Quark propagator in the 2SC phase.

In the lecture, the inverse quark propagator in a two-flavor color superconductor (2SC) with massless quarks was given as

\[ S^{-1}(p) = \left( \begin{array}{cc} \hat{p} + \mu \gamma^0 & \Delta \gamma_5 \tau_2 \lambda_2 \\ \Delta^* \gamma_5 \tau_2 \lambda_2 & \hat{p} - \mu \gamma^0 \end{array} \right), \] (1)

where the explicit 2x2 structure corresponds to the Nambu-Gorkov components of \( S^{-1} \). Each of these components is a matrix in color, flavor, and Dirac space. As usual, \( \tau_2 \) is a Pauli matrix acting in flavor space and \( \lambda_2 \) is a Gell-Mann matrix, acting in color space. Altogether, \( S^{-1} \) is a 48x48 matrix.

1.1 Invert \( S^{-1}(p) \) to calculate the propagator

\[ S(p) = \left( \begin{array}{cc} S^+(p) & T^-(p) \\ T^+(p) & S^-(p) \end{array} \right), \] (2)

with the matrix elements

\[ S^+(p) = \frac{p^2 \hat{p}^+ - |\Delta|^2 \hat{p}^- \hat{\gamma}_5 \hat{\tau}_2 \hat{\lambda}_2}{D(p)} \hat{\gamma}_5 \hat{\tau}_2 \hat{\lambda}_2, \] (3)

\[ T^+(p) = -\frac{\hat{p}^- \hat{p}^+ - |\Delta|^2}{D(p)} \Delta^* \gamma_5 \tau_2 \lambda_2, \] (4)

\[ S^-(p) = \frac{p^2 \hat{p}^- - |\Delta|^2 \hat{p}^+ \hat{\gamma}_5 \hat{\tau}_2 \hat{\lambda}_2}{D(p)} \hat{\gamma}_5 \hat{\tau}_2 \hat{\lambda}_2, \] (5)

\[ T^-(p) = \frac{\hat{p}^+ \hat{p}^- - |\Delta|^2}{D(p)} - \Delta \gamma_5 \tau_2 \lambda_2, \] (6)

where\n
\[ p_\pm = \left( \begin{array}{c} p^0 \pm \mu \\ \vec{p} \end{array} \right), \] (8)

and for the denominator \( D(p) \) holds

\[ D(p) = p^2 p^2 - 2|\Delta|^2 p^- p^+ + |\Delta|^4. \] (9)

1.2 Show that the denominator function \( D(p) \) of Eq. (9) can be rewritten as

\[ D(p) = [p_0^2 - \omega_\pm^2 (\vec{p})][p_0^2 - \omega_\pm^2 (\vec{p})] \] (10)

with the gapped dispersion relations \( \omega_\pm(\vec{p}) = \sqrt{(|\vec{p}| \pm \mu)^2 + |\Delta|^2}. \)