$) in a separate work.

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