Modern Problems of Nuclear Physics I - Exercise sheet 2 - 11.11.2019

Quark propagator in the 2SC phase.
In the lecture, the inverse quark propagator in a two-flavor color superconductor (2SC) with massless quarks was given as

$$S^{-1}(p) = \begin{pmatrix} \not{p} + \mu\gamma^0 & \Delta\gamma_5\tau_2\lambda_2 \\ -\Delta^*\gamma_5\tau_2\lambda_2 & \not{p} - \mu\gamma^0 \end{pmatrix} , \tag{1}$$

where the explicit 2x2 structure corresponds to the Nambu-Gorkov components of S^{-1}. Each of these components is a matrix in color, flavor, and Dirac space. As usual, τ_2 is a Pauli matrix acting in flavor space and λ_2 is a Gell-Mann matrix, acting in color space. Altogether, S^{-1} is a 48x48 matrix.

1.1 Invert $S^{-1}(p)$ to calculate the propagator

$$S(p) = \begin{pmatrix} S^+(p) & T^-(p) \\ T^+(p) & S^-(p) \end{pmatrix} , \tag{2}$$

with the matrix elements

$$S^+(p) = \frac{p_-^2\not{p}_+ - |\Delta|^2\not{p}_-}{D(p)}\hat{P}_{rg} + \frac{\not{p}_+}{p_+^2}\hat{P}_b , \tag{3}$$

$$T^+(p) = -\frac{\not{p}_-\not{p}_+ - |\Delta|^2}{D(p)}\Delta^*\gamma_5\tau_2\lambda_2 , \tag{4}$$

$$S^-(p) = \frac{p_+^2\not{p}_- - |\Delta|^2\not{p}_+}{D(p)}\hat{P}_{rg} + \frac{\not{p}_-}{p_-^2}\hat{P}_b , \tag{5}$$

$$T^-(p) = \frac{\not{p}_+\not{p}_- - |\Delta|^2}{D(p)}\Delta\gamma_5\tau_2\lambda_2 , \tag{6}$$

$$\tag{7}$$

where

$$p_\pm = \begin{pmatrix} p^0 \pm \mu \\ \vec{p} \end{pmatrix} , \tag{8}$$

and for the denominator $D(p)$ holds

$$D(p) = p_-^2 p_+^2 - 2|\Delta|^2 p_- p_+ + |\Delta|^4 . \tag{9}$$

1.2 Show that the denominator function $D(p)$ of Eq. (9) can be rewritten as

$$D(p) = [p_0^2 - \omega_-^2(\vec{p})][p_0^2 - \omega_+^2(\vec{p})] \tag{10}$$

with the gapped dispersion relations $\omega_\pm(\vec{p}) = \sqrt{(|\vec{p}| \pm \mu)^2 + |\Delta|^2}$.