TOWARDS QCD MODELING OF DENSE MATTER IN COMPACT STARS

David Blaschke

Institute for Theoretical Physics, University of Wroclaw, Poland

Institut für Physik, Universität Rostock, Germany

Bogoliubov Laboratory for Theoretical Physics, JINR Dubna, Russia

NUCLEAR PHYSICS & ASTROPHYSICS, ORSAY, 12.04.2007

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David Blaschke; *Uni Wroclaw, JINR Dubna and Uni Rostock*

Contents:

Jules Verne

K. Ziolkowski

- General Principles: QCD modeling @ nonzero T, μ
  Path Integral QCD Partition Function, Bosonization, Condensates, Quasiparticles, Fluctuations
- Present status: Two-phase descriptions
  Phase trans. construction & Masquerade problem
- Future: Nuclear Matter on the quark level
  Hadronic bound states & dissociation: Mott effect
  Condensates: BEC-BCS crossover
  Aims: Masquerade probl.; Hyperons & strangeness

TOWARDS QCD MODELING OF DENSE MATTER IN COMPACT STARS

David Blaschke; *Uni Wroclaw, JINR Dubna and Uni Rostock*

Yuri Gagarin
12. April 1961

Monument
Moscow

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Star Trek - Enterprise

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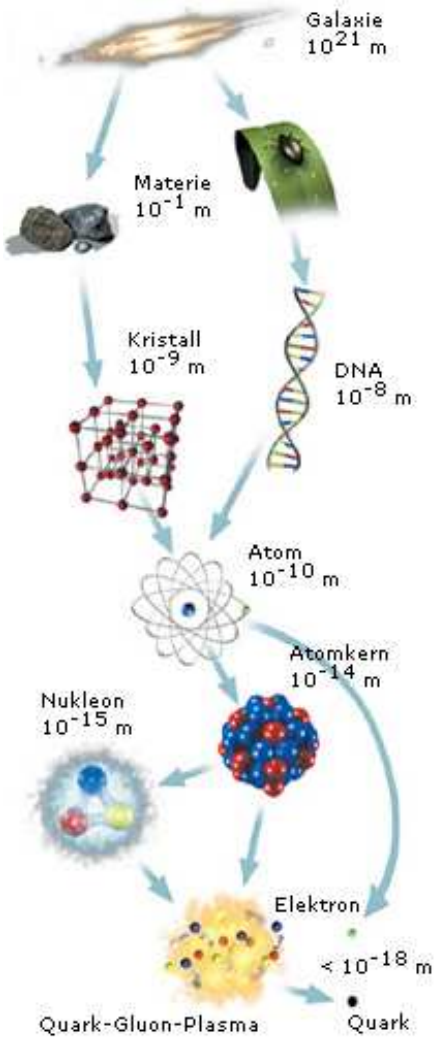
Many Particle Systems & Quantum Field Theory



Elements	Bound states	System
humans, animals	couples, groups, parties	society
molecules, crystals	(bio)polymers	animals, plants
atoms	molecules, clusters, crystals	solids, liquids, ...
ions, electrons	atoms	plasmas
nucleons, mesons	nuclei	nuclear matter
quarks, anti-quarks	nucleons, mesons	quark matter

Highly Compressed Matter $\Leftrightarrow$ Pauli Principle

Partition function: $Z = \mathrm{Tr}\left\{\mathrm{e}^{-\beta(H-\mu_i Q_i)}\right\}$

Partition function for Quantum Chromodynamics (QCD)

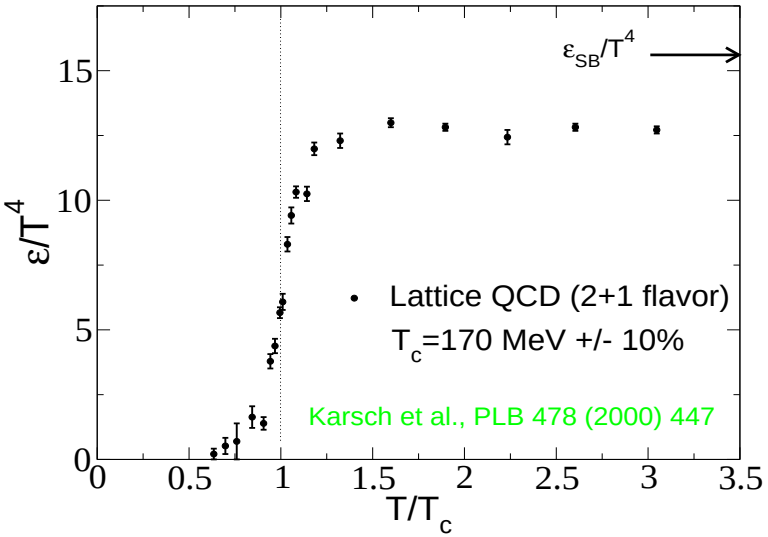
- Partition function as a Path Integral (imaginary time $\tau = i\,t$, $0 \leq \tau \leq \beta = 1/T$)

$$Z[T,V,\mu] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A \exp\left\{-\int_0^\beta d\tau \int_V d^3x\, \mathcal{L}_{QCD}(\psi,\bar{\psi},A)\right\}$$

- QCD Lagrangian, non-Abelian gluon field strength: $F^a_{\mu\nu}(A) = \partial_\mu A^a \nu - \partial_\nu A^a_\mu + g\, f^{abc}[A^b_\mu, A^c_\nu]$

$$\mathcal{L}_{QCD}(\psi,\bar{\psi},A) = \bar{\psi}[i\gamma^\mu(\partial_\mu - igA_\mu) - m - \gamma^0\mu]\psi - \frac{1}{4}F^a_{\mu\nu}(A)F^{a,\mu\nu}(A)$$

- Numerical evaluation: Lattice gauge theory simulations (Bielefeld group)



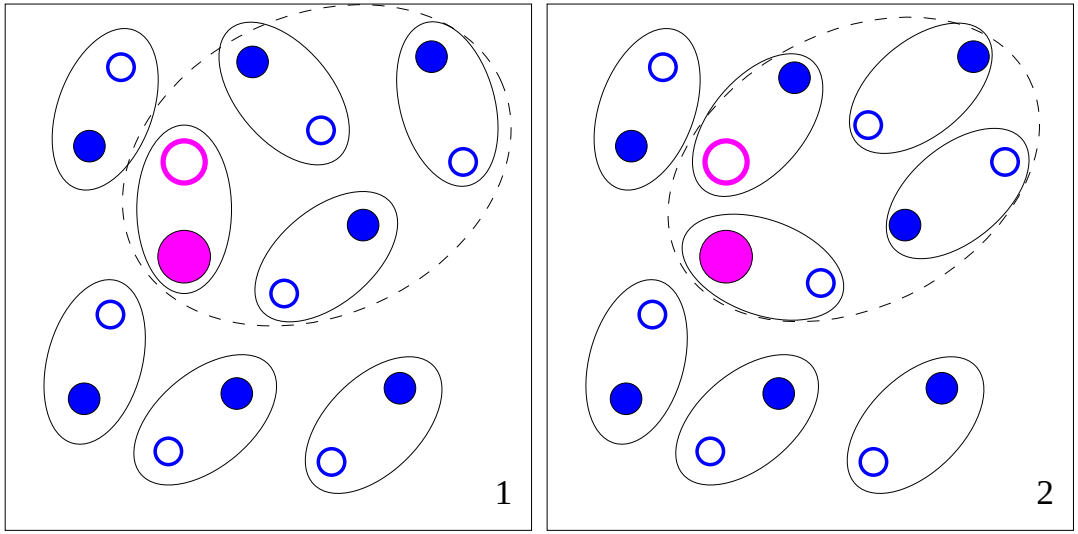
- Equation of state: $\varepsilon(T) = -\partial \ln Z[T,V,\mu]/\partial\beta$
- Phase transition at $T_c = 170$ MeV
- Problem: Interpretation ?

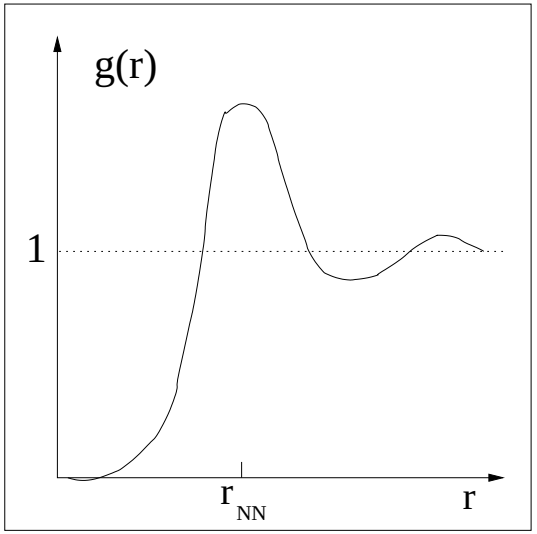
$\varepsilon/T^4 = \frac{\pi^2}{30}N_\pi \sim 1$ (ideal pion gas)

$\varepsilon/T^4 = \frac{\pi^2}{30}(N_G + \frac{7}{8}N_Q) \sim 15.6$ (quarks and gluons)

A SNAPSHOP OF THE sQGP

The Picture: String-flip (Rearrangement) $\Longleftrightarrow$ Pair correlation



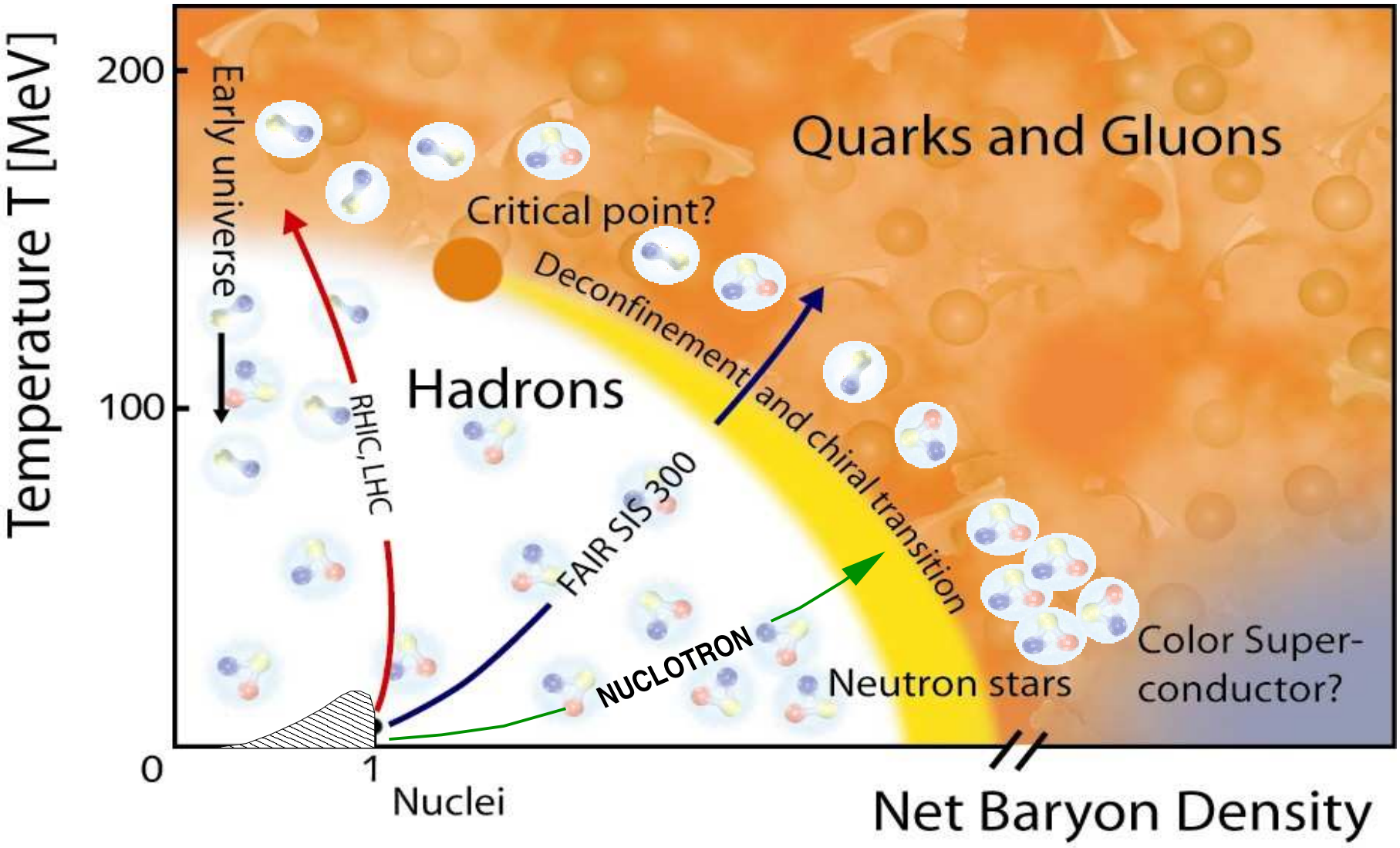


Horowitz et al. PRD (1985), D.B. et al. PLB (1985), Röpke, Blaschke, Schulz, PRD (1986)

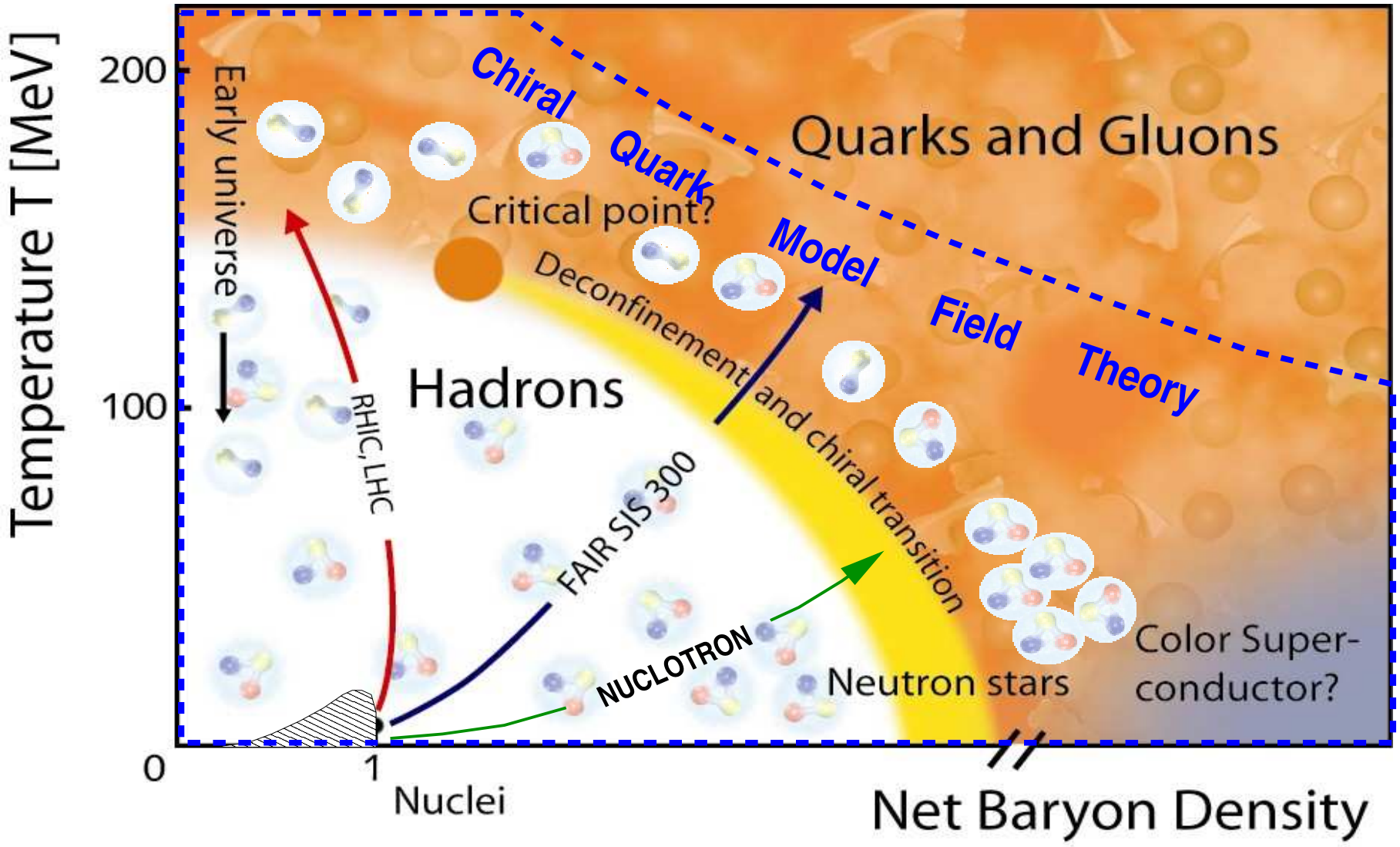
Thoma, Quark Matter '05; [hep-ph/0509154]

- Strong correlations present: hadronic spectral functions above T_c (lattice QCD)
- Finite width due to rearrangement collisions (higher order correlations)
- Liquid-like pair correlation function (nearest neighbor peak)

Quark Substructure and Phase Diagram



PHASE DIAGRAM OF QCD: CHIRAL QUARK MODELS



CHIRAL MODEL FIELD THEORY FOR QUARK MATTER

- Partition function as a Path Integral (imaginary time $\tau = i\,t$)

$$Z[T,V,\mu] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left\{-\int^{\beta} d\tau \int_V d^3x[\bar{\psi}(i\gamma^\mu\partial_\mu - m - \gamma^0\mu)\psi - \mathcal{L}_{\rm int}]\right\}$$

- Current-current interaction (4-Fermion coupling)

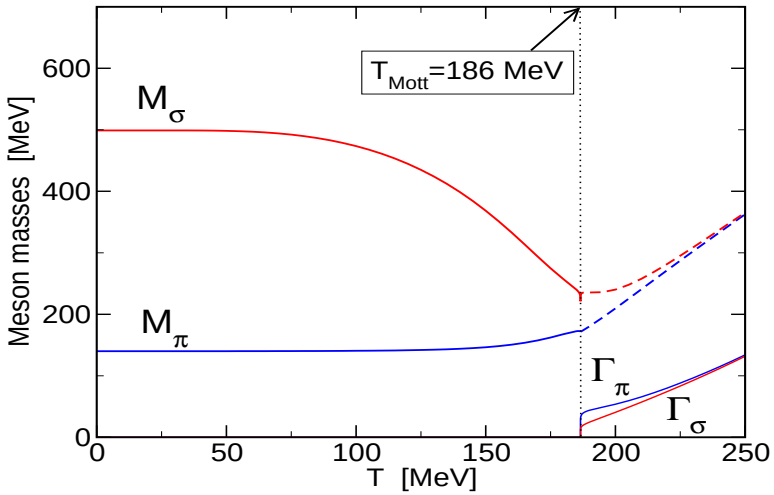
$$\mathcal{L}_{\rm int} = \sum_{M=\pi,\sigma,\ldots} G_M(\bar{\psi}\Gamma_M\psi)^2 + \sum_D G_D(\bar{\psi}^C\Gamma_D\psi)^2$$

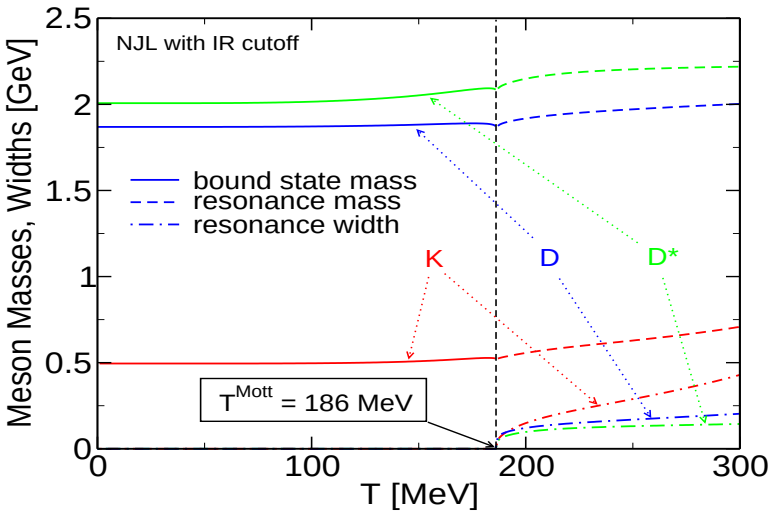
- Bosonization (Hubbard-Stratonovich Transformation)

$$Z[T,V,\mu] = \int \mathcal{D}M_M\mathcal{D}\Delta_D^\dagger\mathcal{D}\Delta_D \exp\left\{-\sum_M \frac{M_M^2}{4G_M} - \sum_D \frac{|\Delta_D|^2}{4G_D} + \frac{1}{2}{\rm Tr}\,\ln S^{-1}[\{M_M\},\{\Delta_D\}]\right\}$$

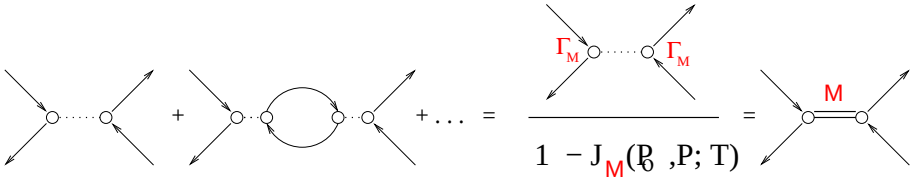
- Collective (stochastic) fields: Mesons (M_M) and Diquarks (Δ_D)
- Systematic evaluation: Mean fields + Fluctuations
  - Mean-field approximation: order parameters for phase transitions (gap equations)
  - Lowest order fluctuations: hadronic correlations (bound & scattering states)
  - Higher order fluctuations: hadron-hadron interactions

MOTT EFFECT: NJL MODEL PRIMER





RPA-type resummation of quark-antiquark scattering in the mesonic channel M,



defi nes Meson propagator

$$D_M(P_0, P; T) \sim [1 - J_M(P_0, P; T)]^{-1},$$

by the complex polarization function J_M
$\rightarrow$ Breit-Wigner type spectral function

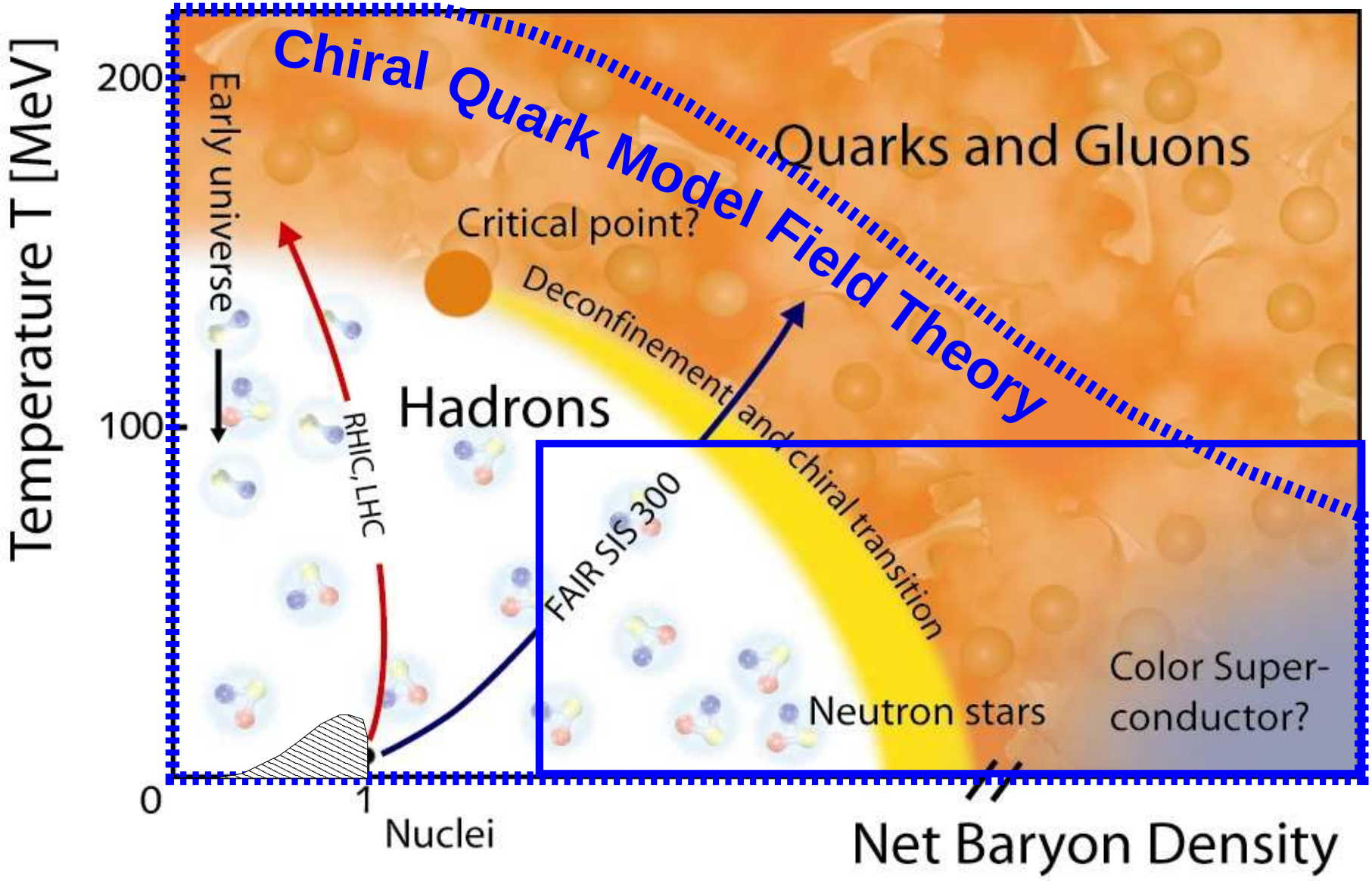
$$\begin{aligned} \mathcal{A}_M(P_0, P; T) &= \frac{1}{\pi} \mathrm{Im}\, D_M(P_0, P; T) \\ &\sim \frac{1}{\pi} \frac{\Gamma_M(T) M_M(T)}{(s - M_M^2(T))^2 + \Gamma_M^2(T) M_M^2(T)} \end{aligned}$$

For $T < T_{\mathrm{Mott}}$: $\Gamma \to 0$, i.e. bound state
$\mathcal{A}_M(P_0, P; T) = \delta(s - M_M^2(T)$

Light meson sector:
Blaschke, Burau, Volkov, Yudichev: EPJA **11** (2001) 319

Charm meson sector:
Blaschke, Burau, Kalinovsky, Yudichev,
Prog. Theor. Phys. Suppl. **149** (2003) 182

PHASEDIAGRAM OF DEGENERATE QUARK MATTER



NJL MODEL FOR NEUTRAL 3-FLAVOR QUARK MATTER

$$\begin{aligned}\Omega(T,\mu) &= \frac{\phi_u^2+\phi_d^2+\phi_s^2}{8G_S}+\frac{|\Delta_{ud}|^2+|\Delta_{us}|^2+|\Delta_{ds}|^2}{4G_D}\\ &- T\sum_n\int\frac{d^3p}{(2\pi)^3}\frac{1}{2}\mathrm{Tr}\ln\left(\frac{1}{T}S^{-1}(i\omega_n,\vec{p})\right)\\ &+ \Omega_e-\Omega_0.\end{aligned}$$

Inverse propagator of Nambu-Gorkov spinors

$$S^{-1}(i\omega_n,\vec{p}) = \begin{bmatrix}\not{p}-M+\mu\gamma^0 & \widehat{\Delta}\\ \widehat{\Delta}^\dagger & \not{p}-M-\mu\gamma^0\end{bmatrix},$$

with diquark gaps ($\Delta_{ur}=\Delta_{ds},\ldots$)

$$\Delta_{k\gamma}=2G_D\langle\bar{q}_{i\alpha}i\gamma_5\epsilon_{\alpha\beta\gamma}\epsilon_{ijk}q^C_{j\beta}\rangle.$$

as elements of the gap matrix

$$\widehat{\Delta} = i\gamma_5\epsilon_{\alpha\beta\gamma}\epsilon_{ijk}\Delta_{k\gamma}.$$

Fermion determinant (Tr ln D = ln det D)

$$\mathrm{lndet}\left(\frac{1}{T}S^{-1}(i\omega_n,\vec{p})\right)=2\sum_{a=1}^{18}\ln\left(\frac{\omega_n^2+\lambda_a(\vec{p})^2}{T^2}\right).$$

Result for thermodynamic potential

$$\begin{aligned}\Omega(T,\mu) &= \frac{\phi_u^2+\phi_d^2+\phi_s^2}{8G_S}+\frac{|\Delta_{ud}|^2+|\Delta_{us}|^2+|\Delta_{ds}|^2}{4G_D}\\ &- \int\frac{d^3p}{(2\pi)^3}\sum_{a=1}^{18}\left(\lambda_a+2T\ln\left(1+e^{-\lambda_a/T}\right)\right)\\ &+ \Omega_e-\Omega_0.\end{aligned}$$

Neutrality conditions: $n_Q=n_8=n_3=0$,

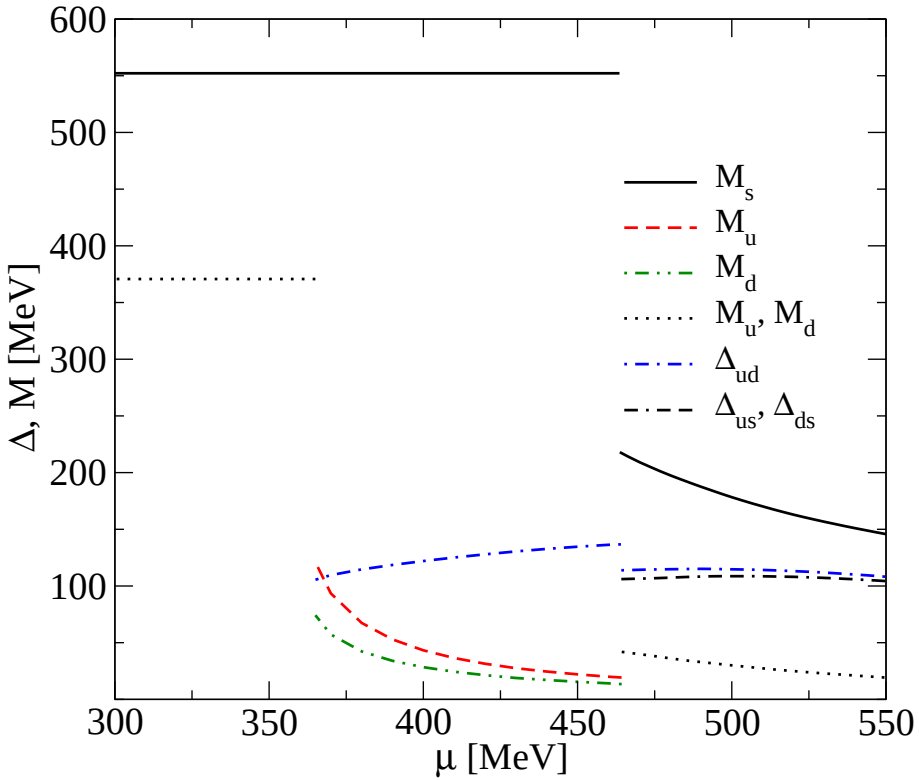
$$n_i = -\frac{\partial\Omega}{\partial\mu_i}=0,$$

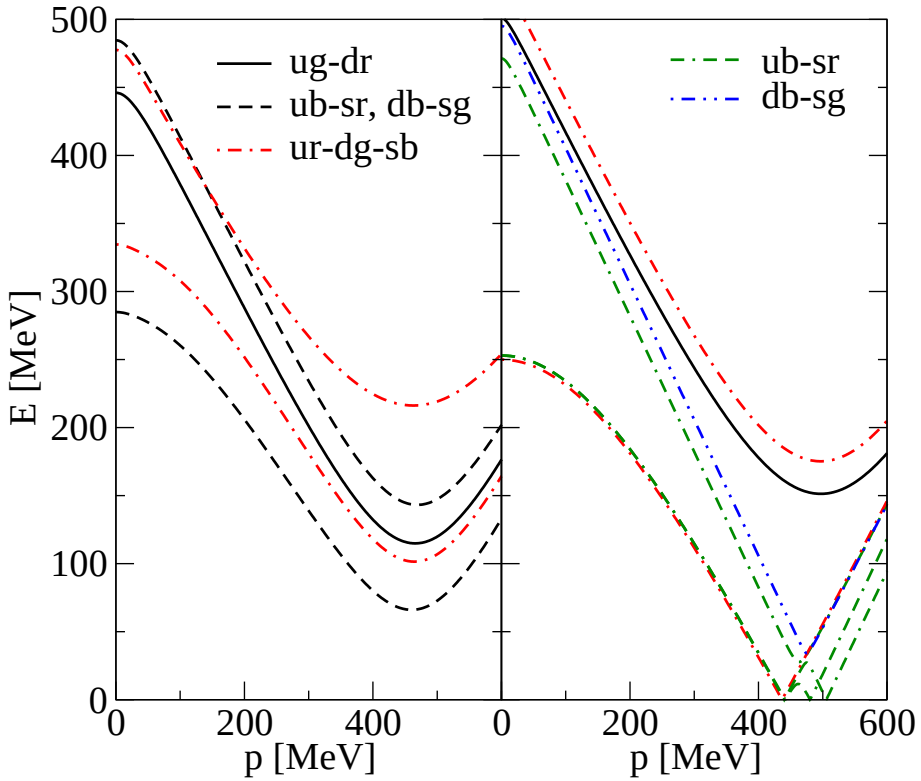
Equation of state: $P=-\Omega$, etc.

Order Parameters: Masses and Diquark Gaps

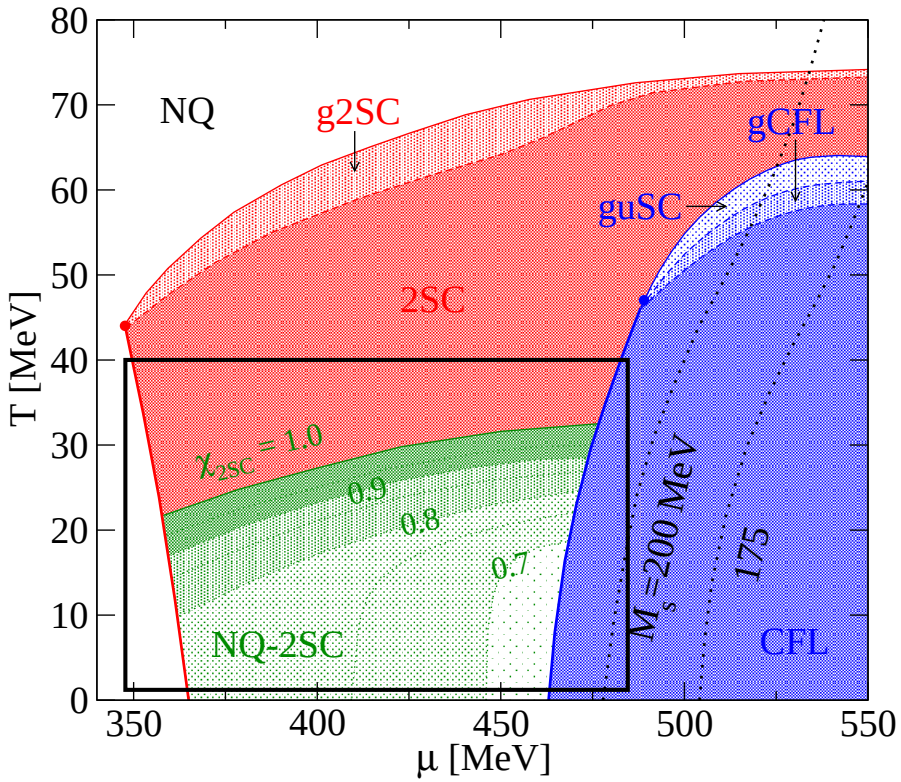
Masses (M) and Diquark gaps (Δ) as a function of the chemical potential at $T = 0$

Left: Gap in excitation spectrum ($T = 0$)
Right: 'Gapless' excitations ($T = 60$ MeV)





QUARK MATTER IN COMPACT STARS



Rüster et al: PRD 72 (2005) 034004
Blaschke et al: PRD 72 (2005) 065020
Abuki, Kunihiro: NPA 768 (2006) 118

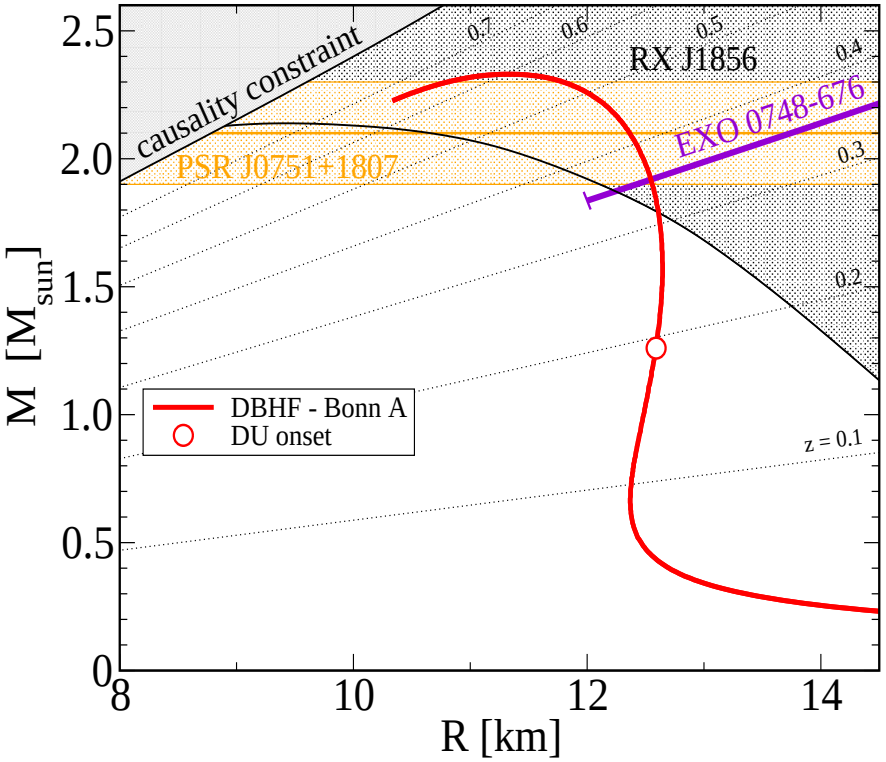
The phases are characterized by 3 gaps:

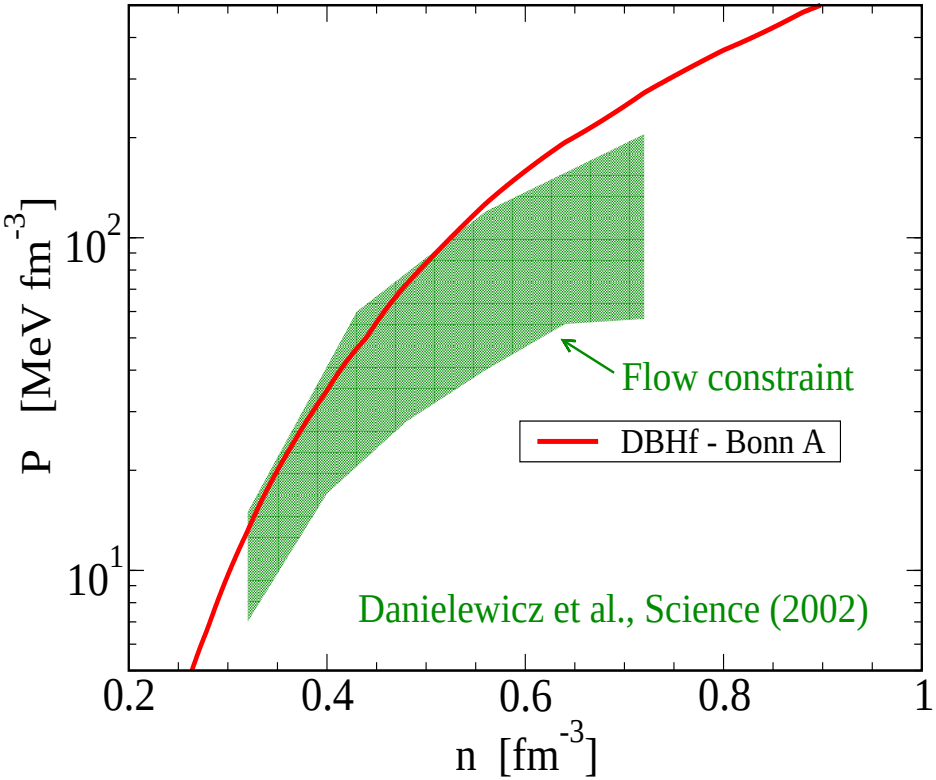
- NQ: $\Delta_{ud} = \Delta_{us} = \Delta_{ds} = 0$;
- NQ-2SC: $\Delta_{ud} \neq 0$, $\Delta_{us} = \Delta_{ds} = 0$, $0 \leq \chi_{2SC} \leq 1$;
- 2SC: $\Delta_{ud} \neq 0$, $\Delta_{us} = \Delta_{ds} = 0$;
- uSC: $\Delta_{ud} \neq 0$, $\Delta_{us} \neq 0$, $\Delta_{ds} = 0$;
- CFL: $\Delta_{ud} \neq 0$, $\Delta_{ds} \neq 0$, $\Delta_{us} \neq 0$;

Result:

- Gapless phases only at high T,
- CFL only at high chemical potential,
- At T $\leq$25-30 MeV: mixed NQ-2SC phase,
- Critical point (T_c,μ_c)=(48 MeV, 353 MeV),
- Strong coupling, $\eta = 1$, changes?.

Mass-Radius constraint and Flow constraint (I)

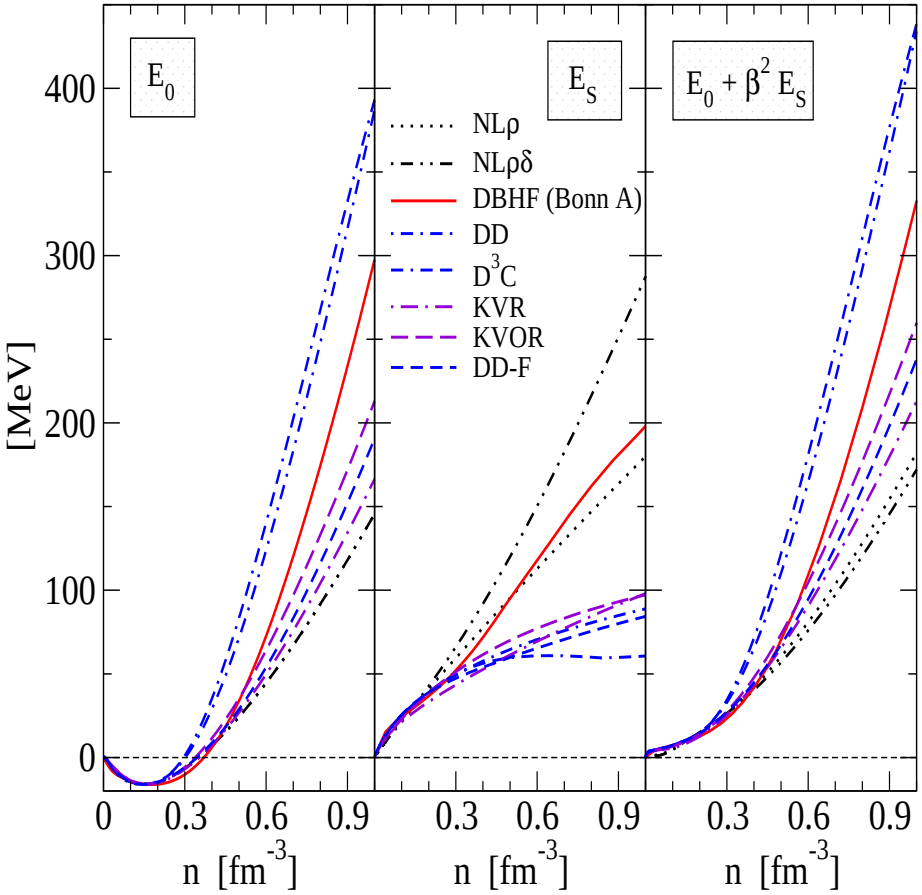


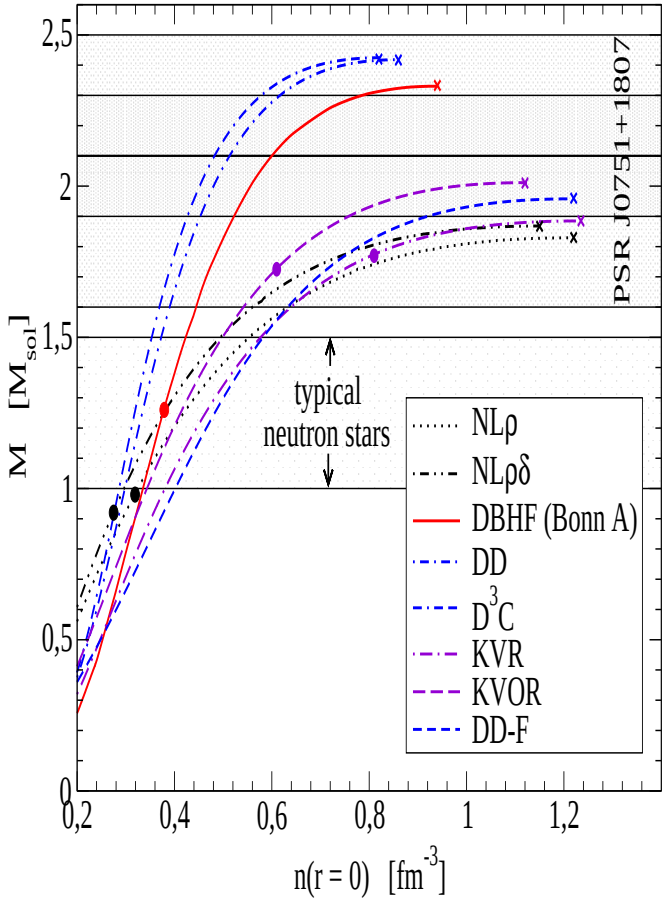


- Large Mass ($\sim 2\ M_\odot$) and radius ($R \geq 12$ km) $\Rightarrow$ stiff EoS;
  But: DBHF has DU onset in typical mass regon!
- Flow in Heavy-Ion Collisions $\Rightarrow$ not too stiff EoS ! But: DBHF violates at high densities

Klähn, D.B., Sandin, Fuchs, Faessler, Grigorian, Röpke, Trümper, [arxiv:nucl-th/0609067]

DU THRESHOLD AND 'HADRONIC' NEUTRON STARS

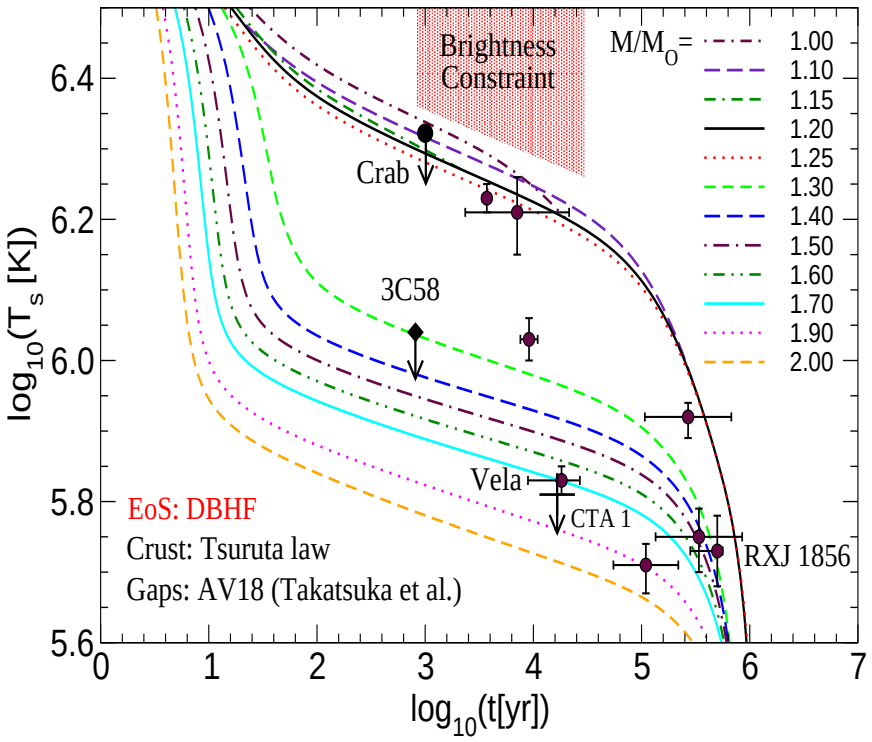


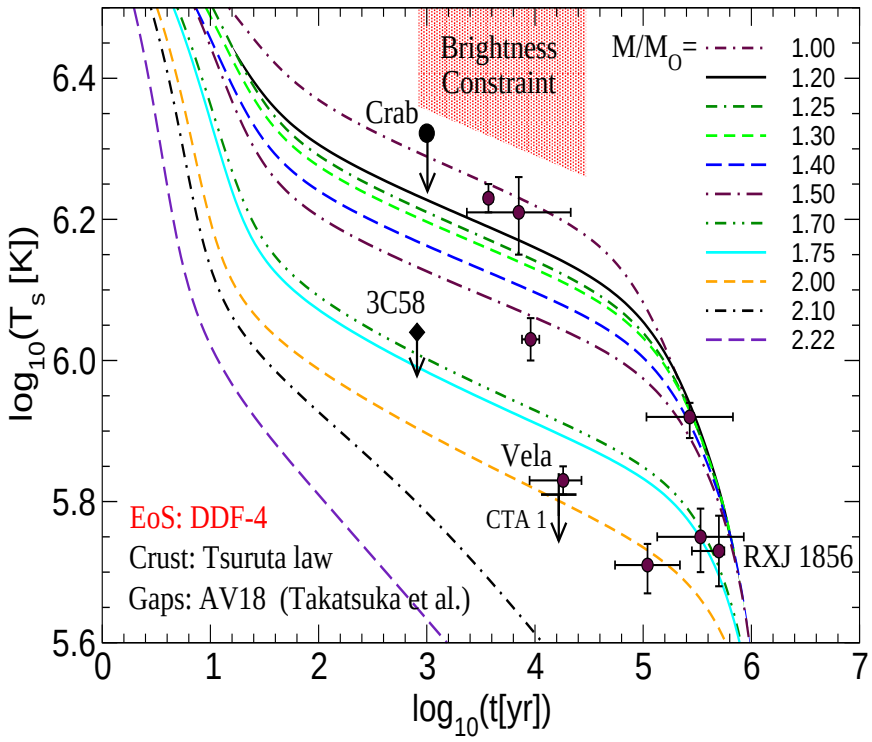


DU threshold for most hadronic EoS active in neutron stars with typical masses !

Klähn, et al., PRC 74, 035802 (2006); [nucl-th/0602038]

DU THRESHOLD AND 'HADRONIC' NEUTRON STARS (II)

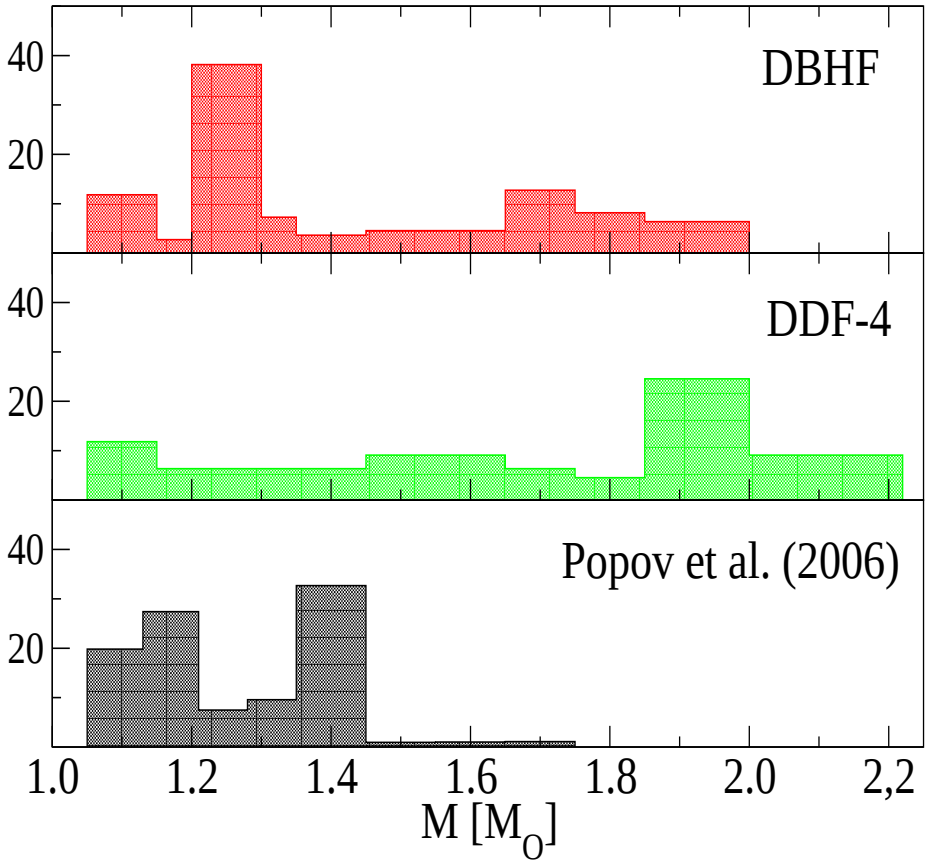




- DU threshold $\Rightarrow$ sensitivity to tiny mass variations;
- Description of Vela not possible with typical masses !

S. Popov et al., PRC 74 (2006); D.B. and H. Grigorian, PPNP (2007) [astro-ph/0612092]

DU THRESHOLD AND 'HADRONIC' NEUTRON STARS (III)

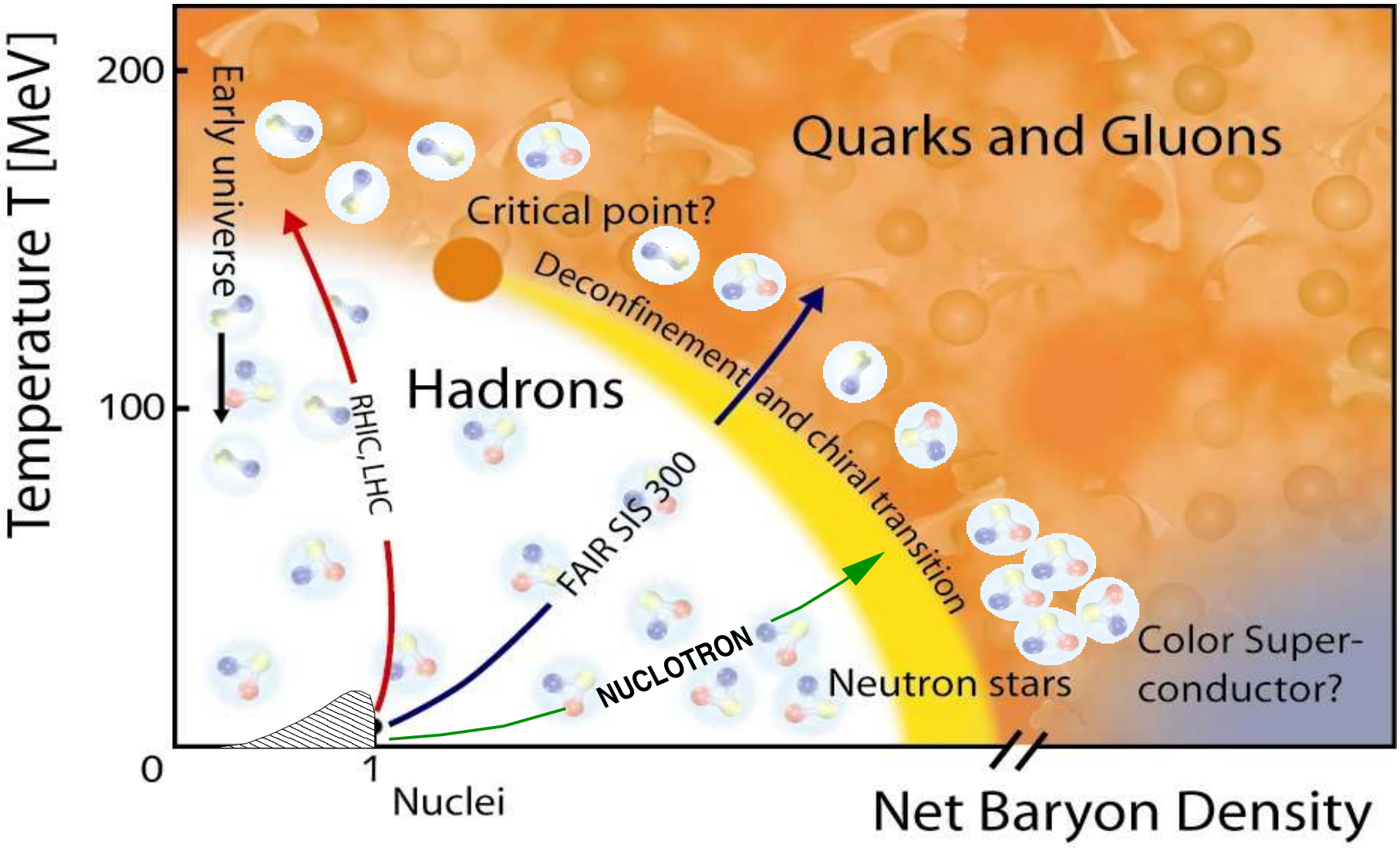


Mass population from Cooling

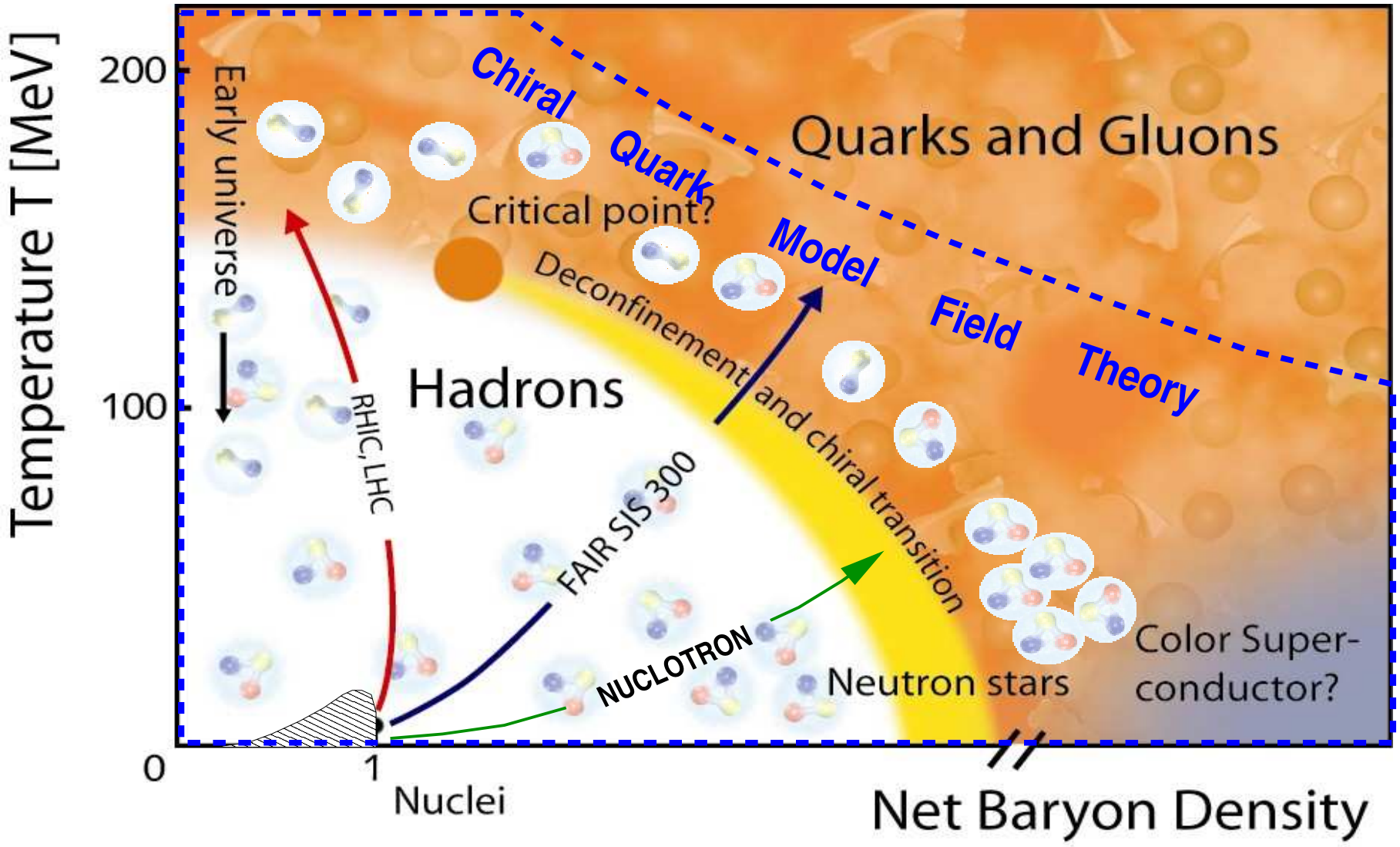
- DU threshold: overpopulation of a small mass window;
- Hadronic cooling not fast enough to describe Vela with $M < 1.5\ M_\odot$!

D.B. and H. Grigorian, PPNP (2007); [astro-ph/0612092]

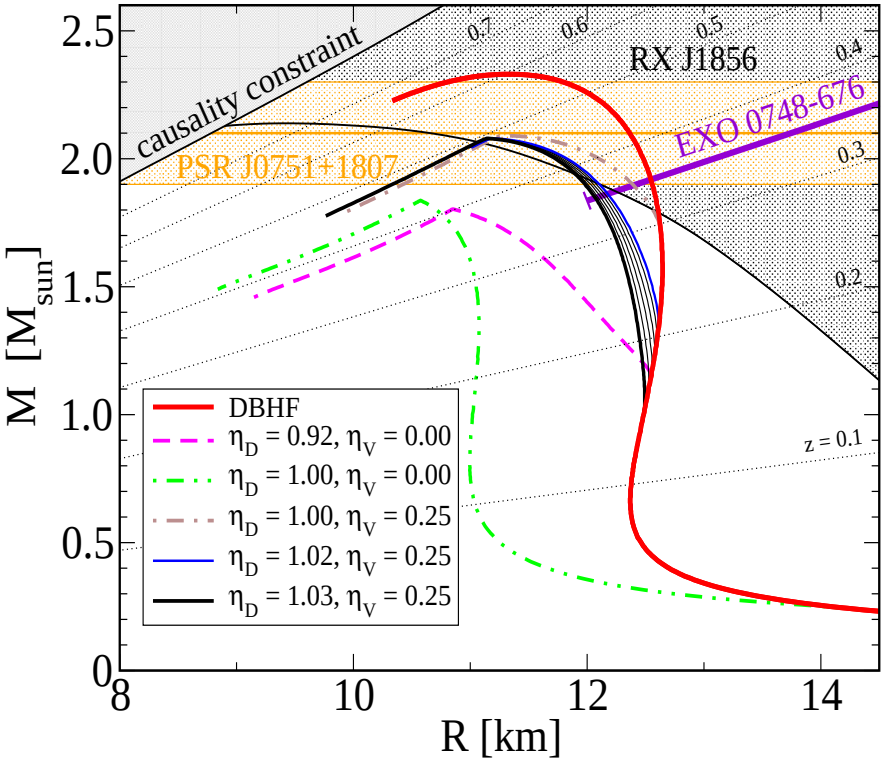
Quark Substructure and Phase Diagram

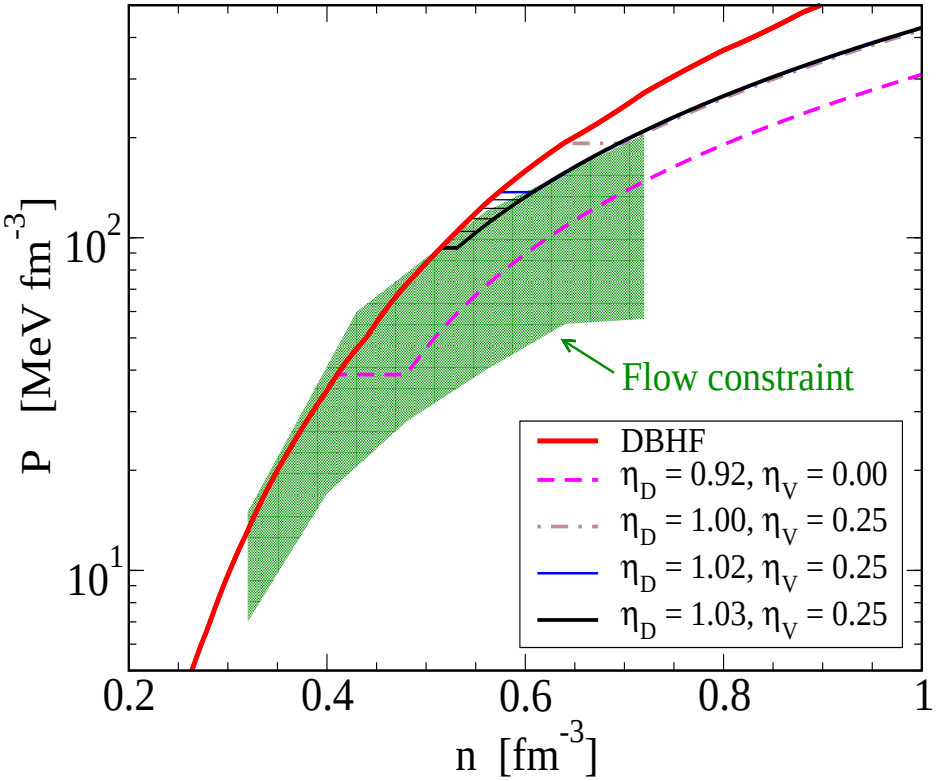


PHASE DIAGRAM OF QCD: CHIRAL QUARK MODELS



Mass-Radius constraint and Flow constraint (II)

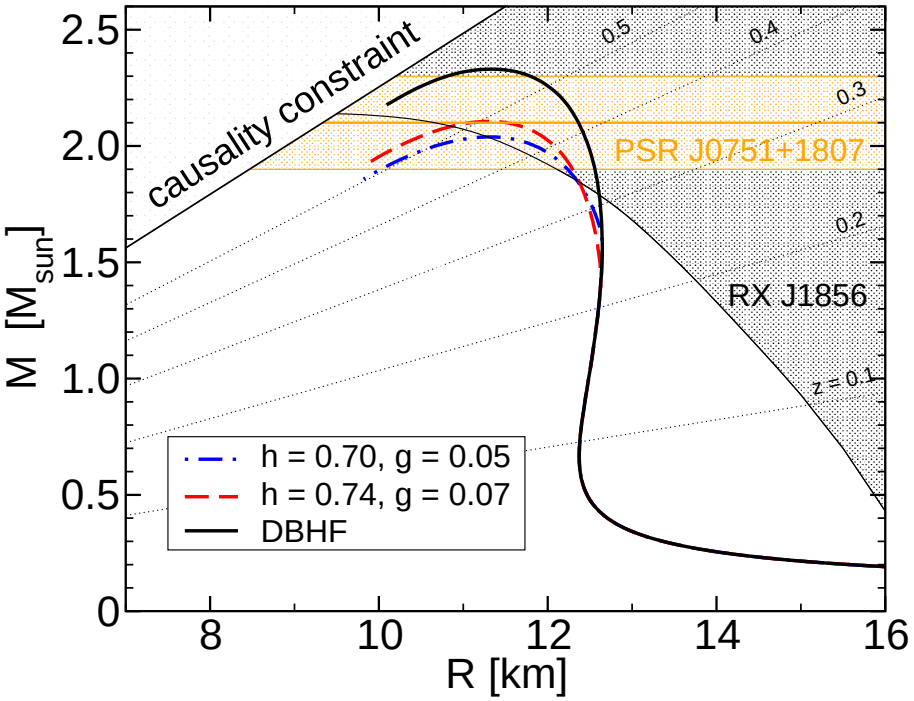


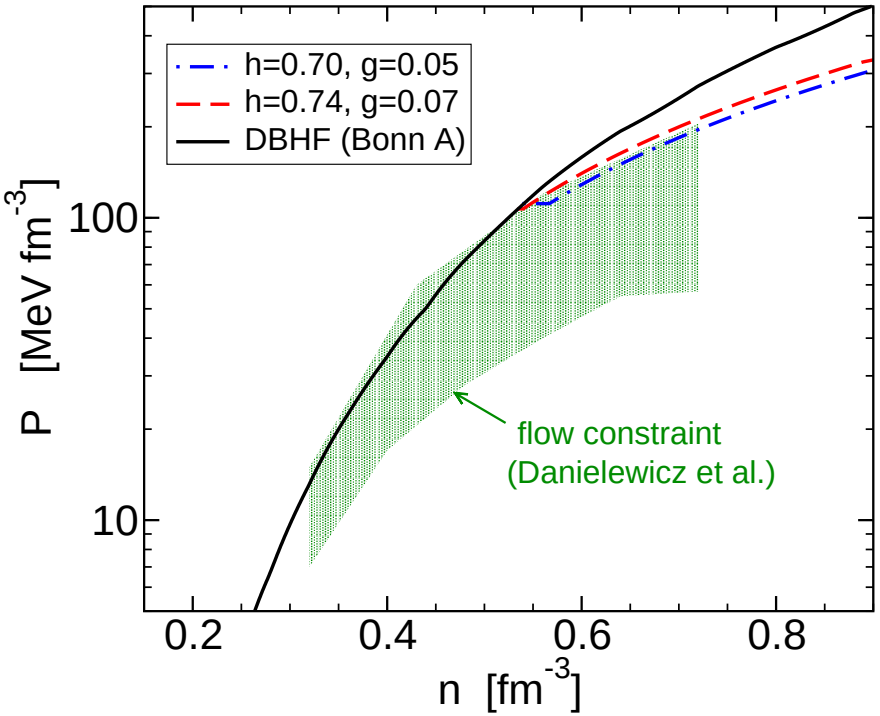


- Large Mass ($\sim 2\ M_\odot$) and radius ($R \geq 12$ km) $\Rightarrow$ stiff quark matter EoS;
  Note: DU problem of DBHF removed by deconfinement! and: CFL core Hybrids unstable!
- Flow in Heavy-Ion Collisions $\Rightarrow$ not too stiff EoS !
  Note: Quark matter removes violation by DBHF at high densities

Klähn, D.B., Sandin, Fuchs, Faessler, Grigorian, Röpke, Trümper, [arxiv:nucl-th/0609067]

HYBRID-EOS ROBUST? CONSTRAINTS & COVARIANT NCQM





- Covariant, nonlocal interaction model:

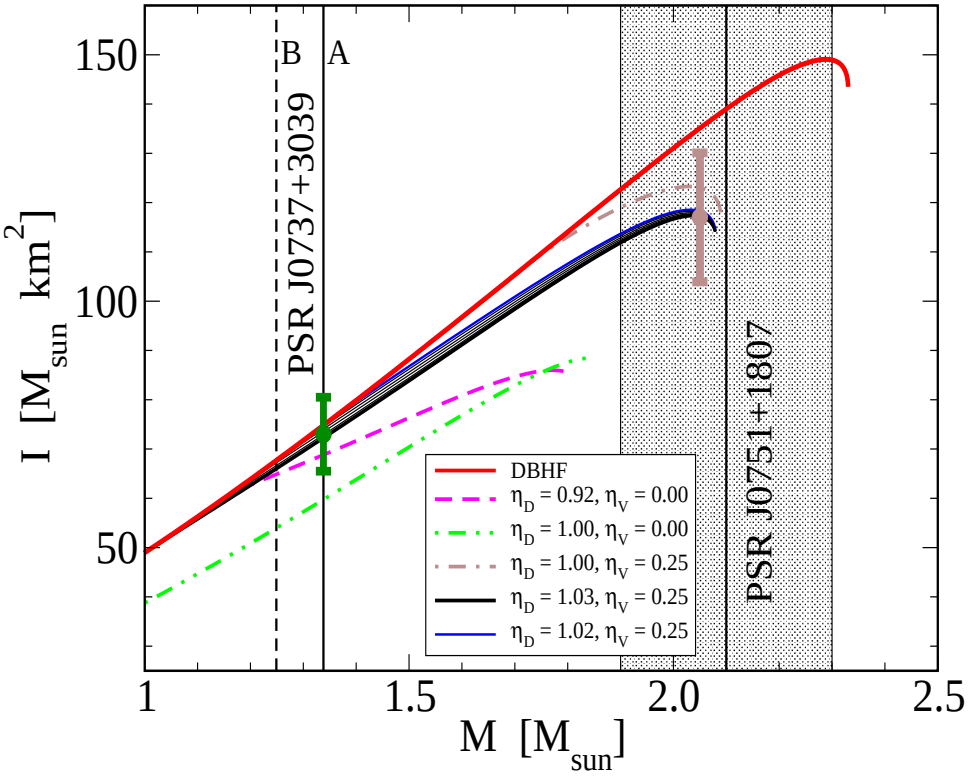
$$\mathcal{L}_{\rm int} = -\int d^4x \, \left\{ \frac{G_S}{2} j_S^f(x) j_S^f(x) + \frac{H}{2} \left[j_D^a(x) \right]^\dagger j_D^a(x) + \frac{G_V}{2} j_V^\mu(x) \, j_V^\mu(x) \right\}$$

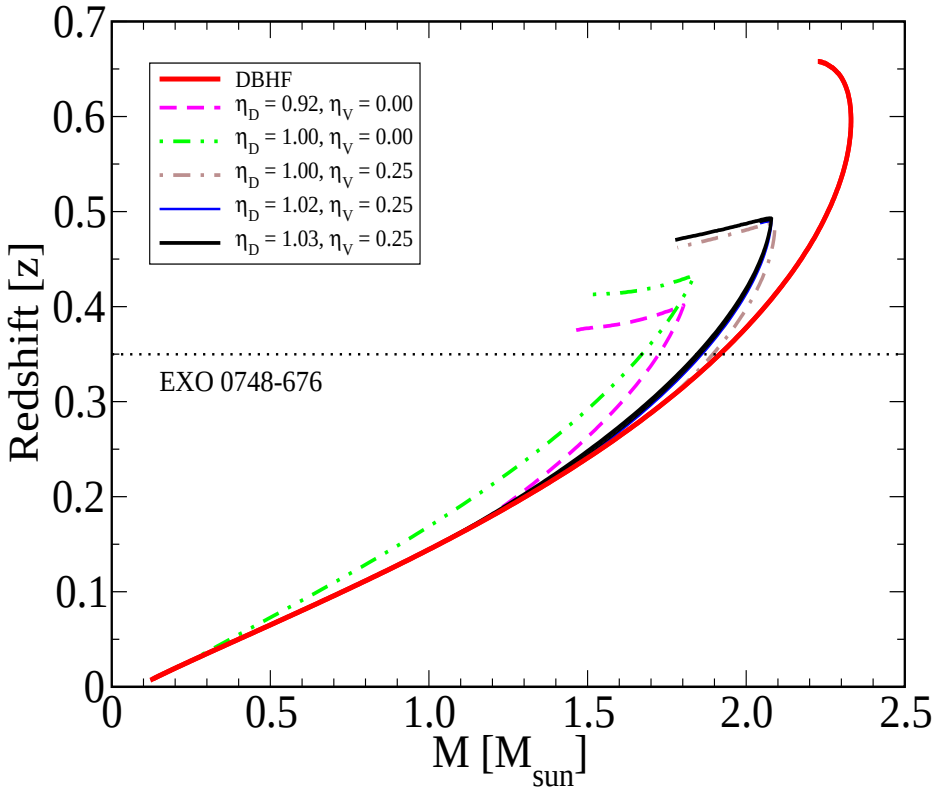
- Nonlocal currents, e.g.

$$j_S^f(x) = \int d^4z \; g(z) \; \bar{\psi}(x + \tfrac{z}{2}) \; \Gamma_f \, \psi(x - \tfrac{z}{2}) \, ,$$

D.B., Gomez-Dumm, Grunfeld, Klähn, Scoccola; [arxiv:nucl-th/0703088]

HYBRID STARS THAT MASQUERADE AS NEUTRON STARS*





- Moment of Inertia $\Rightarrow$ objects with large masses necessary
- Surface redshift $\Rightarrow$ large values (> 0.5) troublesome for quark matter

* **Alford et al., ApJ 629, 969 (2005); Klähn et al., nucl-th/0609067**

HYBRID STAR COOLING WITH 2SC QUARK MATTER

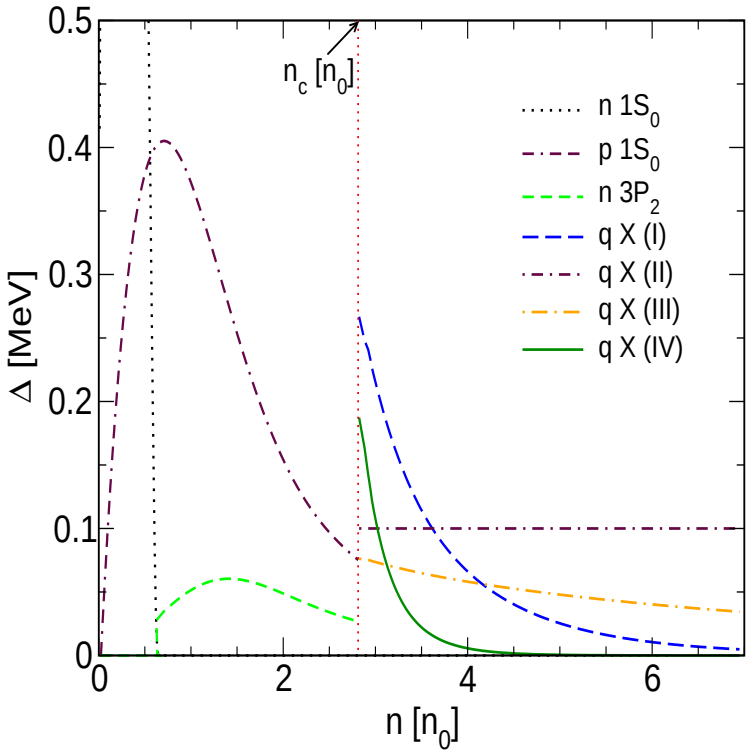
2SC phase: 1 color (blue) is unpaired (mixed superconductivity)

Ansatz 2SC + X phase:

$$\Delta_X(\mu) = \Delta_0 \exp[\alpha(1 - \mu/\mu_c)]$$

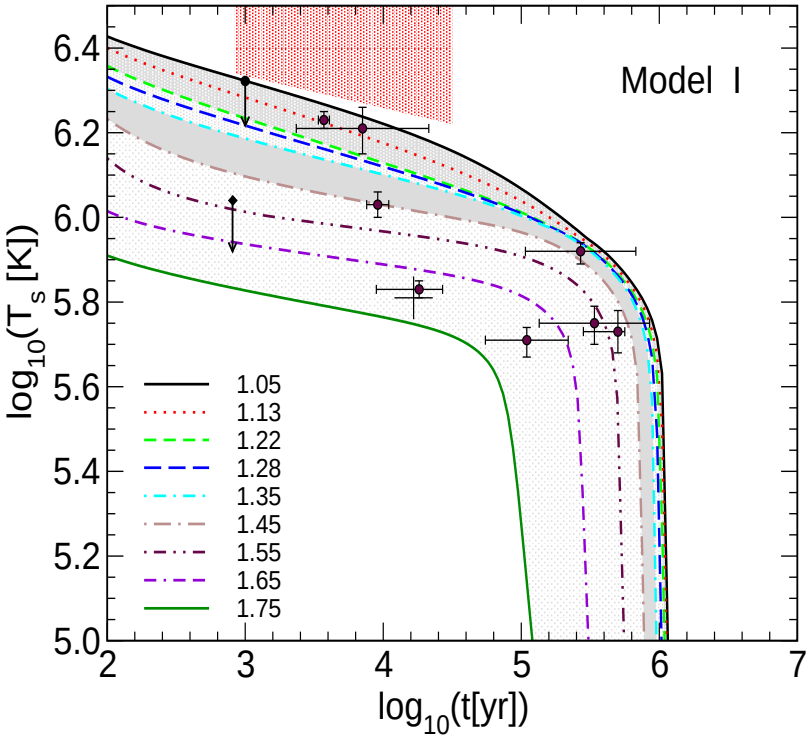
Model	Δ_0 [MeV]	α
I	1	10
II	0.1	0
III	0.1	2
IV	5	25

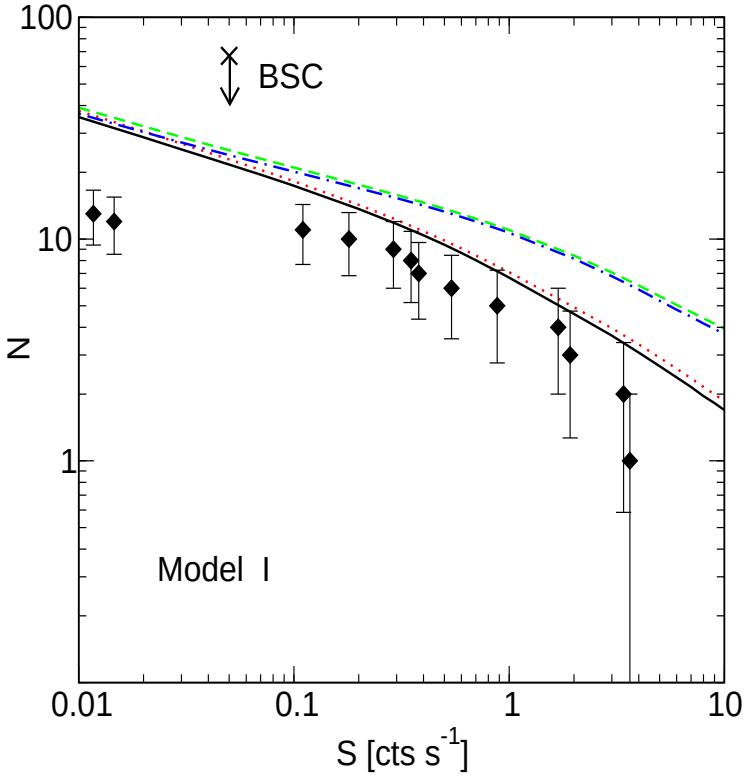
Popov, Grigorian, D.B., PRC 74 (2006)



Pairing gaps for hadronic phase (**AV18 - Takatsuka et al. (2004)**) and 2SC + X phase

HYBRID STAR COOLING WITH 2SC QUARK MATTER (II)





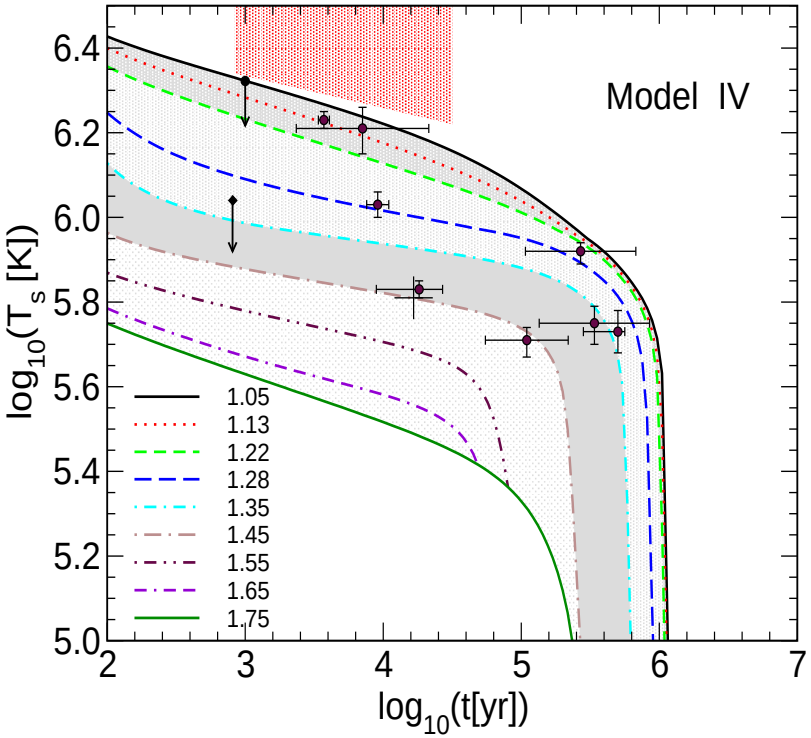
2SC + X phase, $\Delta_0 = 1$ MeV, $\alpha = 10$

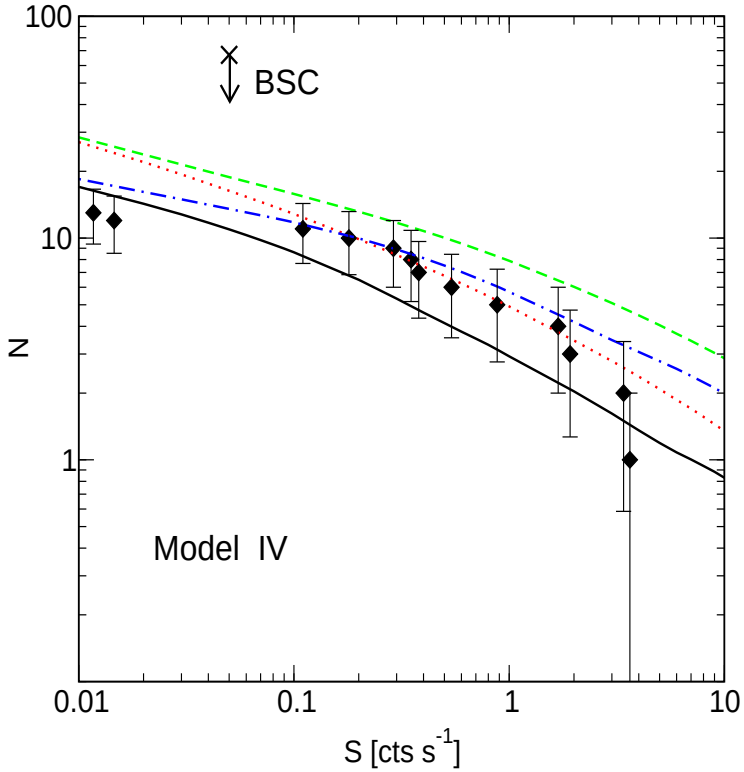
Too large mass for Vela required

Log N - Log S test fails

Popov, Grigorian, D.B., PRC 74 (2006)

HYBRID STAR COOLING WITH 2SC QUARK MATTER (III)





2SC + X phase, $\Delta_0 = 5$ MeV, $\alpha = 25$

Temperature-age and Vela mass OK

Log N - Log S test passed

Popov, Grigorian, D.B., PRC 74 (2006)

HYBRID STAR COOLING WITH 2SC QUARK MATTER (IV)

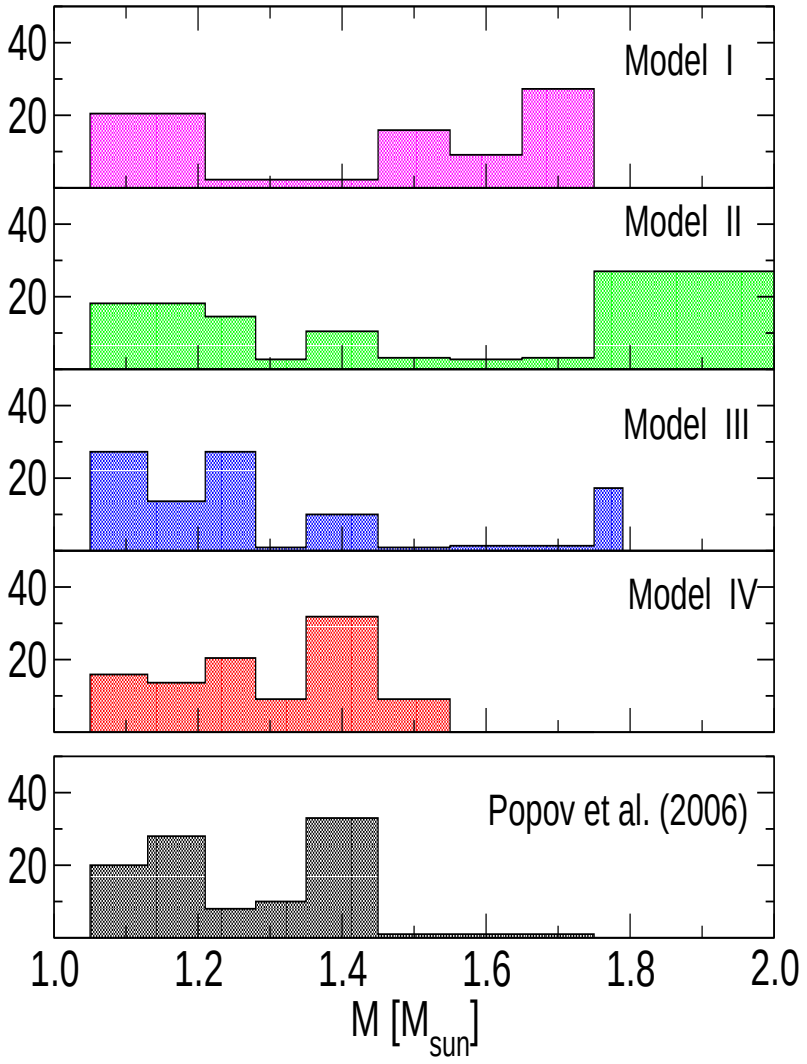
Hybrid star cooling passes all modern tests:

- Temperature - age
- Log N - Log S
- Brightness constraint
- Vela mass (Population sysnthesis)

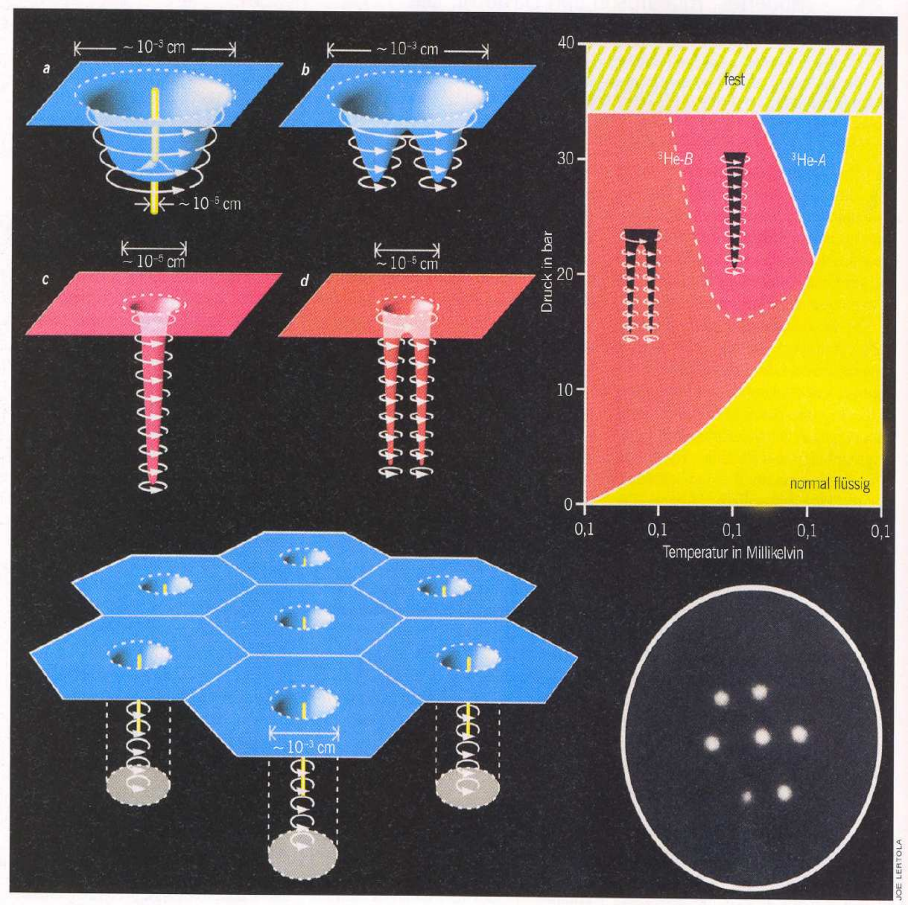
Popov, Grigorian, D.B., PRC 74 (2006)

D.B., H. Grigorian, D.B., PPNP (2007)

astro-ph/0612092



Spin-1 (iso-CSL) Phase for Compact Stars



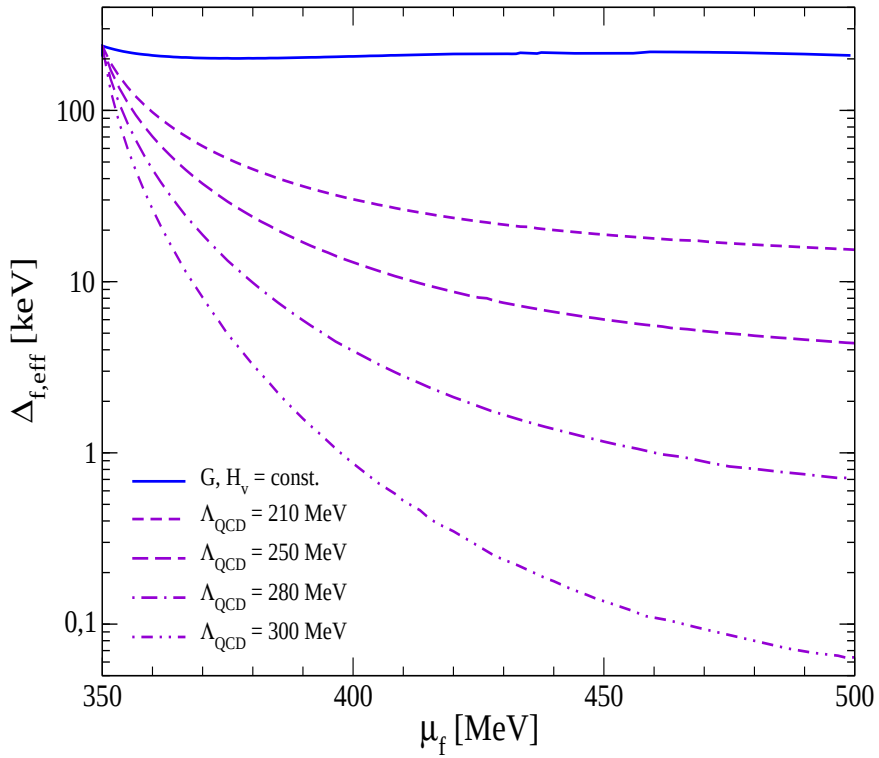
Phases of superfluid ^{3}He

Color-spin-locking (CSL) condensate

Aguilera et al., PRD 72 (2005) 034008

PRD 74 (2006) 114005

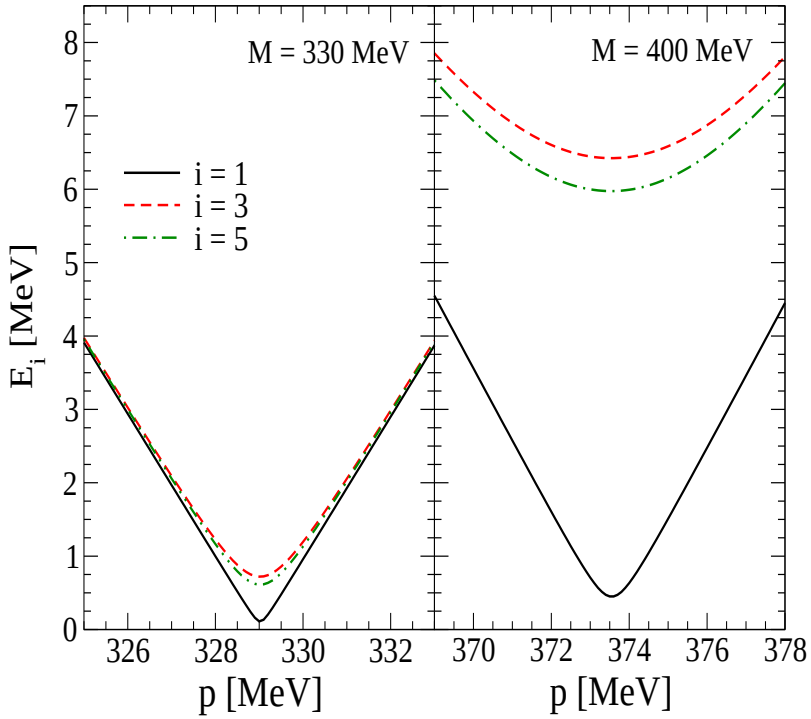
$$\eta_f \equiv \langle q_f^T \, C(\gamma^1\lambda_7 + \gamma^2\lambda_5 + \gamma^3\lambda_2) \, q_f \rangle$$

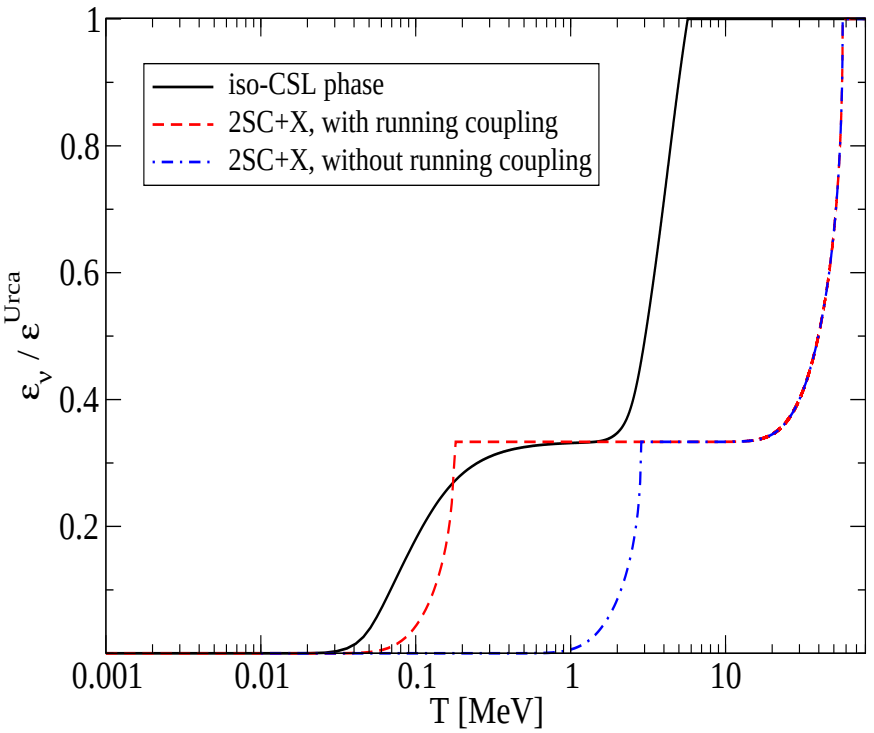


Neutrino Emissivities in the iso-CSL Phase

iso-CSL phase: Dispersion relations
Aguilera et al., PRD 74 (2006) 114005

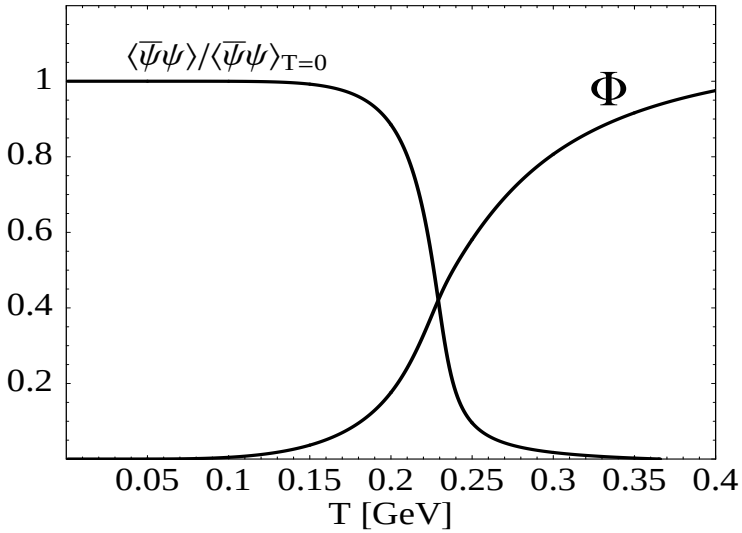
Emissivities from J. Berdermann,
PhD Thesis (U Rostock, 2007).

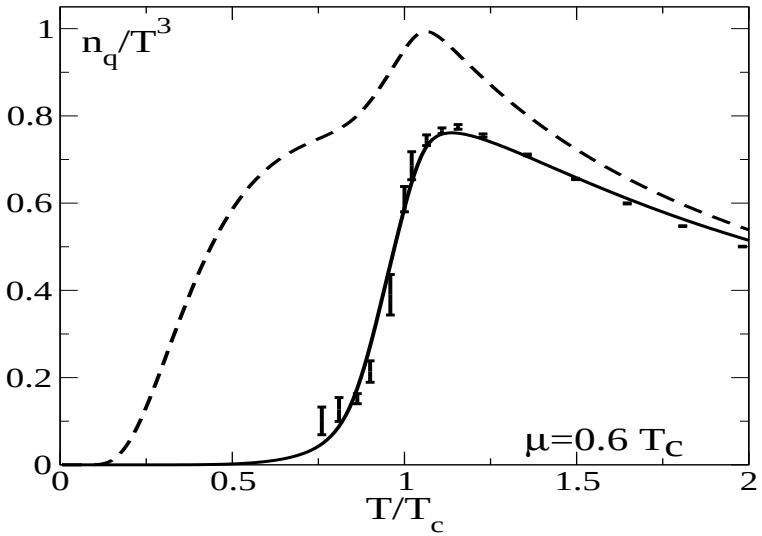




Possible microscopic explanation of the X-gap as the iso-CSL phase!

CONFINEMENT: POLYAKOV LOOP CHIRAL QUARK MODEL





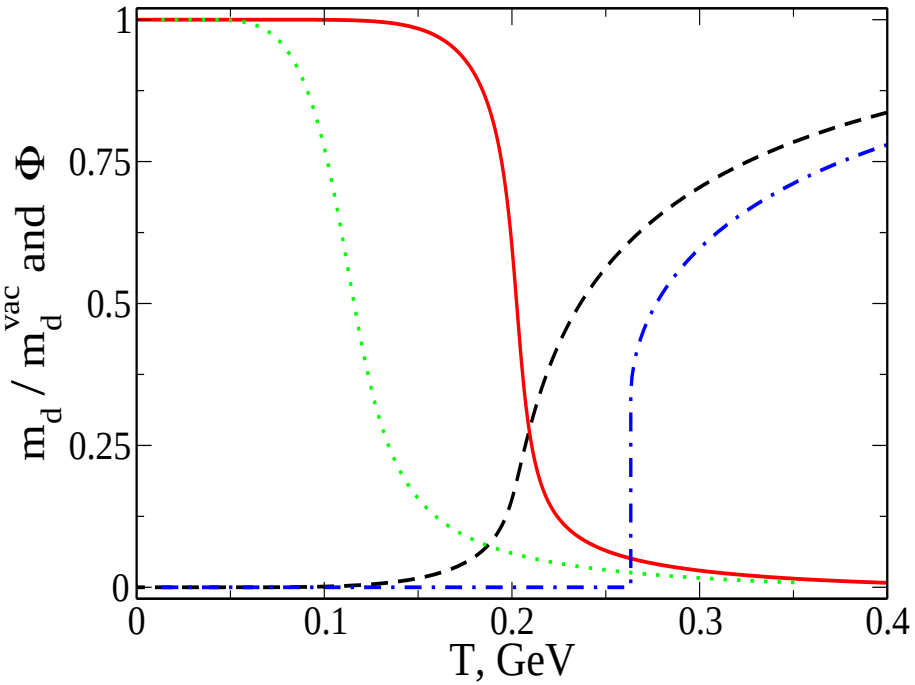
$$
\begin{aligned}
\Omega &= \mathcal{U}\left(\Phi,\bar{\Phi},T\right) + \frac{\sigma^2}{2G} \\
&- 2N_f\,T\int\frac{\mathrm{d}^3p}{(2\pi)^3}\Big\{\mathrm{Tr}_c\ln\left[1+L\,\mathrm{e}^{-\left(E_p-\mu\right)/T}\right] \\
&+ \mathrm{Tr}_c\ln\left[1+L^\dagger\,\mathrm{e}^{-\left(E_p+\mu\right)/T}\right]\Big\} \\
&- 6N_f\int\frac{\mathrm{d}^3p}{(2\pi)^3}E_p\,\theta\left(\Lambda^2-\vec{p}^{\,2}\right)
\end{aligned}
$$

Appearance of quarks below T_c largely suppressed
Accordance with QCD lattice susceptibilities!

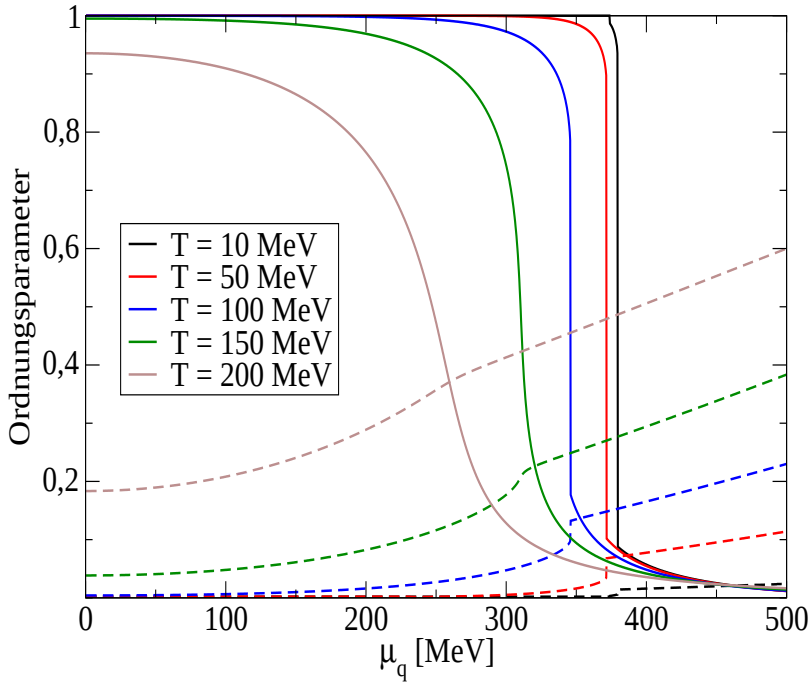
$$
\begin{aligned}
&\ln\det\left[1+L\,\mathrm{e}^{-\left(E_p-\mu\right)/T}\right] + \ln\det\left[1+L^\dagger\,\mathrm{e}^{-\left(E_p+\mu\right)/T}\right] \\
&= \ln\left[1+3\left(\Phi+\bar{\Phi}\mathrm{e}^{-\left(E_p-\mu\right)/T}\right)\mathrm{e}^{-\left(E_p-\mu\right)/T}+\mathrm{e}^{-3\left(E_p-\mu\right)/T}\right] \\
&+ \ln\left[1+3\left(\bar{\Phi}+\Phi\mathrm{e}^{-\left(E_p+\mu\right)/T}\right)\mathrm{e}^{-\left(E_p+\mu\right)/T}+\mathrm{e}^{-3\left(E_p+\mu\right)/T}\right]
\end{aligned}
$$

Ratti, Thaler, Weise, PRD 73 (2006) 014019.

NONLOCAL POLYAKOV LOOP CHIRAL QUARK MODEL

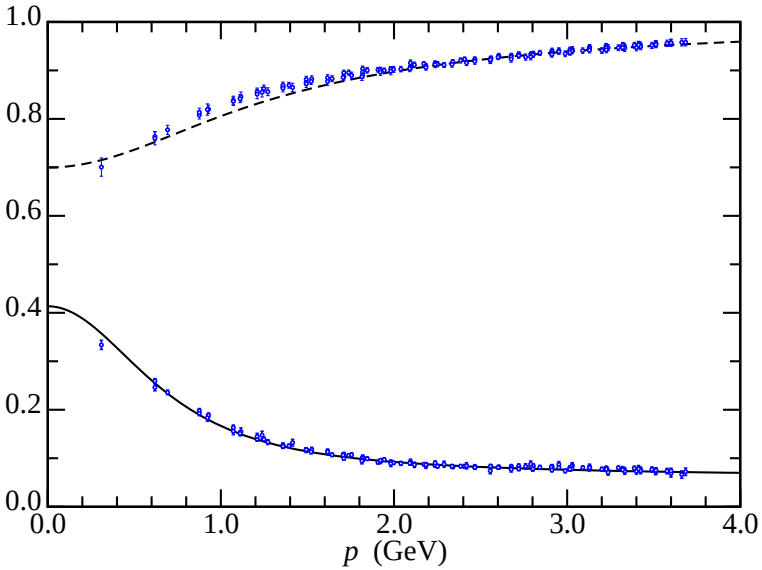


D.B., Buballa, Radzhabov, Volkov, in prep.



Berdermann, D.B., Scoccola, in prep.

COMPLEX MASS POLE FIT TO LATTICE PROPAGATOR



BHAGWAT, PICHOWSKY, ROBERTS, TANDY, PHYS. REV. **C68** (2003) 015203

$$S(p)^{-1} = i\not{p} A(p^2) + B(p^2) \,,$$

$$M(p^2) = B(p^2)/A(p^2)$$

$$Z(p^2) = 1/A(p^2)$$

$S(p)$ sum of N pairs of complex conj. mass poles

$$S(p) = \sum_{i=1}^{N} \frac{1}{Z_2} \left\{ \frac{z_i}{i\not{p} + m_i} + \frac{z_i^*}{i\not{p} + m_i^*} \right\} = -i\not{p}\sigma_V(p^2) + \sigma_S(p^2)$$

Representation of the scalar amplitude

$$\sigma_S(p^2) = \sum_{i=1}^{N} Z_2^{-1} \left\{ \frac{z_i\, m_i}{p^2 + m_i^2} + \frac{z_i^*\, m_i^*}{p^2 + m_i^{*2}} \right\}$$

"Derivation" of the equivalent separable model (in Feynman-like gauge) $D_{\mu\nu}(p-q) = \delta_{\mu\nu}\, D(p,q)$ and

$$\begin{aligned} D(p,q) &= f_0(p^2)\, f_0(q^2) + f_1(p^2)\, p\cdot q\, f_1(q^2) \\ f_1(p^2) &= \frac{A(p^2)-1}{a} \quad ; \quad f_0(p^2) = \frac{B(p^2)-m_c}{b} \end{aligned}$$

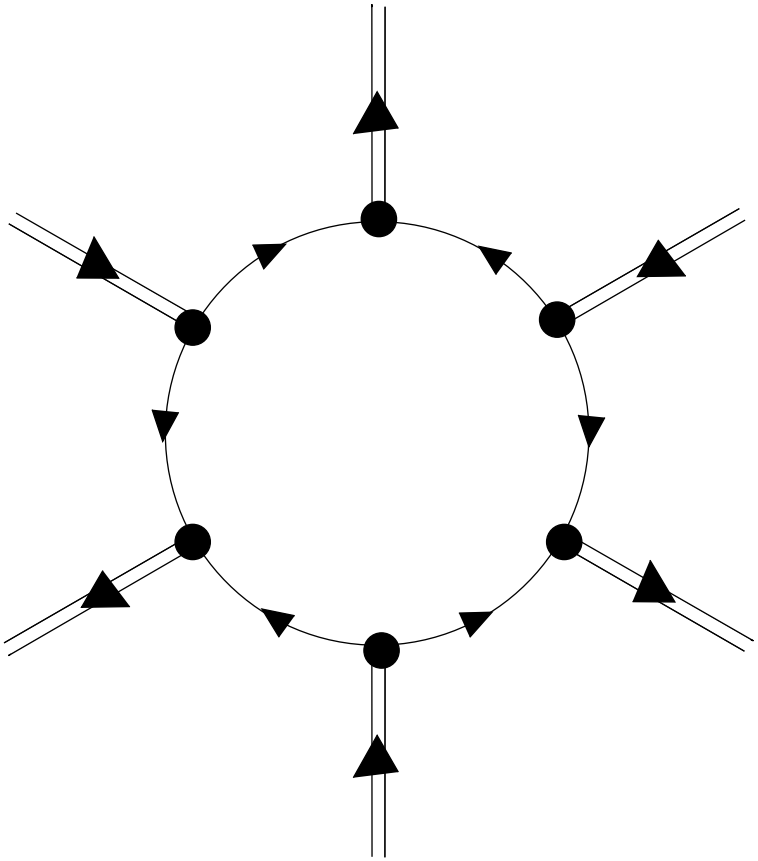
$$b^2 = \frac{16}{3} \int_q^{\Lambda} [B(q^2) - m_c]\sigma_s(q^2)$$

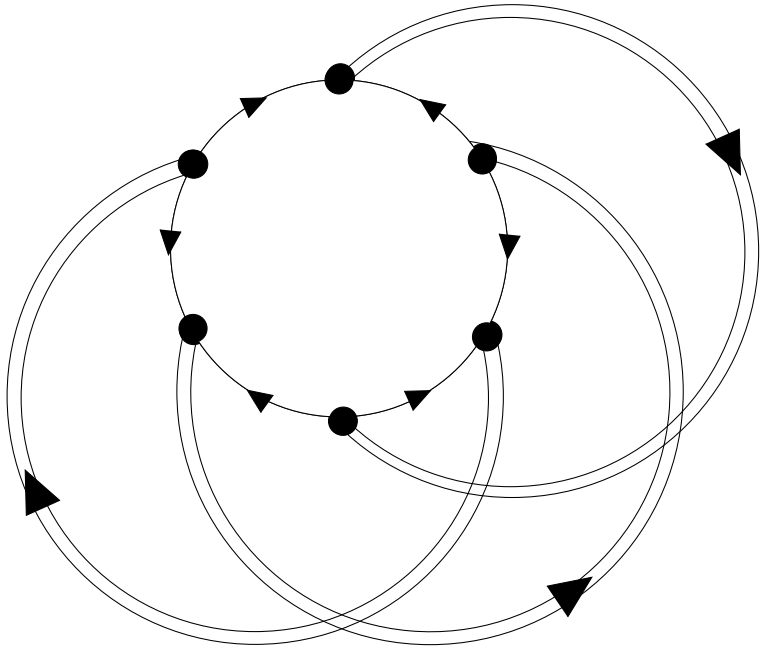
$$a^2 = \frac{8}{3} \int_q^{\Lambda} [A(q^2) - 1] \frac{q^2}{4} \sigma_v(q^2)$$

NUCLEONS IN THE NONLOCAL CHIRAL QUARK MODEL

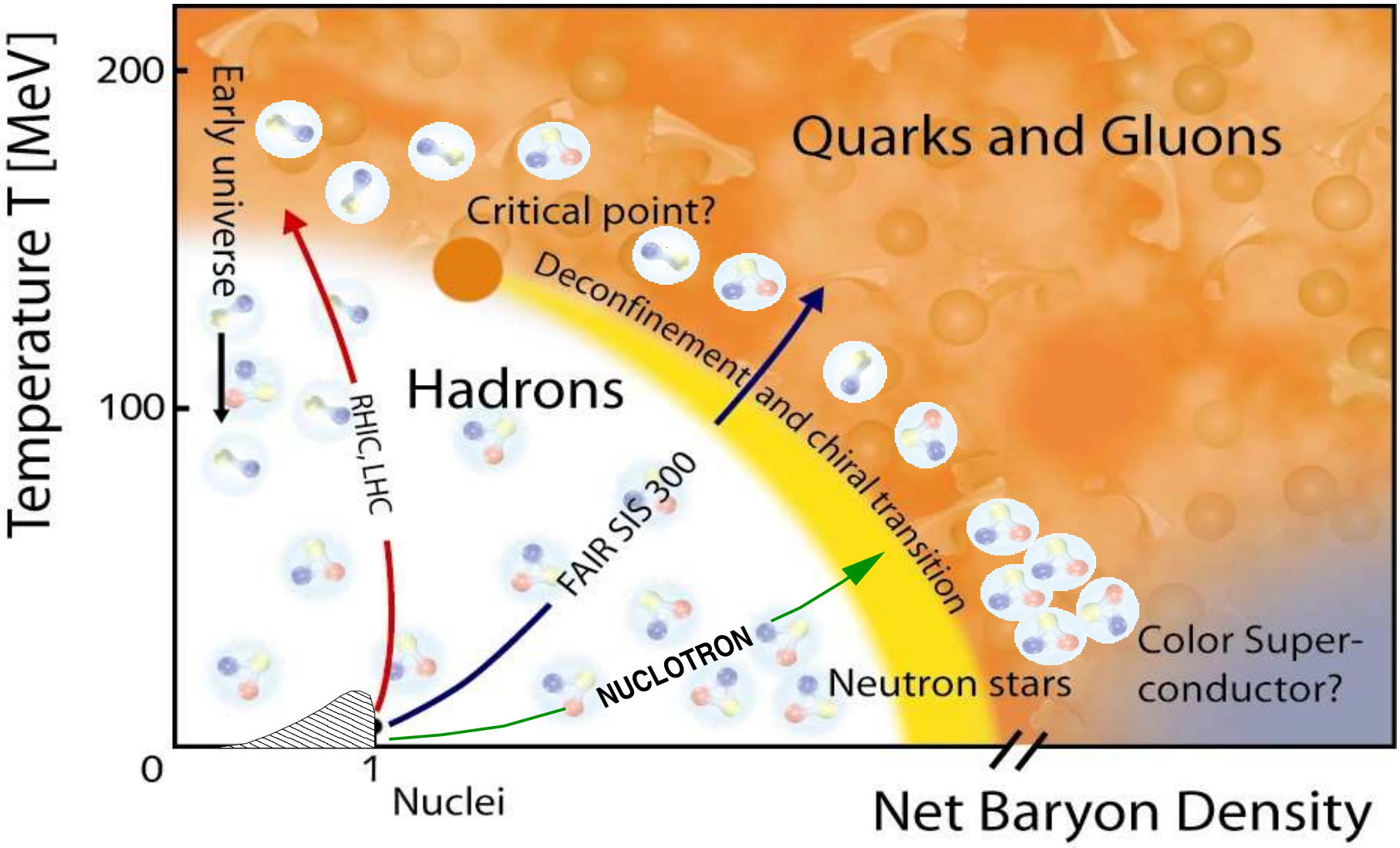
$$Z_{\rm fluct} = \int D\Delta^\dagger D\Delta \exp\{-\frac{|\Delta|^2}{4G_D} - Tr\ lnS^{-1}[\Delta, \Delta^\dagger]\}$$

Cahill, Roberts, Prashifka: Aust. J. Phys. 42 (1989) 129, 161
Cahill, ibid, 171; Reinhardt: PLB 244 (1990) 316; Buck, Alkofer, Reinhardt: PLB 286 (1992) 29

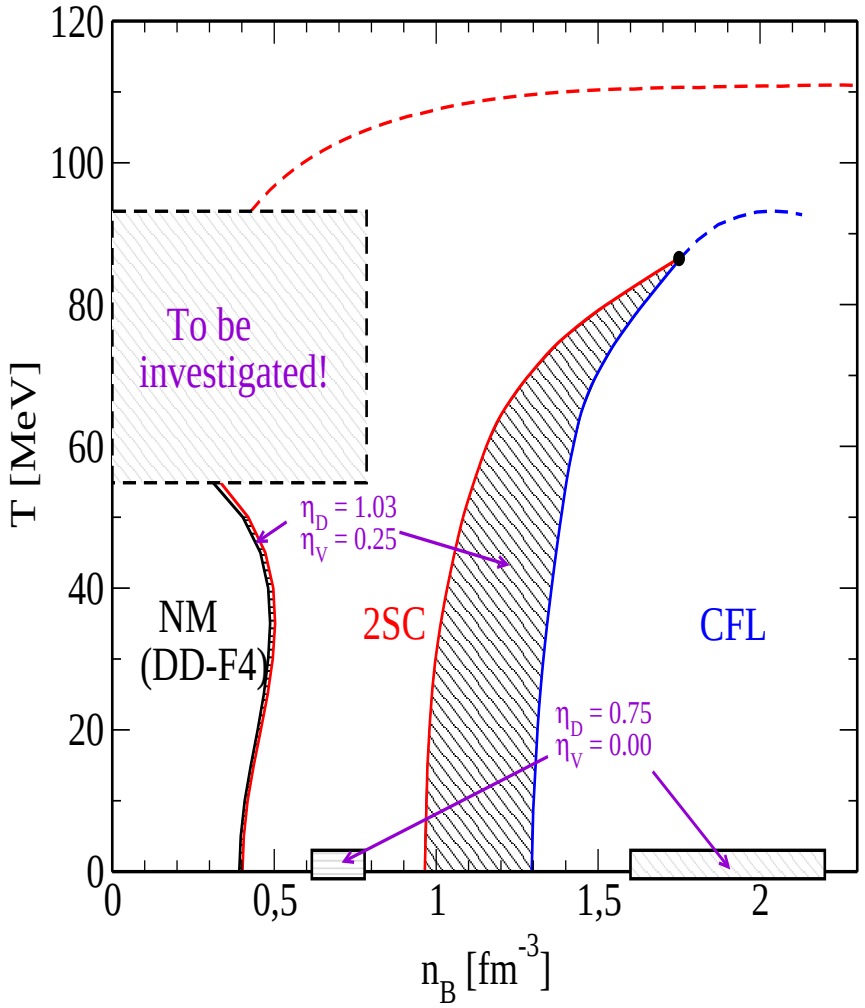




Quark Substructure and Phase Diagram



PHASE DIAGRAM FOR SYMMETRIC MATTER (HIC)



- Critical density for deconfi nement in heavy-ion collisions $n_{dec} \geq 3\ n_0$
- Almost crossover (masquerade!), i.e. no density jump, no latent heat/ time delay!
- 2SC - CFL phase transition at $n \geq 6\ n_0$ with density jump and latent heat/ time delay!
  Provided the temperature can be kept low $T \leq 70$ MeV

Klahn, Sandin, Typel, D.B., in preparation

SUMMARY

- Large neutron star masses: EoS at high densities needs to be stiff! QM - superconducting!
- Hybrid stars 'masquerade' as neutron stars: Cooling observables can tell!

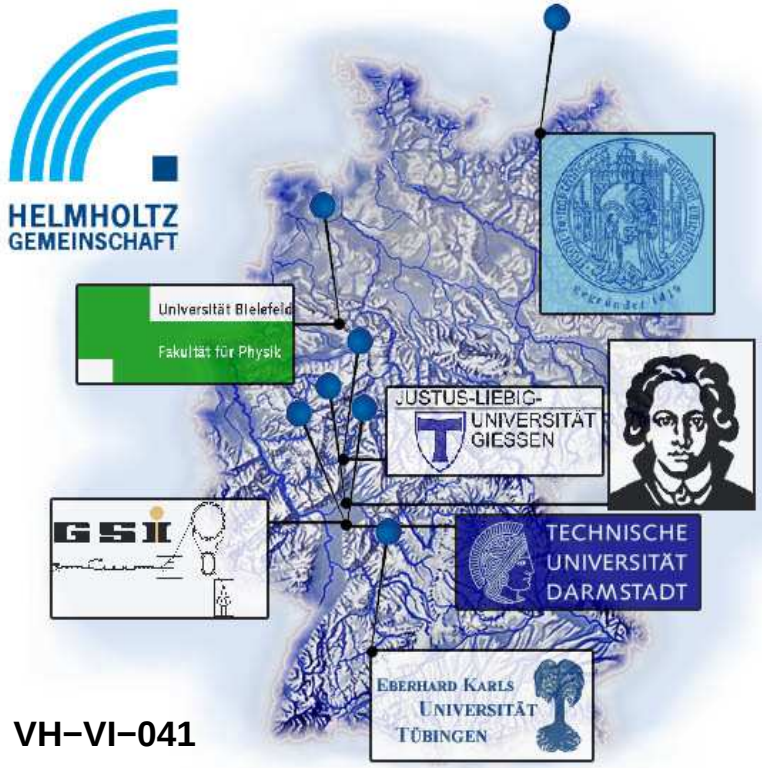
PLANS FOR THE FUTURE

- Mott effect in dense baryonic matter: nucleon dissociation!
- EoS for hot, dense matter with Mott-effect, encoded in hadronic spectral functions

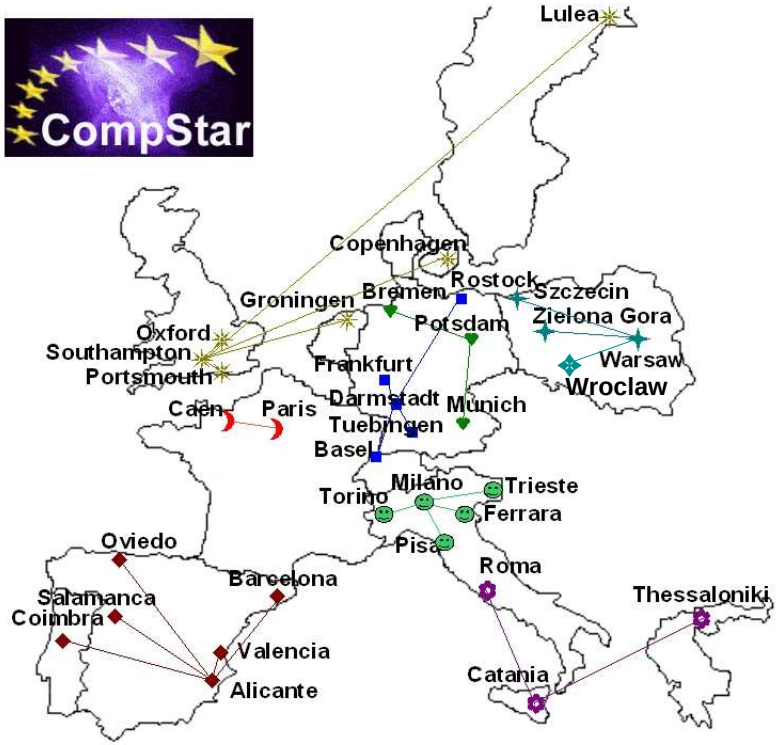
COLLABORATIONS

Virtual Institute of the Helmholtz−Association

Dense Hadronic Matter and QCD Phase Transition

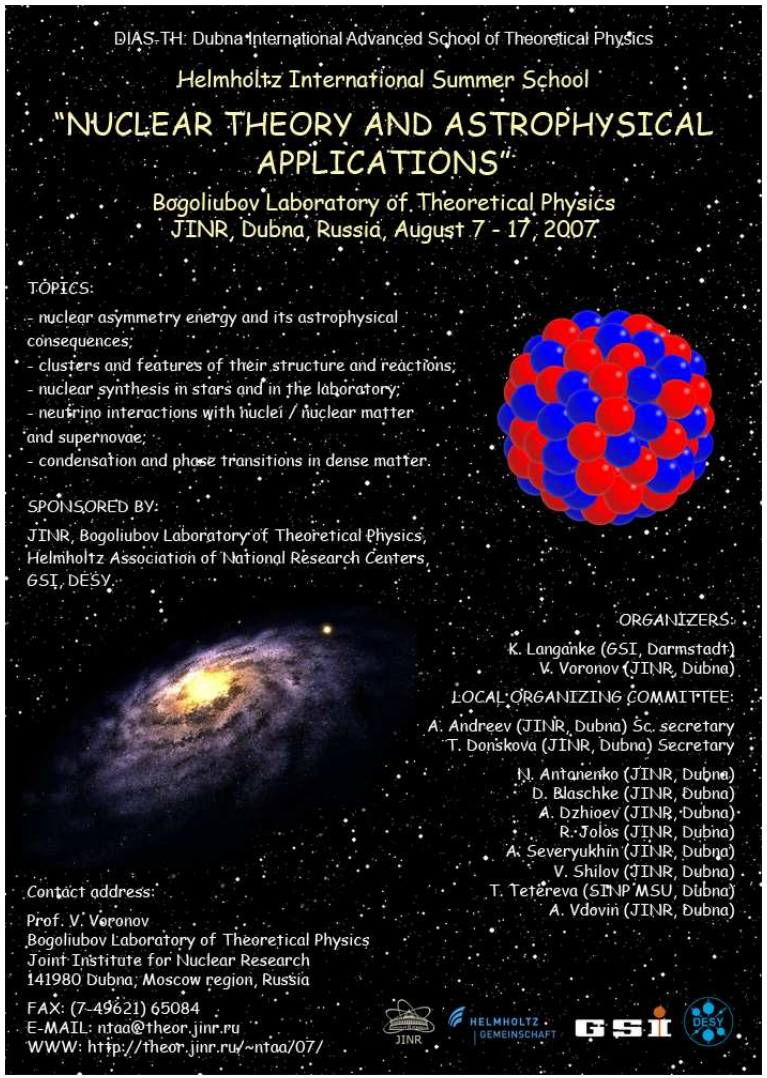


VH−VI−041



European Network Initiative

INVITATIONS ...



Helmholtz International Summer School

"Nuclear Structure and Astrophysical Applications",

Dubna, Russia, August 7 - 17, 2007

http://theor.jinr.ru/meetings/2007

ECT* Training Programme:

"Physics of Compact Stars"

Trento, Italy, August 20 - October 5, 2007

http://www.ect.it

ESF Research Training Network (2008-2012)

(under refereeing)

THANKS ...

... to the organizers !!

... and to the patient audience !