Neutrino emissivities and bulk viscosity
in neutral two-flavor quark matter

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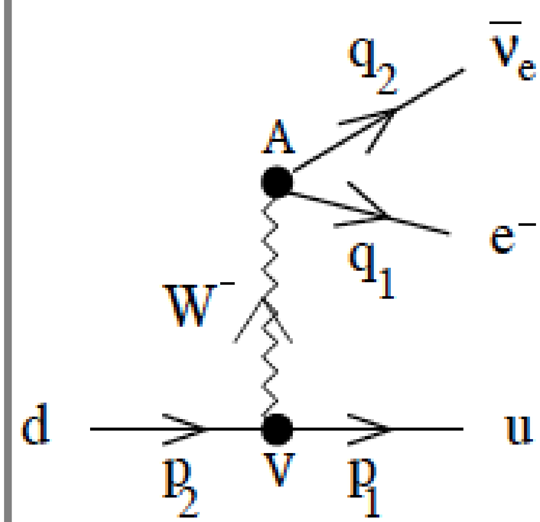
Direct URCA process:



1. Direct URCA process → Emissivity
2. Kinetic Equation → Kadanoff-Baym formulation
3. Color superconductivity → Nambu-Gorkov formalism
4. Bulk viscosity & application to 2SC + CSL matter

Based on: J. Berdermann et al., Phys. Rev. D94, 123010 (2016)
arXiv:1609.05201 [astro-ph.HE]

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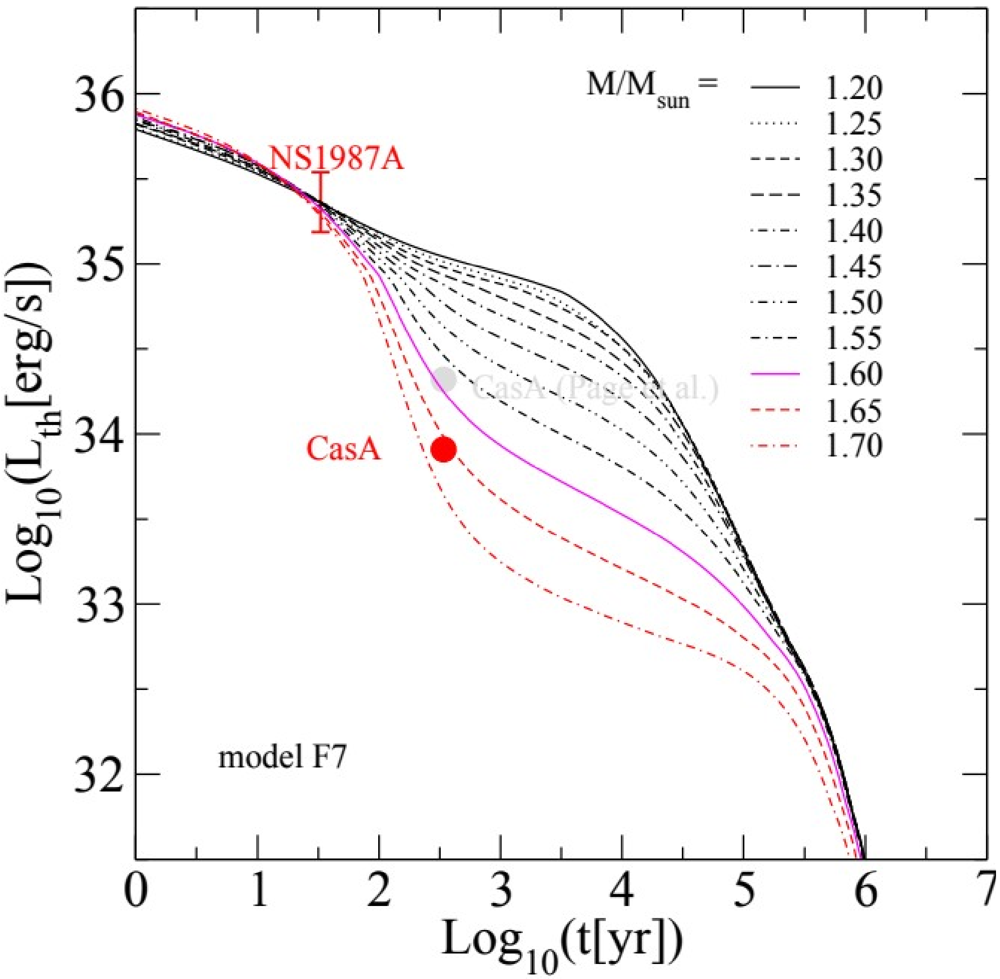
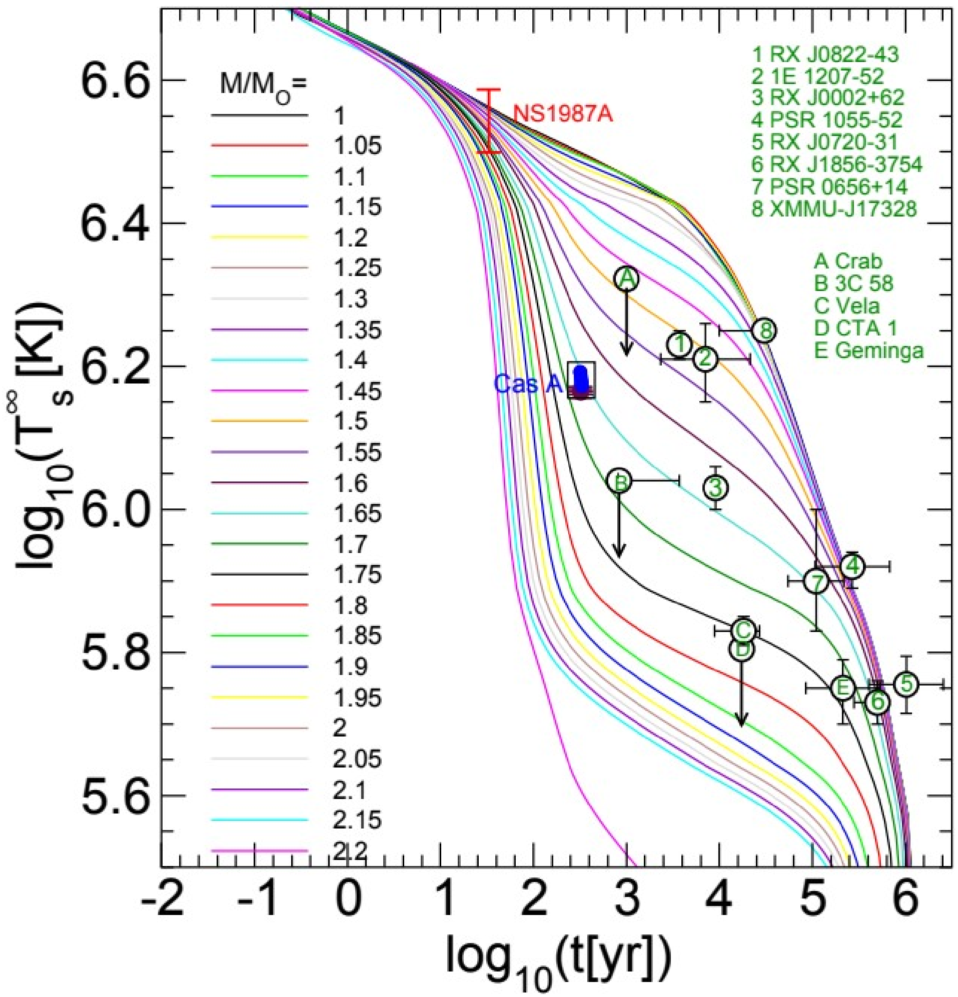
Motivation: Cooling of Neutron Stars – hot actual topic !!

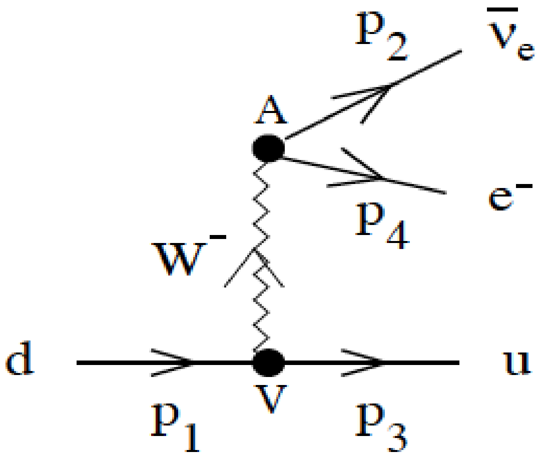
Photon luminosity for the neutron star in supernova SN1987A
[Cigan et al., arxiv:1910.02960; Page et al., arxiv:2004.06078]



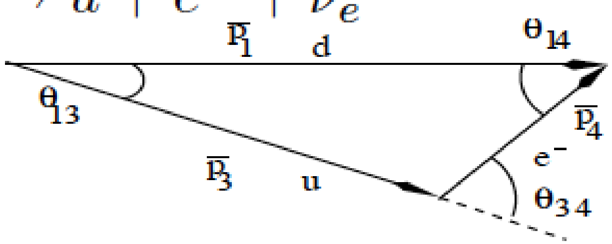


D. Blaschke, H. Grigorian and D.N. Voskresensky, in preparation (2020)

1. Direct URCA process emissivity: Synopsis



$$d \longrightarrow u + e^- + \bar{\nu}_e$$



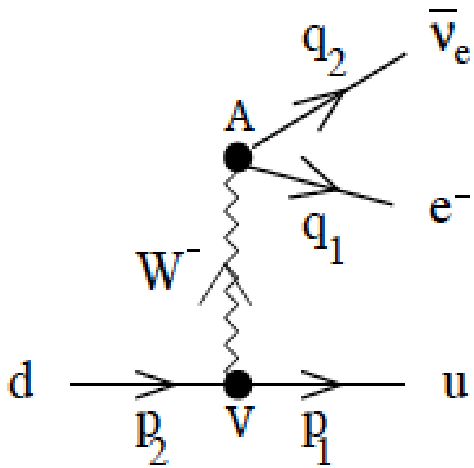
$$\begin{aligned}
\varepsilon &= 6 \prod_{i=1}^{4} \int \frac{d^3 p_i}{(2\pi)^3\, 2E_i}\, E_2\, W_{fi}\, n(\mathbf{p_1})[1-n(\mathbf{p_3})][1-n(\mathbf{p_4})] \\
W_{fi} &= (2\pi)^4 \delta^{(4)}(p_1 - p_2 - p_3 - p_4) \mid M \mid^2 \\
\mid M \mid^2 &\equiv \frac{1}{2} \sum_{\sigma_d, \sigma_u, \sigma_{e^-}} \mid M_{fi} \mid^2 = 64\, G^2 \cos^2\theta_c\, (p_1 \cdot p_2)\, (p_3 \cdot p_4) \\
&= 64\, G^2 \cos^2\theta_c\, E_1 E_2 E_3 E_4 (1 - \cos\theta_{34})
\end{aligned}$$

$$\begin{aligned}
\mu_i &= p_F^i[1 + (2/3\pi)\alpha_s], \qquad i = u, d \\
\mu_i &\simeq p_F^i[1 + 0.5(m_i/p_F^i)^2], \qquad i = u, d, e
\end{aligned}$$

$$\begin{aligned}
p_F^d - p_F^u - p_F^e &\simeq -\frac{1}{2} p_F^e \theta_{14}^2, \qquad \theta_{14} \simeq \theta_{34} \\
p_F^d - p_F^u - p_F^e &\simeq -p_F^e (2/3\pi)\alpha_s, \qquad \alpha_s = g^2/4 \\
p_F^d - p_F^u - p_F^e &\simeq -\frac{1}{2} p_F^e \left[\left(\frac{p_F^d}{p_F^e}\right) \left(\frac{m_d}{p_F^d}\right)^2 - \left(\frac{p_F^u}{p_F^e}\right) \left(\frac{m_u}{p_F^u}\right)^2 - \left(\frac{m_e}{p_F^e}\right)^2 \right]
\end{aligned}$$

$$\begin{aligned}
\varepsilon^{\alpha_s} &\simeq (457/630)\, G^2 \cos^2\theta_c\, \alpha_s\, p_F^d\, p_F^u\, p_F^e\, T^6 \\
\varepsilon^{m} &\simeq (457\pi/1680)\, G^2 \cos^2\theta_c\, m_d^2\, f\, p_F^u\, T^6, f \equiv 1 - (m_u/m_d)^2 (p_F^d/p_F^u) - (m_e/m_d)^2 (p_F^d/p_F^e)
\end{aligned}$$

Evaluation of Feynman diagram for direct URCA process



Vertices:

$$-i\frac{g}{\sqrt{2}}\gamma^\mu \quad \text{für vectorboson}$$

$$-i\frac{g}{\sqrt{2}}\gamma^\mu\gamma_5 \quad \text{für axialvector boson}$$

Leptons:

$$\bar{l}[\gamma^\mu \frac{1}{2}(1-\gamma_5)]\nu \;=\; \frac{1}{2}\,\bar{l}\;\hat{o}^\mu\;\nu$$

Quarks:

$$\bar{d}[\gamma_\mu \frac{1}{2}(g_V + g_A\gamma_5)]u \;=\; \frac{1}{2}\,\bar{d}\;\breve{o}_\mu\;u$$

$$\boxed{-(\frac{g^2}{8})^2[\bar{l}(\gamma_\nu + \gamma_\nu\gamma_5)\nu][\bar{d}(G_V\gamma_\mu + G_A\gamma_\mu\gamma_5)u]}$$

Fermion propagator:

$$G^0 \;=\; \frac{i}{\not{p}-m} = \frac{i(\not{p}+m)}{p_0^2 - \mathrm{p}^2 - m^2}$$

Gauge boson propagator:

$$D^0 \;=\; \frac{i}{p_0^2 - \mathrm{p}^2 - M_W^2} \qquad M_W \;=\; 80.3\ \mathrm{GeV}$$

Squared matrix element --> integration over closed line momenta, trace over Dirac indices:

$$\frac{1}{M_W^4}\sum_{\mathrm{Spins}}\int \frac{d^3q_1}{(2\pi)^3\,2E_e}\int \frac{d^3q_2\,E_{\bar{\nu}}}{(2\pi)^3\,2E_{\bar{\nu}}}\int \frac{d^3p_1}{(2\pi)^3\,2E_u}\int \frac{d^3p_2}{(2\pi)^3\,2E_d}$$

Multiply with neutrino energy for energy loss!

4-momentum conservation: $\boxed{(2\pi)^4\delta^{(4)}(p_2 - q_2 - p_1 - q_1)}$ Factor: $(-1)^L(2s+1)$

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$$
\begin{aligned}
\epsilon_{\bar{\nu}} &= 6\left(\frac{g^2}{8M_W^2}\right)^2 \sum_{\text{Spins}} \int \frac{d^3p_2}{(2\pi)^3\, 2E_d} \int \frac{d^3q_2\; E_{\bar{\nu}}}{(2\pi)^3\, 2E_{\bar{\nu}}} \int \frac{d^3p_1}{(2\pi)^3\, 2E_u} \int \frac{d^3q_1}{(2\pi)^3\, 2E_e} \\
&\times\ (2\pi)^4 \delta^4(p_2 - q_2 - p_1 - q_1)[\bar{l}(\gamma_\nu + \gamma_\nu\gamma_5)\nu][\bar{d}(G_V\gamma_\mu + G_A\gamma_\mu\gamma_5)u] \\
&\times\ f(\mathbf{p_2})[1 - f(\mathbf{p_1})][1 - f(\mathbf{q_1})].
\end{aligned}
$$

statistical factor $S = f(\mathbf{p_2})[1 - f(\mathbf{p_1})][1 - f(\mathbf{q_1})]$; neutrino blocking $[1 - f(\mathbf{q_2})]$ is neglected

Spin sums --> separate leptonic and hadronic (quark) part:

$$
|M|^2 = \sum_{\text{Spins}} |M_{fi}|^2 = \sum_{\text{Spins}} \left(\frac{G}{\sqrt{2}}\right)^2 (l^\mu h_\mu)(l^\nu h_\nu)^\dagger = \sum_{\text{Spins}} \left(\frac{G}{\sqrt{2}}\right)^2 (l^\mu l^{\nu\dagger})(h_\mu h_{\nu\dagger}) = \sum_{\text{Spins}} \left(\frac{G}{\sqrt{2}}\right)^2 l^{\mu\nu} h_{\mu\nu}
$$

Leptonic part:

$$
l^{\mu\nu} = \mathrm{Tr}[\hat{o}^\mu(\not{q}_1 + m_e)\hat{o}^\nu(\not{q}_2 + m_{\bar{\nu}})] \text{ ,where } m_e, m_\nu = 0
$$

Hadronic part:

$$
h_{\mu\nu} = \mathrm{Tr}[\breve{o}_\mu(\not{p}_1 + m_u)\breve{o}_\nu(\not{p}_2 + m_d)] = \mathrm{Tr}[(\gamma_\mu + g_A\gamma_\mu\gamma_5)(\not{p}_1 + m_u)(\gamma_\nu + g_A\gamma_\nu\gamma_5)(\not{p}_2 + m_d)]
$$

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A: Dirac trace over leptonic tensor part:

$$
\begin{aligned}
\mathrm{Tr}[\gamma^\mu(1-\gamma_5)\not{q}_1\gamma^\nu(1-\gamma_5)\not{q}_2] &= \mathrm{Tr}[\gamma^\mu(1-\gamma_5)\not{q}_1(1+\gamma_5)\gamma^\nu\not{q}_2] \\
&= \mathrm{Tr}[\gamma^\mu(1-\gamma_5)(1-\gamma_5)\not{q}_1\gamma^\nu\not{q}_2] \\
&= 2\mathrm{Tr}[\gamma^\mu(1-\gamma_5)\not{q}_1\gamma^\nu\not{q}_2] \\
&= 2[\mathrm{Tr}(\gamma^\mu\not{q}_1\gamma^\nu\not{q}_2)+\mathrm{Tr}(\gamma_5\gamma^\mu\not{q}_1\gamma^\nu\not{q}_2)] \\
&= 2[\mathrm{Tr}(\gamma^\mu\gamma^\alpha q_{1\alpha}\gamma^\nu\gamma^\beta q_{2\beta})+\mathrm{Tr}(\gamma_5\gamma^\mu\gamma^\alpha q_{1\alpha}\gamma^\nu\gamma^\beta q_{2\beta})] \\
&= 8[g^{\mu\alpha}g^{\nu\beta}q_{1\alpha}q_{2\beta}-g^{\mu\nu}g^{\alpha\beta}q_{1\alpha}q_{2\beta}+g^{\mu\beta}g^{\alpha\nu}q_{1\alpha}q_{2\beta}-i\varepsilon^{\mu\alpha\nu\beta}q_{1\alpha}q_{2\beta}] \\
&= 8[q_1^\mu q_2^\nu-g^{\mu\nu}(q_1\cdot q_2)+q_1^\nu q_2^\mu-i\varepsilon^{\mu\alpha\nu\beta}q_{1\alpha}q_{2\beta}]
\end{aligned}
$$

Neutrino emissivity for direct URCA process

B: Dirac trace over hadronic (quark) tensor part:

$$h_{\mu\nu} = \mathrm{Tr}[\breve{o}_\mu(\not{p}_1 + m_u)\breve{o}_\nu(\not{p}_2 + m_d)] = \mathrm{Tr}[(\gamma_\mu + g_A\gamma_\mu\gamma_5)(\not{p}_1 + m_u)(\gamma_\nu + g_A\gamma_\nu\gamma_5)(\not{p}_2 + m_d)]$$

$$= \mathrm{Tr}[\gamma_\mu(\not{p}_1 + m_u)\gamma_\nu(\not{p}_2 + m_d) + g_A^2\gamma_\mu\gamma_5(\not{p}_1 + m_u)\gamma_\nu\gamma_5(\not{p}_2 + m_d)] +$$
$$\mathrm{Tr}\, g_A[\gamma_\mu(\not{p}_1 + m_u)\gamma_\nu\gamma_5(\not{p}_2 + m_d) + \gamma_\mu\gamma_5(\not{p}_1 + m_u)\gamma_\nu(\not{p}_2 + m_d)].$$

Upper line describes decoupled vector and axialvector contributions:

$$\begin{aligned}
& \mathrm{Tr}\gamma_\mu\gamma_\alpha\gamma_\nu\gamma_\beta p_1^\alpha p_2^\beta + \mathrm{Tr}\gamma_\mu\gamma_\nu m_u m_d + \mathrm{Tr}\gamma_\mu\gamma_\alpha\gamma_\nu p_1^\alpha m_d \\
+ \quad & \mathrm{Tr}\gamma_\mu\gamma_\nu\gamma_\beta m_u p_2^\beta + g_A^2[\mathrm{Tr}\gamma_\mu\gamma_5\gamma_\alpha\gamma_\nu\gamma_5\gamma_\beta p_1^\alpha p_2^\beta + \mathrm{Tr}\gamma_\mu\gamma_5\gamma_\nu\gamma_5 m_u m_d \\
+ \quad & \mathrm{Tr}\gamma_\mu\gamma_5\gamma_\alpha\gamma_\nu\gamma_5 p_1^\alpha m_d + \mathrm{Tr}\gamma_\mu\gamma_5\gamma_\nu\gamma_5\gamma_\beta m_u p_2^\beta] \\
= \quad & \mathrm{Tr}\gamma_\mu\gamma_\alpha\gamma_\nu\gamma_\beta p_1^\alpha p_2^\beta + \mathrm{Tr}\gamma_\mu\gamma_\nu m_u m_d + g_A^2\mathrm{Tr}\gamma_\mu\gamma_\alpha\gamma_\nu\gamma_\beta p_1^\alpha p_2^\beta - g_A^2\mathrm{Tr}\gamma_\mu\gamma_\nu m_u m_d
\end{aligned}$$

Lower line describes the mixing of vector and axialvector contributions:

$$\begin{aligned}
& g_A\mathrm{Tr}[\gamma_5\gamma_\beta\gamma_\mu\gamma_\alpha\gamma_\nu p_1^\alpha p_2^\beta + \gamma_5\gamma_\beta\gamma_\mu\gamma_\nu p_2^\beta m_u + \gamma_5\gamma_\mu\gamma_\alpha\gamma_\nu p_1^\alpha m_d + \gamma_5\gamma_\mu\gamma_\nu m_u m_d \\
- \quad & (\gamma_5\gamma_\mu\gamma_\alpha\gamma_\nu\gamma_\beta p_1^\alpha p_2^\beta + \gamma_5\gamma_\mu\gamma_\alpha\gamma_\nu p_1^\alpha m_d + \gamma_5\gamma_\mu\gamma_\nu\gamma_\beta p_2^\beta m_u + \gamma_5\gamma_\mu\gamma_\nu m_u m_d)] \\
= \quad & g_A\mathrm{Tr}(\gamma_5\gamma_\beta\gamma_\mu\gamma_\alpha\gamma_\nu)p_1^\alpha p_2^\beta - \mathrm{Tr}(\gamma_\beta\gamma_5\gamma_\mu\gamma_\alpha\gamma_\nu)p_1^\alpha p_2^\beta \\
= \quad & 2g_A\mathrm{Tr}(\gamma_5\gamma_\beta\gamma_\mu\gamma_\alpha\gamma_\nu)p_1^\alpha p_2^\beta.
\end{aligned}$$

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Both contributions together, applying Dirac trace rules:

$$
\begin{aligned}
& (1+g_A^2)\mathrm{T}\gamma_\mu\gamma_\alpha\gamma_\nu\gamma_\beta p_1^\alpha p_2^\beta + (1-g_A^2)\mathrm{Tr}\gamma_\mu\gamma_\nu m_u m_d + 2g_A\mathrm{Tr}(\gamma_5\gamma_\beta\gamma_\mu\gamma_\alpha\gamma_\nu)p_1^\alpha p_2^\beta \\
= \;& 4[(1+g_A^2)(g_{\mu\alpha}g_{\nu\beta}p_1^\alpha p_2^\beta - g_{\mu\nu}g_{\alpha\beta}p_1^\alpha p_2^\beta + g_{\mu\beta}g_{\alpha\nu}p_1^\alpha p_2^\beta) + (1-g_A^2)g_{\mu\nu}m_u m_d - 2ig_A\varepsilon_{\beta\mu\alpha\nu}p_1^\alpha p_2^\beta] \\
= \;& 4[(1+g_A^2)(p_{1\mu}p_{2\nu} - g_{\mu\nu}p_1p_2 + p_{1\nu}p_{2\mu}) + (1-g_A^2)g_{\mu\nu}m_u m_d - 2ig_A\varepsilon_{\beta\mu\alpha\nu}p_1^\alpha p_2^\beta].
\end{aligned}
$$

Multiplying leptonic (A) and hadronic (B) tensors and contract Dirac indices:

$$
\begin{aligned}
& 8[q_1^\mu q_2^\nu - g^{\mu\nu}q_1q_2 + q_1^\nu q_2^\mu - i\varepsilon^{\mu\alpha\nu\beta}q_{1\alpha}q_{2\beta}] \\
\times \;& 4[(1+g_A^2)(p_{1\mu}p_{2\nu} - g_{\mu\nu}p_1p_2 + p_{1\nu}p_{2\mu}) + (1-g_A^2)g_{\mu\nu}m_u m_d - 2ig_A\varepsilon_{\beta\mu\alpha\nu}p_1^\alpha p_2^\beta] \\
= \;& 32[(1+g_A^2)\{(q_1\cdot p_1)(q_2\cdot p_2) - (q_1\cdot q_2)(p_1\cdot p_2) + (q_1\cdot p_2)(q_2\cdot p_1) + (q_1\cdot q_2)(p_1\cdot p_2) \\
+ \;& 4(q_1\cdot q_2)(p_1\cdot p_2) - (q_1\cdot q_2)(p_1\cdot p_2) + (q_1\cdot p_2)(q_2\cdot p_1) - (q_1\cdot q_2)(p_1\cdot p_2) \\
+ \;& (q_1\cdot p_1)(q_2\cdot p_2) - i\varepsilon^{\mu\alpha\nu\beta}(p_{1\mu}p_{2\nu} - p_{1\nu}p_{2\mu})q_{1\alpha}q_{2\beta} + i\varepsilon_{\mu\alpha\nu\beta}q_{1\alpha}q_{2\beta}g_{\mu\nu}(p_1p_2) \\
+ \;& (1-g_A^2)\{(q_1\cdot q_2)m_u m_d - 4(q_1\cdot q_2)m_u m_d + (q_1\cdot q_2)m_u m_d - i\varepsilon^{\mu\alpha\nu\beta}q_{1\alpha}q_{2\beta}g_{\mu\nu}m_u m_d\} \\
- \;& 2g_A\{i\varepsilon_{\beta\mu\alpha\nu}(q_1^\mu q_2^{nu} - q_1^\nu q_2^\mu)p_1^\alpha p_2^\beta - i\varepsilon_{\mu\alpha\nu\beta}g^{\mu\nu}p_1^\alpha p_2^\beta(q_1\cdot q_2) + \varepsilon_{\mu\alpha\nu\beta}\varepsilon^{\mu\alpha\nu\beta}p_1^\alpha p_2^\beta q_{1\alpha}q_{2\beta}\} \\
= \;& 32[(1+g_A^2)\{2(q_1\cdot p_1)(q_2\cdot p_2) + 2(q_1\cdot p_2)(q_2\cdot p_1)\} - (1-g_A^2)2(q_1\cdot q_2)m_u m_d \\
- \;& 2g_A\varepsilon_{\mu\alpha\nu\beta}\varepsilon^{\mu\alpha\nu\beta}p_1^\alpha p_2^\beta q_{1\alpha}q_{2\beta}]
\end{aligned}
$$

Neutrino emissivity for direct URCA process

Nonrelativistic limit:

In this case is $p_1, p_2 \longrightarrow m_u, m_d$ and $q_1, q_2 \longrightarrow q_1^0, q_2^0 \longrightarrow E_e, E_\nu$ with the consequence of two zero leptonic components. The Levi-Civita symbol has therefore to identical indices and vanishes. The mixed term did not contribute in the nonrelativistic case. Furthermore one has:

$$\begin{aligned} (p_1 \cdot q_1) &= m_d\ q_1^0 = m_d\ E_e \qquad (p_1 \cdot q_2) = m_d\ q_2^0 = m_d\ E_\nu \\ (p_2 \cdot q_1) &= m_u\ q_1^0 = m_u\ E_e \qquad (p_2 \cdot q_2) = m_u\ q_2^0 = m_u\ E_\nu, \end{aligned}$$

with the result:

$$\begin{aligned} |M|^2 &= 64[(1+g_A^2)E_eE_\nu m_d m_u + (1+g_A^2)E_eE_\nu m_d m_u + (-1+g_A^2)E_eE_\nu m_d m_u] \\ &= 64[1+3g_A^2]E_eE_\nu m_d m_u. \end{aligned}$$

Ultrarelativistic limit:

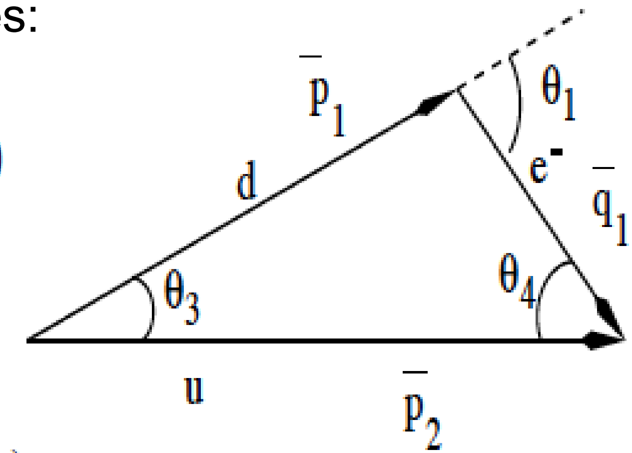
$$\begin{aligned} &= 64[(1+g_A^2)\{(q_1 \cdot p_1)(q_2 \cdot p_2) + (q_1 \cdot p_2)(q_2 \cdot p_1)\} - g_A \varepsilon_{\alpha\beta\mu\nu}\varepsilon^{\bar{\alpha}\bar{\beta}\mu\nu} p_1^\alpha p_2^\beta q_{1\alpha} q_{2\beta}] \\ &= 64\{(1+g_A^2)\{(q_1 \cdot p_1)(q_2 \cdot p_2) + (q_1 \cdot p_2)(q_2 \cdot p_1)\} - 2g_A(\delta_\beta^{\bar{\alpha}}\delta_\alpha^{\bar{\beta}} - \delta_\alpha^{\bar{\alpha}}\delta_\beta^{\bar{\beta}})p_1^\alpha p_2^\beta q_{1\alpha} q_{2\beta}] \\ &= 64[(1+g_A^2)\{(q_1 \cdot p_1)(q_2 \cdot p_2) + (q_1 \cdot p_2)(q_2 \cdot p_1)\} - 2g_A\{(q_1 \cdot p_2)(q_2 \cdot p_1) - (q_1 \cdot p_1)(q_2 \cdot p_2)\}] \\ &= 64[(1+g_A^2+2g_A)(q_1 \cdot p_1)(q_2 \cdot p_2) + (1+g_A^2-2g_A)(q_1 \cdot p_2)(q_2 \cdot p_1)] \\ &= 64[(1+g_A)^2(q_1 \cdot p_1)(q_2 \cdot p_2) + (1-g_A)^2(q_1 \cdot p_2)(q_2 \cdot p_1)]. \end{aligned}$$

Neglect $(1-g_A)^2 \sim 0.048$ against $(1+g_A)^2 \sim 4.92$, obtain:

$$|M|^2 = 64\ G^2\ \cos^2\ \theta_C(q_1 \cdot p_1)(q_2 \cdot p_2)$$

Neutrino emissivity for direct URCA process

Representation with momentum vectors and enclosed angles:



$$|M|^2 = 64\, G^2 \cos^2\theta_C\; E_e E_u E_{\bar\nu} E_d (1 - \frac{\mathbf{q_2 p_2}}{E_{\bar\nu} E_d}\cos\theta_2)(1 - \frac{\mathbf{q_1 p_1}}{E_e E_u}\cos\theta_1)$$

Nonvanishing for low quark mass and perturbative corr.:

$$\mu_i = p_F(i)\left[1 + \frac{1}{2}\left(\frac{m^2}{p_F}\right)\right] = \left(1 + \frac{8}{3\pi}a_c\right)p_F(i) \qquad i = \mathrm{u, d}$$

$$(a_c = g^2/16\pi)$$

Beta-equilibrium: $\mu_d = \mu_u + \mu_e.$

$$p_F(d) - p_F(u) - p_F(e) \simeq -(8/3\pi)\,\alpha_c\, p_F(e)$$

$$\begin{aligned} p_F(u)\cos\theta_3 + p_F(e)\cos\theta_4 &= p_F(d) \\ p_F(u)\sin\theta_3 &= p_F(e)\sin\theta_4, \end{aligned}$$

$$\cos\theta_3 = \frac{\mathbf{q_1 \cdot p_2}}{|q_1||p_2|} \text{ and } \cos\theta_4 = \frac{\mathbf{p_2 \cdot p_1}}{|p_2||p_1|} \qquad \theta_3, \theta_4 \ll 1$$

$$p_F(d) - p_F(u) - p_F(e) \simeq -\frac{1}{2}p_F(e)\theta_4^2. \longrightarrow \theta_4^2 \simeq (16/3\pi)\alpha_c$$

$$(q_1 \cdot p_1)(q_2 \cdot p_2) = E_e E_u E_{\bar\nu} E_d [1 - \frac{\mathbf{p_2}}{E_d}\cos\theta_2](16\frac{\alpha_s}{3\pi})$$

$$\boxed{\begin{aligned} \epsilon_{\bar\nu} &= 24\, G^2 \cos^2\theta_C \int \frac{d^3p_1}{(2\pi)^3}\int \frac{d^3q_2\; E_{\bar\nu}}{(2\pi)^3}\int \frac{d^3p_1}{(2\pi)^3}\int \frac{d^3q_1}{(2\pi)^3}(1 - \frac{\mathbf{p_2}}{E_d}\cos\theta_2)(\frac{16\alpha_c}{3\pi}) \\ &\times (2\pi)^4\delta^3(\mathbf{p_2} - \mathbf{p_1} - \mathbf{q_1})\delta(E_d - E_{\bar\nu} - E_u - E_e) f(\mathbf{p_1})[1 - f(\mathbf{p_1})][1 - f(\mathbf{p_2})] \end{aligned}}$$

Neutrino emissivity for direct URCA process

Further evaluation of the general expression: $$\epsilon_{\bar{\nu}} \simeq \frac{\alpha_c}{2\pi^9} G^2 \cos^2\theta_c \, I_\Omega \, I_E$$

Requires angular and momentum-energy integrations:

$$I_\Omega = \int d\Omega_d \int d\Omega_e \int d\Omega_u \int d\Omega_{\bar{\nu}} (1 - \frac{\mathbf{p_2}}{E_d} \cos\theta_2) \delta^{(3)}(\mathbf{p_2} - \mathbf{p_1} - \mathbf{q_1})$$

$$I_E = \int_0^\infty p_F(d)^2 dE_d \int_0^\infty E_{\bar{\nu}}^3 dE_{\bar{\nu}} \int_0^\infty p_F(u)^2 dE_u \int_0^\infty p_F(e)^2 \, S \, \delta(E_d - E_{\bar{\nu}} - E_u - E_e).$$

Angular integration:

$$\delta^{(3)}(\mathbf{p_2} - \mathbf{p_1} - \mathbf{q_1}) = \int \frac{d^3x}{(2\pi)^3} e^{i(\mathbf{p_2} - \mathbf{p_1} - \mathbf{q_1})\cdot x} \qquad d\Omega = \sin\theta \, d\theta \, d\phi = d(\cos\theta) d\phi.$$

$$\begin{aligned}
I_\Omega &= \frac{2^5\pi^4}{(2\pi)^3} \int d^3x \int_{-1}^{+1} dy_d \, e^{i|p_2||x|y_d} \int_{-1}^{+1} dy_u \, e^{-i|p_1||x|y_u} \int_{-1}^{+1} dy_e \, e^{-i|q_1||x|y_e} \\
&= \frac{2^5\pi^4}{(2\pi)^3} \int d^3x \Big[\frac{1}{i|p_2||x|} e^{i|p_2||x||y_d|}\Big]_{-1}^{1} \Big[\frac{1}{-i|p_1||x|} e^{-i|p_1||x||y_u|}\Big]_{-1}^{1} \Big[\frac{1}{-i|q_1||x|} e^{-i|q_1||x||y_e|}\Big]_{-1}^{1} \\
&= \frac{2^5\pi^4}{(2\pi)^3} \int d^3x \frac{2\sin|p_2||x|}{|p_2||x|} \, \frac{2\sin|p_1||x|}{|p_1||x|} \, \frac{2\sin|q_1||x|}{|q_1||x|} \\
&= \frac{2^7\pi^2}{|p_2||p_1||q_1|} \int_0^\infty dx \frac{1}{|x|} \sin(|p_2||x|) \sin(|p_1||x|) \sin(|q_1||x|) \quad = \frac{32\pi^3}{p_F(d) p_F(u) p_F(e)}
\end{aligned}$$

Neutrino emissivity for direct URCA process

For the energy integrations we introduce new variables:

$$z = E/k_BT, \; x_1 = (E_d - \mu_d)/k_BT, \; x_2 = -(E_u - \mu_u)/k_BT \text{ and } x_3 = -(E_e - \mu_e)/k_BT$$

$$\begin{aligned} I_{x_i} &\simeq T^3 \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 \int_{-\infty}^{\infty} dx_3 f(x_1) f(x_2) f(x_3) \delta(x_1 + x_2 + x_3 - z) \\ &= T^3 \int_{-\infty}^{\infty} e^{izt} \frac{dt}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-(1+it)x_1}}{1+e^{-x_1}} dx_1 \int_{-\infty}^{\infty} \frac{e^{-(1+it)x_2}}{1+e^{-x_2}} dx_2 \int_{-\infty}^{\infty} \frac{e^{-(1+it)x_3}}{1+e^{-x_3}} dx_3 \\ &= T^3 \int_{-\infty}^{\infty} e^{izt} \left(\frac{i\pi}{\sinh(\pi t)} \right)^3 \frac{dt}{2\pi} \end{aligned}$$

, where we used: $\int_{-\infty}^{\infty} \frac{e^{-(1+it)x}}{1+e^{-x}} dx = \pi \mathrm{cosec}[(1+it)\pi] = \frac{i\pi}{\sinh(\pi t)}$.

$$= \frac{T^3}{2} \frac{\pi^2 + z^2}{1+e^z}.$$

The remaining integration over the neutrino energy (z) can be performed now with the result:

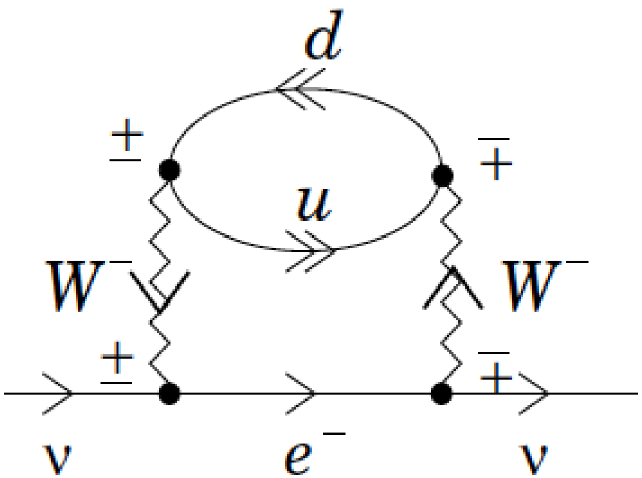
$$\frac{T^6}{2} \int_0^{\infty} dz z^3 \frac{z^2 + \pi^2}{1+e^z} = \frac{T^6}{2} \left[\int_0^{\infty} \frac{z^5}{1+e^z} dz + \pi^2 \int_0^{\infty} \frac{z^3}{1+e^z} dz \right] = T^6 \frac{457\pi^6}{5040}.$$

The final result for the DU emissivity is:

$$\boxed{\epsilon_{\bar{\nu}} \simeq \frac{914}{315} G^2 \cos^2\theta_c \, \alpha_s \, p_F(d) \, p_F(u) \, p_F(e) \, T^6.}$$

Kadanoff-Baym formulation of neutrino kinetic equation

$$i\partial_x^\mu \mathrm{Tr}[\gamma_\mu G_\nu^<(X,q_2)] = -\mathrm{Tr}[G_\nu^>(X,q_2)\Sigma_\nu^<(X,q_2) - \Sigma_\nu^>(X,q_2)G_\nu^<(X,q_2)], \quad X = (t,\mathbf{x})$$



$$\begin{aligned}
\Sigma_\nu^<(t,q_2) &= \frac{G_F^2}{2}\int \frac{d^4\, q_1}{(2\pi^4)}\gamma^\mu(1-\gamma_5)(\gamma^\alpha q_{1,\alpha}+\mu_e\gamma_0)\gamma^\nu(1-\gamma_5 \\
&\quad \times \Pi^>_{\mu\nu}(q_1-q_2)\frac{\pi}{q_1}f_e(t,\mathbf{q_1})\delta(q_1^0+\mu_e-|\mathbf{q}_1|), \\
\Sigma_\nu^>(t,q_2) &= \frac{G_F^2}{2}\int \frac{d^4\, q_1}{(2\pi^4)}\gamma^\mu(1-\gamma_5)(\gamma^\alpha q_{1,\alpha}+\mu_e\gamma_0)\gamma^\nu(1-\gamma_5 \\
&\quad \times \Pi^<_{\mu\nu}(q_1-q_2)\frac{\pi}{q_1}[1-f_e(t,\mathbf{q_1})]\delta(q_1^0+\mu_e-|\mathbf{q}_1
\end{aligned}$$

$$\begin{aligned}
iG_\nu^<(t,q_2) &= -(\gamma^\beta q_{2,\beta}+\mu_\nu\gamma_0)\frac{\pi}{q_2}\{f_\nu(t,\mathbf{q_2})\delta(p_2^0+\mu_\nu-|\mathbf{q_2}|) \\
&\quad -[1-f_{\bar\nu}(t,-\mathbf{q_2})]\delta(q_2^0+\mu_\nu+|\mathbf{q_2}|)\} \\
iG_\nu^>(t,q_2) &= (\gamma^\beta q_{2,\beta}+\mu_\nu\gamma_0)\frac{\pi}{q_2}\{[1-f_\nu(t,\mathbf{q_2})]\delta(q_2^0+\mu_\nu-|\mathbf{q_2}|) \\
&\quad -f_{\bar\nu}(t,-\mathbf{q_2})\delta(q_2^0+\mu_\nu+|\mathbf{q_2}|)\}
\end{aligned}$$

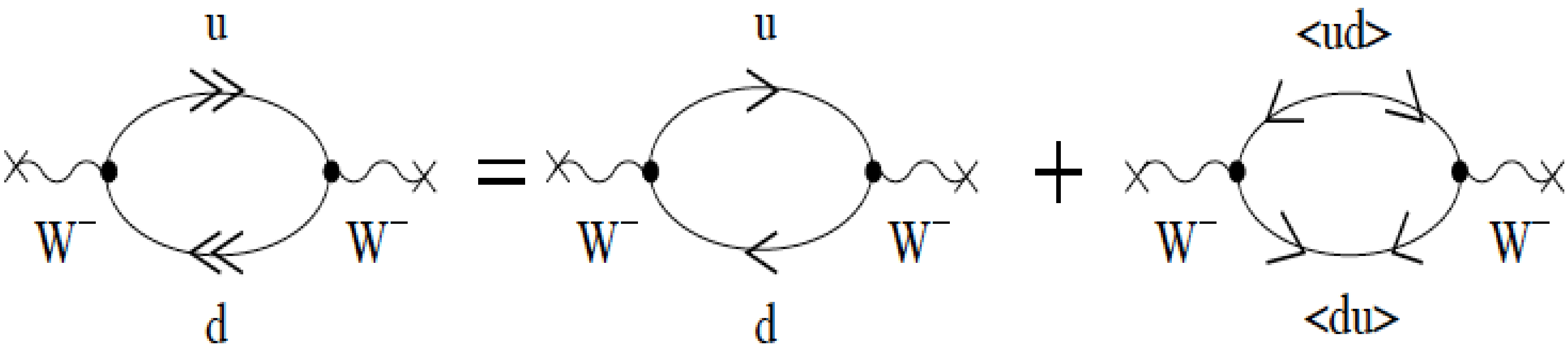
Kadanoff-Baym formulation of neutrino kinetic equation

$$\begin{aligned}
\frac{\partial}{\partial t} f_\nu(t,\mathbf{q_2}) &= -i\frac{G_F^2}{16}\int \frac{d^3\mathbf{q_1}}{(2\pi)^3|\mathbf{q_1}||\mathbf{q_2}|}\mathcal{L}^{\mu\nu}(q_1,q_2)\{[1-f_\nu(t,\mathbf{q_2})]f_e(t,\mathbf{q_1})\Pi^{>}_{\mu\nu}(q) \\
&\quad - f_\nu(t,\mathbf{q_2})[1-f_e(t,\mathbf{q_1})]\Pi^{<}_{\mu\nu}(q)\} \\
&= \frac{G_F^2}{8}\int \frac{d^3\mathbf{q_1}}{(2\pi)^3|\mathbf{q_1}||\mathbf{q_2}|}\mathcal{L}^{\mu\nu}(q_1,q_2)\, n_F(|\mathbf{q_1}|-\mu_e)n_B(|\mathbf{q_2}|+\mu_e-|\mathbf{q_1}|)\mathrm{Im}\Pi^R_{\mu\nu}
\end{aligned}$$

$$\begin{aligned}
\Pi^{>}(q) &= -2i[1+n_B(q_0)]\mathrm{Im}\Pi_R(q) \\
\Pi^{<}(q) &= -2i\, n_B(q_0)\mathrm{Im}\Pi_R(q) \\
n_B(\omega) &\equiv 1/(e^{\omega/T}-1) \quad \text{(Bose)} \\
n_F(\omega) &\equiv 1/(e^{\omega/T}+1) \quad \text{(Fermi)}
\end{aligned}$$

$$\mathcal{L}^{\mu\nu}(q_1,q_2) = \mathrm{Tr}[\gamma^\mu(1-\gamma_5)\ \not{q}_1\gamma^\nu(1-\gamma_5)\ \not{q}_2] = 8[q_1^\mu q_2^\nu - g^{\mu\nu}(q_1\cdot q_2) + q_1^\nu q_2^\mu - i\epsilon^{\mu\alpha\nu\beta}q_{1\alpha}q_{2\beta}]$$

Kadanoff-Baym formulation of neutrino kinetic equation



$$\Pi_{\mu\nu}(q) = -\frac{i}{2}\int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}_{\mathrm{Z,D}}\,[\Gamma^Z_\mu\; S_p\; \Gamma^Z_\nu\; S_{p+q}], \qquad u,d \to p, p+q \to \hat{p}, \hat{k}$$

$$\Gamma^Z_i = \begin{pmatrix} \Gamma^-_i & 0 \\ 0 & \Gamma^+_i \end{pmatrix} \quad \Gamma^\pm_i = \gamma_i(1 \pm g_A \gamma_5) \quad i = \mu, \nu \;;\; S_j = \begin{pmatrix} G^+_j & F^-_j \\ F^+_j & G^-_j \end{pmatrix} \quad j = p, p+q.$$

$$S^{-1} S = \mathbf{1} \quad \Rightarrow \quad \begin{pmatrix} [S_0^+]^{-1} & \Delta^- \\ \Delta^+ & [S_0^-]^{-1} \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Kadanoff-Baym formulation of neutrino kinetic equation

$$
\begin{aligned}
G^{\pm} &= [(S_0^{\pm})^{-1} - \Delta^{\mp} S_0^{\mp} \Delta^{\pm}]^{-1} = \frac{p_0 + E_p^{\mp}}{p_0^2 - (\xi_p^{\mp})^2} \gamma_0 \tilde{\Lambda}_p^{-} + \frac{p_0 - E_p^{\pm}}{p_0^2 - (\xi_p^{\pm})^2} \gamma_0 \tilde{\Lambda}_p^{+} \\
F^{\pm} &= -S_0^{\mp} \Delta^{\pm} G^{\pm} = \frac{\Delta^{\pm}}{p_0^2 - (\xi_p^{\pm})^2} \tilde{\Lambda}_p^{+} + \frac{\Delta^{\pm}}{p_0^2 - (\xi_p^{\mp})^2} \tilde{\Lambda}_p^{-}
\end{aligned}
$$

- Pole $p_0 = \pm \xi_p^-$ und $p_0 = \mp \xi_p^+$ mit $(\xi_p^{\pm})^2 = (E_p^{\pm})^2 + \Delta^2$
  (Quasiparticle/Quasihole- and Quasiantiparticle/Quasiantihole excitation energies)
- Energy projectors

$$
\left.\begin{aligned}
\Lambda_p^{\pm} &= \tfrac{1}{2}(1 \pm \gamma_0\, \mathcal{S}_p^{+}) \\
\tilde{\Lambda}_p^{\pm} &= \tfrac{1}{2}(1 \pm \gamma_0\, \mathcal{S}_p^{-})
\end{aligned}\right\} \quad \mathcal{S}_p^{\pm} = \vec{\gamma}\, \hat{p} \pm \hat{m}, \quad \hat{p} = \frac{\mathbf{p}}{E_p} \quad \hat{m} = \frac{m}{E_p}
$$

Kadanoff-Baym formulation of neutrino kinetic equation

$$\Pi_{\mu\nu}(q) = -i\frac{T}{2}\sum_n \int \frac{d^3p}{(2\pi)^3}\mathrm{Tr}_{\mathrm{Z,D}}\,[\Gamma^Z_\mu\ S_p\ \Gamma^Z_\nu\ S_{p+q}]$$

$$\begin{aligned}
&\mathrm{Tr}_{\mathrm{D}}[\Gamma^-_\mu G^+_p \Gamma^-_\nu G^+_{p+q} + \Gamma^+_\mu G^-_p \Gamma^+_\nu G^-_{p+q} + \Gamma^-_\mu F^-_p \Gamma^+_\nu F^+_{p+q} + \Gamma^+_\mu F^+_p \Gamma^-_\nu F^-_{p+q}] \quad = \\
&\frac{(p_0+E^-_p)(p_0+q_0+E^-_k)}{[p_0^2-(\xi^-_p)^2][(p_0+q_0)^2-(\xi^-_k)^2]}\{\mathcal{T}^+_{\mu\nu}(\hat{p},\hat{k}) + g_A^2\widetilde{\mathcal{T}}^+_{\mu\nu}(\hat{p},\hat{k}) - g_A[\widetilde{\mathcal{W}}^+_{\mu\nu}(\hat{p},\hat{k}) + \mathcal{W}^+_{\mu\nu}(\hat{p},\hat{k})]\} \quad + \\
&\frac{(p_0-E^-_p)(p_0+q_0-E^-_k)}{[p_0^2-(\xi^-_p)^2][(p_0+q_0)^2-(\xi^-_k)^2]}\{\mathcal{T}^-_{\mu\nu}(\hat{p},\hat{k}) + g_A^2\widetilde{\mathcal{T}}^-_{\mu\nu}(\hat{p},\hat{k}) + g_A[\widetilde{\mathcal{W}}^-_{\mu\nu}(\hat{p},\hat{k}) + \mathcal{W}^-_{\mu\nu}(\hat{p},\hat{k})]\} \quad - \\
&\frac{\Delta^2}{[p_0^2-(\xi^-_p)^2][(p_0+q_0)^2-(\xi^-_k)^2]}\{[\mathcal{T}^-_{\mu\nu}(\hat{p},\hat{k}) + \mathcal{T}^+_{\mu\nu}(\hat{p},\hat{k})] + g_A^2[\widetilde{\mathcal{T}}^-_{\mu\nu}(\hat{p},\hat{k}) + \widetilde{\mathcal{T}}^+_{\mu\nu}(\hat{p},\hat{k})] \quad - \\
&\qquad g_A[\widetilde{\mathcal{W}}^+_{\mu\nu}(\hat{p},\hat{k}) + \mathcal{W}^+_{\mu\nu}(\hat{p},\hat{k}) - \widetilde{\mathcal{W}}^-_{\mu\nu}(\hat{p},\hat{k}) - \mathcal{W}^-_{\mu\nu}(\hat{p},\hat{k})]\}
\end{aligned}$$

- $p_0 = i(2n+1)\pi T$, $q_0 = i2m\pi T$ fermionic and bosonic Matsubara frequencies
- $\widetilde{\mathcal{T}}^\pm_{\mu\nu}(\hat{p},\hat{k}) = \mathrm{Tr}[\gamma_0\gamma_\mu\widetilde{\Lambda}^\pm_p\gamma_0\gamma_\nu\Lambda^\pm_k]$, $\widetilde{\mathcal{W}}^\pm_{\mu\nu}(\hat{p},\hat{k}) = \mathrm{Tr}[\gamma_0\gamma_\mu\widetilde{\Lambda}^\pm_p\gamma_0\gamma_\nu\Lambda^\pm_k\gamma_5]$,
  $\mathcal{T}^\pm_{\mu\nu}(\hat{p},\hat{k}) = \mathrm{Tr}[\gamma_0\gamma_\mu\Lambda^\pm_p\gamma_0\gamma_\nu\Lambda^\pm_k]$, $\mathcal{W}^\pm_{\mu\nu}(\hat{p},\hat{k}) = \mathrm{Tr}[\gamma_0\gamma_\mu\Lambda^\pm_p\gamma_0\gamma_\nu\Lambda^\pm_k\gamma_5]$
  (hadronic tensors)

Kadanoff-Baym formulation of neutrino kinetic equation

$$
\begin{aligned}
\Pi_{\mu\nu}(q_0,\mathbf{q}) \;\; = \;\; & -\frac{i}{2}\int \frac{\mathrm{d}^3\mathrm{p}}{(2\pi)^3} A^+(E_p,E_k)\{\mathcal{T}^+_{\mu\nu}(\hat{p},\hat{k}) + \widetilde{\mathcal{T}}^+_{\mu\nu}(\hat{p},\hat{k}) - [\widetilde{\mathcal{W}}^+_{\mu\nu}(\hat{p},\hat{k}) + \mathcal{W}^+_{\mu\nu}(\hat{p},\hat{k}) \\
& + A^-(E_p,E_k)\{\mathcal{T}^-_{\mu\nu}(\hat{p},\hat{k}) + \widetilde{\mathcal{T}}^-_{\mu\nu}(\hat{p},\hat{k}) + [\widetilde{\mathcal{W}}^-_{\mu\nu}(\hat{p},\hat{k}) + \mathcal{W}^-_{\mu\nu}(\hat{p},\hat{k})]\} \\
& - \Delta^2 B(E_p,E_k)\{\mathcal{T}^-_{\mu\nu}(\hat{p},\hat{k}) + \mathcal{T}^+_{\mu\nu}(\hat{p},\hat{k}) + \widetilde{\mathcal{T}}^-_{\mu\nu}(\hat{p},\hat{k}) + \widetilde{\mathcal{T}}^+_{\mu\nu}(\hat{p},\hat{k}) \\
& - [\widetilde{\mathcal{W}}^+_{\mu\nu}(\hat{p},\hat{k}) + \mathcal{W}^+_{\mu\nu}(\hat{p},\hat{k}) - \widetilde{\mathcal{W}}^-_{\mu\nu}(\hat{p},\hat{k}) - \mathcal{W}^-_{\mu\nu}(\hat{p},\hat{k})]\}
\end{aligned}
$$

$$
\begin{aligned}
A^{\pm}(E_p,E_k) \;\; &= \;\; -\frac{1}{2\xi_p^- 2\xi_k^-}\sum_{s_1 s_2=\pm}\frac{(\xi_p^- + s_1 E_p^-)(\xi_k^- + s_2 E_k^-)}{q_0 \pm s_1\xi_p^- \mp s_2\xi_k^-}\,\frac{n_F(\pm s_1\xi_p^-)n_F(\mp s_2\xi_k^-)}{n_B(\pm s_1\xi_p^- \mp s_2\xi_k^-)} \\
B(E_p,E_k) \;\; &= \;\; -\frac{1}{2\xi_p^- 2\xi_k^-}\sum_{s_1 s_2=\pm}\frac{1}{q_0 + s_1\xi_p^- - s_2\xi_k^-}\,\frac{n_F(s_1\xi_p^-)n_F(-s_2\xi_k^-)}{n_B(s_1\xi_p^- - s_2\xi_k^-)}
\end{aligned}
$$

Kadanoff-Baym formulation of neutrino kinetic equation

$$\varepsilon_\nu \equiv -\frac{\partial}{\partial t}\int \frac{\mathrm{d}^3\mathbf{q_2}}{(2\pi)^3}|\mathbf{q_2}|[f_\nu(t,\mathbf{q_2}) + f_{\bar\nu}(t,\mathbf{q_2})] = -2\frac{\partial}{\partial t}\int \frac{\mathrm{d}^3\mathbf{q_2}}{(2\pi)^3}\, p_{F,\nu}\; f_\nu(t,\mathbf{q_2})$$

$$\frac{\partial}{\partial t} f_\nu(t,\mathbf{q_2}) = \frac{G_F^2}{8}\int \frac{d^3\mathbf{q_1}}{(2\pi)^3\, p_{F,e}\, p_{F,\nu}} \mathcal{L}^{\mu\nu}(q_1,q_2)\, n_F(p_{F,e}-\mu_e) n_B(p_{F,\nu}+\mu_e-p_{F,e}) \mathrm{Im}\Pi^R_{\mu\nu}(q)$$

$$\varepsilon_\nu = \frac{\pi}{8} G_F^2\cos^2\theta_c \int \frac{d^3\,\mathbf{q_2}}{(2\pi)^3}\, p_{F,\nu} \sum_{s_1 s_2=\pm}\int \frac{d^3\mathbf{q_1}}{(2\pi)^3\, p_{F,e}\, p_{F,\nu}} \int \frac{d^3\mathbf{p}}{(2\pi)^3}\, n_F(p_{F,e}-\mu_e)$$
$$\times\, n_B(p_{F,\nu}+\mu_e-p_{F,e}) \left[2\mathcal{B}_p^{s_1}\mathcal{B}_k^{s_2}\mathcal{L}^{\mu\nu}(q_1,q_2)\,\mathcal{H}^{(n)}_{\mu\nu}(\hat p,\hat k) - \frac{\Delta^2}{2\xi_p^- 2\xi_k^-}\mathcal{L}^{\mu\nu}(q_1,q_2)\,\mathcal{H}^{(a)}_{\mu\nu}(\hat p,\hat k) \right.$$
$$\times\, \delta(q_0 + s_1\xi_p^- - s_2\xi_k^-)\frac{n_F(s_1\xi_p^-)n_F(-s_2\xi_k^-)}{n_B(s_1\xi_p^- - s_2\xi_k^-)}.$$

$$\mathcal{L}^{\mu\nu}(q_1,q_2)\mathcal{H}^{(\mathrm{n})}_{\mu\nu}(\hat p,\hat k) = 64 q_1^0 q_2^0 (1-\hat q_1\cdot\hat p)(1-\hat q_2\cdot\hat k)$$

Kadanoff-Baym formulation of neutrino kinetic equation

$$\begin{aligned}\Pi_{\mu\nu}(q_0,\mathbf{q}) \quad &= \quad -\frac{i}{2}\int\frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}A^+(E_p,E_k)\{\mathcal{T}^+_{\mu\nu}(\hat{p},\hat{k})+\widetilde{\mathcal{T}}^+_{\mu\nu}(\hat{p},\hat{k})-[\widetilde{\mathcal{W}}^+_{\mu\nu}(\hat{p},\hat{k})+\mathcal{W}^+_{\mu\nu}(\hat{p},\hat{k})]\}\\
&+A^-(E_p,E_k)\{\mathcal{T}^-_{\mu\nu}(\hat{p},\hat{k})+\widetilde{\mathcal{T}}^-_{\mu\nu}(\hat{p},\hat{k})+[\widetilde{\mathcal{W}}^-_{\mu\nu}(\hat{p},\hat{k})+\mathcal{W}^-_{\mu\nu}(\hat{p},\hat{k})]\}\\
&-\Delta^2 B(E_p,E_k)\{\mathcal{T}^-_{\mu\nu}(\hat{p},\hat{k})+\mathcal{T}^+_{\mu\nu}(\hat{p},\hat{k})+\widetilde{\mathcal{T}}^-_{\mu\nu}(\hat{p},\hat{k})+\widetilde{\mathcal{T}}^+_{\mu\nu}(\hat{p},\hat{k})\\
&-[\widetilde{\mathcal{W}}^+_{\mu\nu}(\hat{p},\hat{k})+\mathcal{W}^+_{\mu\nu}(\hat{p},\hat{k})-\widetilde{\mathcal{W}}^-_{\mu\nu}(\hat{p},\hat{k})-\mathcal{W}^-_{\mu\nu}(\hat{p},\hat{k})]\}\end{aligned}$$

$$A^\pm(E_p,E_k) \quad = \quad -\frac{1}{2\xi_p^- 2\xi_k^-}\sum_{s_1 s_2=\pm}\frac{(\xi_p^-+s_1E_p^-)(\xi_k^-+s_2E_k^-)}{q_0\pm s_1\xi_p^-\mp s_2\xi_k^-}\,\frac{n_F(\pm s_1\xi_p^-)n_F(\mp s_2\xi_k^-)}{n_B(\pm s_1\xi_p^-\mp s_2\xi_k^-)}$$

$$B(E_p,E_k) \quad = \quad -\frac{1}{2\xi_p^- 2\xi_k^-}\sum_{s_1 s_2=\pm}\frac{1}{q_0+s_1\xi_p^--s_2\xi_k^-}\,\frac{n_F(s_1\xi_p^-)n_F(-s_2\xi_k^-)}{n_B(s_1\xi_p^--s_2\xi_k^-)}$$

$$\mathrm{Im}\Pi_{\mu\nu}(q_0,\mathbf{q})=\frac{\pi}{2}\cos^2\theta_c\int\frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\left(2\,A^*(E_p,E_k)\mathcal{H}^{(\mathrm{n})}_{\mu\nu}-\Delta^2\,B^*(E_p,E_k)\mathcal{H}^{(\mathrm{a})}_{\mu\nu}\right),$$

$$A^*(E_p,E_k) \quad = \quad -\sum_{s_1 s_2=\pm}\left(\frac{\xi_p^-+s_1E_p^-}{2\xi_p^-}\right)\left(\frac{\xi_k^-+s_2E_k^-}{2\xi_k^-}\right)\delta(q_0+s_1\xi_p^--s_2\xi_k^-)\,\frac{n_F(s_1\xi_p^-)n_F(-}{n_B(s_1\xi_p^--s}$$

$$B^*(E_p,E_k) \quad = \quad -\frac{1}{2\xi_p^- 2\xi_k^-}\sum_{s_1 s_2=\pm}\delta(q_0+s_1\xi_p^--s_2\xi_k^-)\frac{n_F(s_1\xi_p^-)n_F(-s_2\xi_k^-)}{n_B(s_1\xi_p^--s_2\xi_k^-)}$$

Results for neutrino processes in color superconducting QM

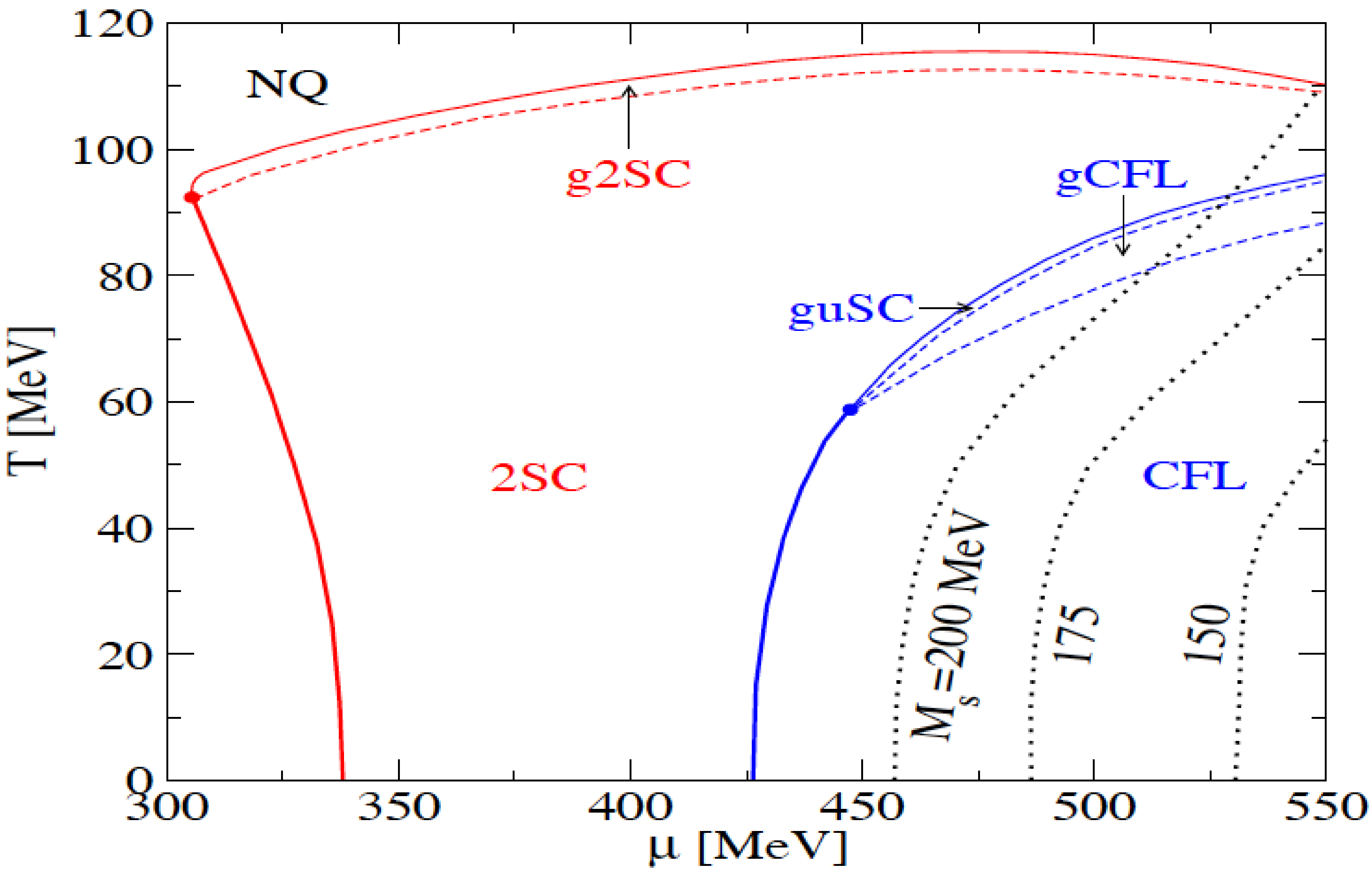
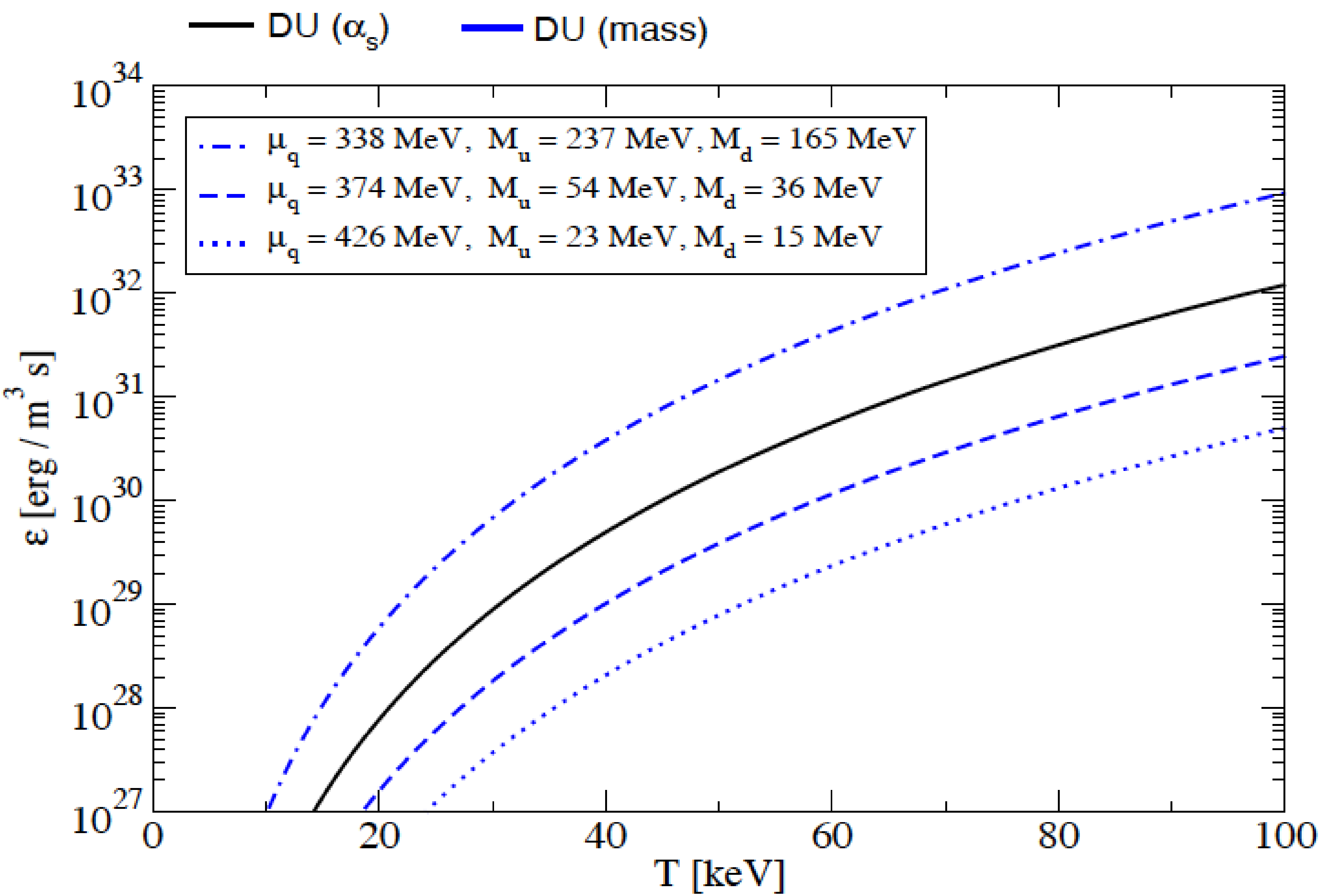


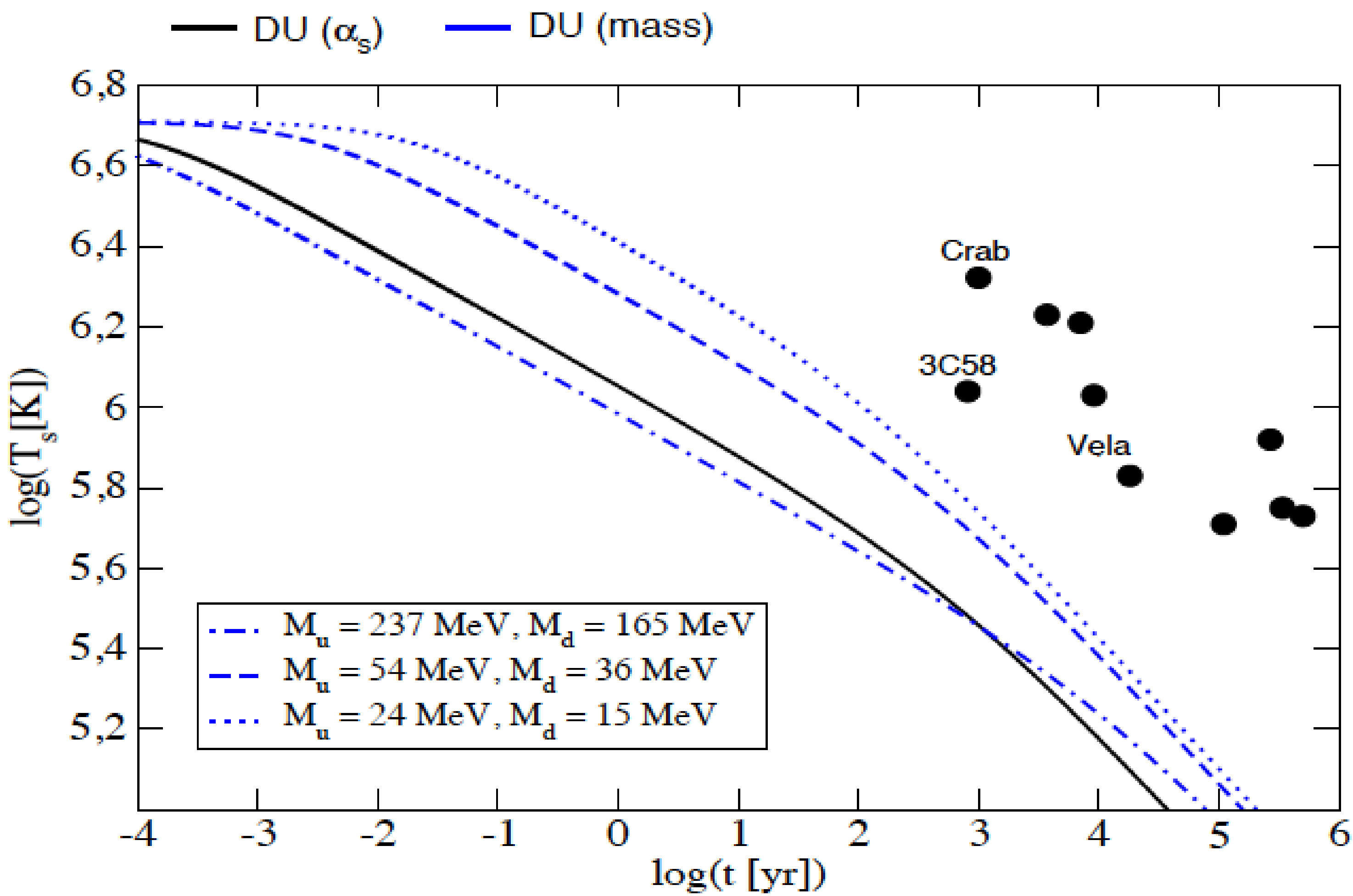
Figure F.Sandin [Blaschke et. al. Phys.Rev D 72,065020 (2005)]

Kadanoff-Baym formulation of neutrino kinetic equation

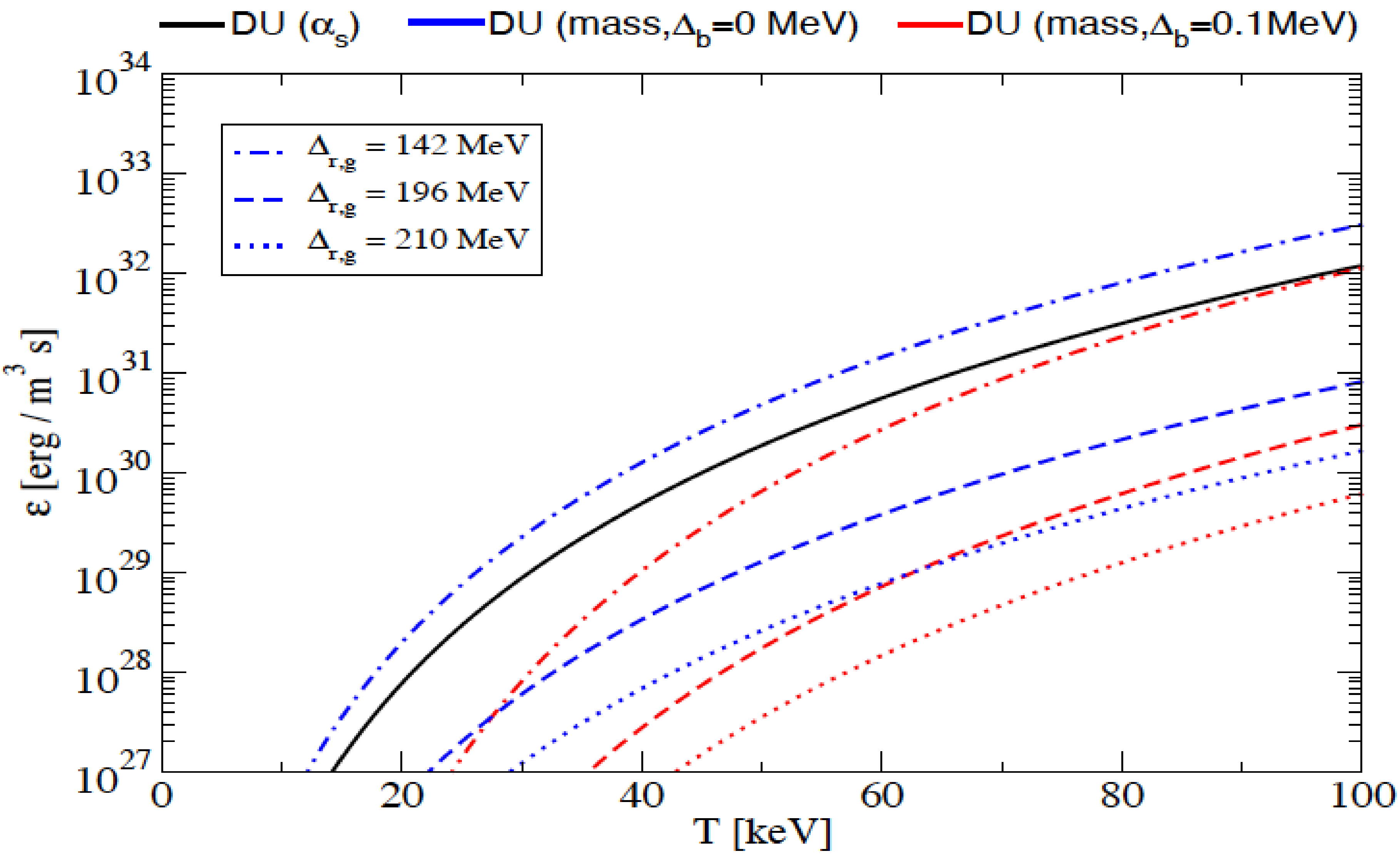


Kadanoff-Baym formulation of neutrino kinetic equation

$$t = -\int_{T_i}^{T_f} \sum_{i,j} \frac{C_v^i(T)}{L_j(T)}\, \mathrm{d}T; \qquad i = \mathrm{quark}, e^-, \gamma, \mathrm{gluon}, \nu; \qquad j = \nu, \gamma$$



Kadanoff-Baym formulation of neutrino kinetic equation



Thermodynamics of iso-CSL and 2SC phases

$$\Omega(\mu_B, \mu_Q, \mu_8, T) = \frac{\bar\sigma_u^2 + \bar\sigma_d^2}{8G_S} + \frac{\Delta_u^2 + \Delta_d^2}{8G_D} - 2\int \frac{d^3p}{(2\pi)^3} \sum_{i=1}^{12} \left[\frac{\lambda_i}{2} + T\ln(1 + e^{-\lambda_i/T})\right] + \Omega_e - \Omega_0,$$

$$\Omega_e = -\mu_Q^4/12\pi^2 - \mu_Q^2 T^2/6 - 7\pi^2 T^4/180$$

λ_i are the excitation energies of the corresponding modes

Iso-CSL phase

$$\hat\Delta = \Delta(\gamma_3\lambda_2 + \gamma_2\lambda_5 + \gamma_1\lambda_7)$$

color
spin

$$\lambda_1^2 = (\epsilon_{u,\text{eff}}(p) - \mu_{u,\text{eff}}(p))^2 + \Delta_{u,\text{eff}}^2(p),$$

$$\lambda_{3,5}^2(p) = (\epsilon_u(p) - \mu_u)^2 + a_{u,(3,5)}(p)\Delta_u^2,$$

$$\epsilon_{u,\text{eff}} = \sqrt{p^2 + M_{u,\text{eff}}^2(p)}, \qquad \epsilon_u(p) = \sqrt{p^2 + M_u^2(p)}.$$

$$M_{u,\text{eff}}(p) = \frac{\mu_u}{\mu_{u,\text{eff}}(p)} M_u(p),$$

$$\mu_{u,\text{eff}}(p) = \mu_u\sqrt{1 + \Delta_u^2/(\mu_u)^2}, \quad \Delta_{u,\text{eff}}^2(p) = a_{u,1}\Delta_u^2$$

$$a_{u,1}(p) = \frac{M_u^2(p)}{\mu_{u,\text{eff}}^2(p)}$$

$$a_{u,(3,5)}(p) = \frac{1}{2}\left[5 - \frac{p^2}{\epsilon_u(p)\mu_u} \pm \sqrt{\left(1 - \frac{p^2}{\epsilon(p)\mu_u}\right)^2 + \frac{8M_u^2(p)}{\epsilon_u^2(p)}}\right],$$

D. N. Aguilera, D. Blaschke, M. Buballa, and V. L. Yudichev, Phys. Rev. D **72**, 034008 (2005).

2SC phase

$$\hat\Delta = \Delta(i\gamma_5\tau_2\lambda_2)$$

$\lambda_{1..4} = \epsilon_f(p) \pm \mu_{fb}$, ungapped bue quarks, λ_{5-12} , gapped red & green

$$\mathcal{M}_s = \begin{vmatrix} -\mu_{d,r} + M_d & p & 0 & -\Delta \\ p & -\mu_{d,r} - M_d & \Delta & 0 \\ 0 & \Delta & \Delta\mu_{u,r} + M_u & p \\ -\Delta & 0 & p & \mu_{u,r} - M_u \end{vmatrix},$$

$\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0$, char. equation with coefficients

$$\begin{aligned} a_0 &= \Delta^4 + 2\Delta^2(M_d M_u + \mu_{d,r}\mu_{u,r} + p^2) \\ &\quad + [M_d^2 - (\mu_{d,r})^2 + p^2][M_u^2 - (\mu_{u,r})^2 + p^2], \\ a_1 &= -2(M_u^2\mu_{d,r} + \Delta^2(\mu_{d,r} - \mu_{u,r}) - M_d^2\mu_{u,r} \\ &\quad + (\mu_{d,r} - \mu_{u,r})(\mu_{d,r}\mu_{u,r} + p^2)), \\ a_2 &= -2\Delta^2 - M_d^2 - M_u^2 + (\mu_{d,r})^2 - 4\mu_{d,r}\mu_{u,r} \\ &\quad + (\mu_{u,r})^2 - 2p^2, \\ a_3 &= 2(\mu_{d,r} - \mu_{u,r}). \end{aligned}$$

$$u^3 - a_2u^2 + (a_1a_3 - 4a_0)u - (a_1^2 + a_0a_3^2 - 4a_0a_2) = 0.$$

Thermodynamics of iso-CSL and 2SC phases

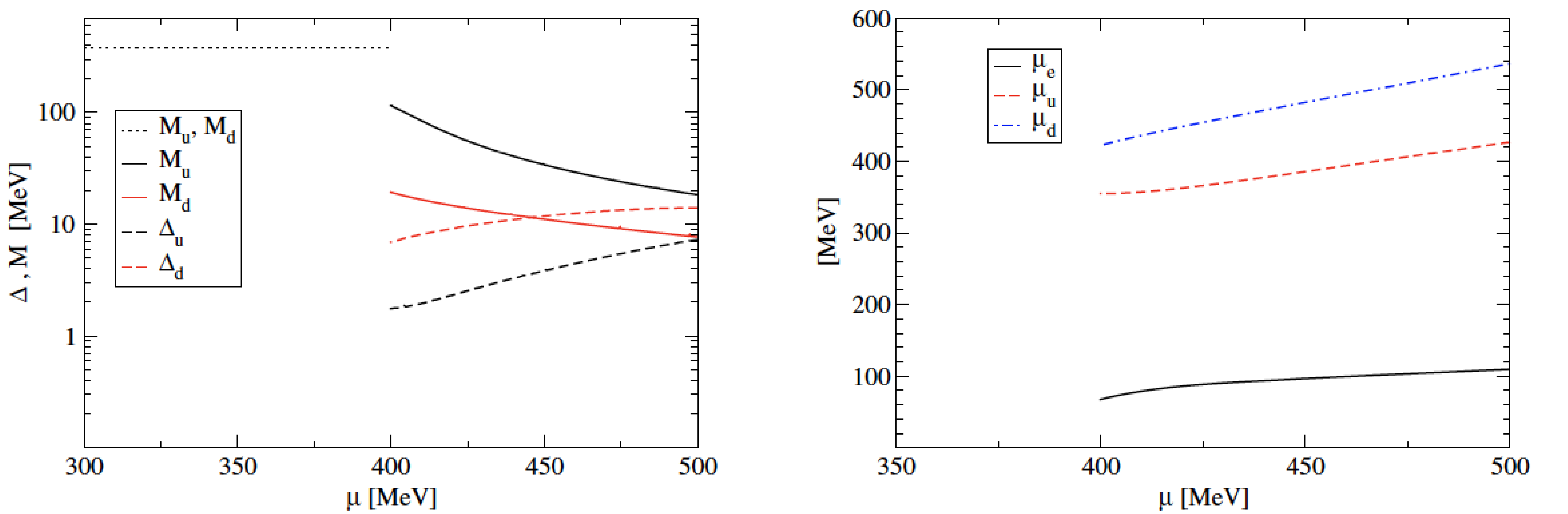


FIG. 1. The left panel shows the dynamical quark masses and pairing gaps ($\eta_D = 3/8$) for the iso-CSL phase as a function of the quark-chemical potential and the corresponding chemical potentials for quarks and electrons are given in the right panel.

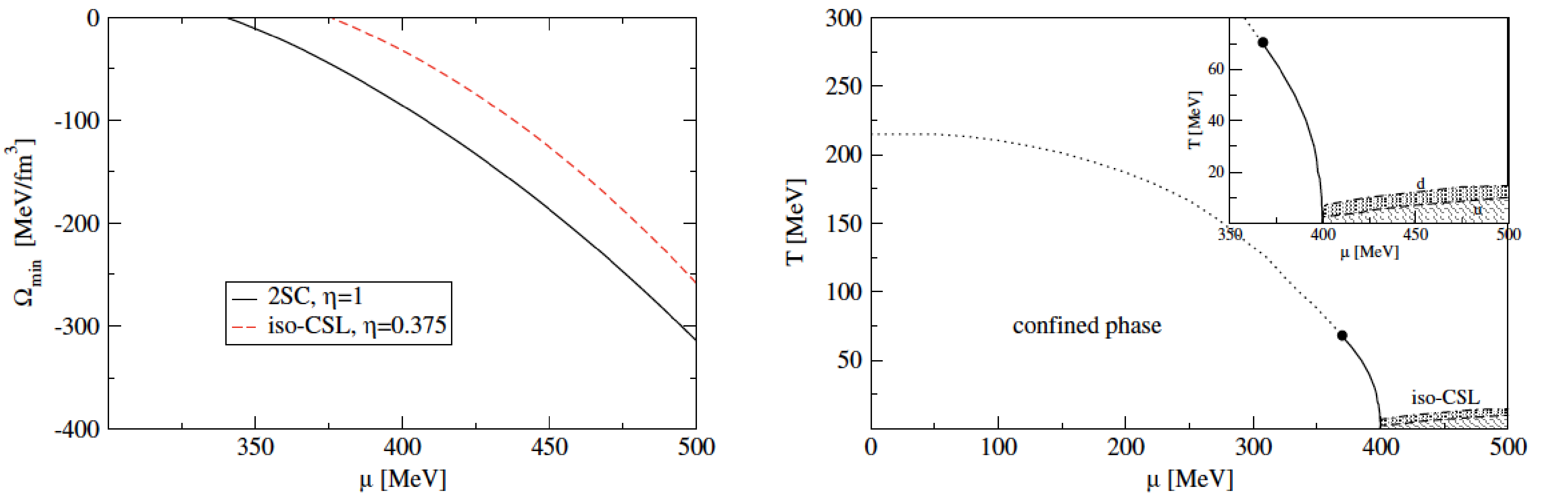


FIG. 2. The left panel displays the minima of the thermodynamical potential for 2SC and iso-CSL quark matter at $T = 0$ as a function of the quark chemical potential. In the right panel the iso-CSL phase diagram calculated with the NJL form factor and a diquark coupling of $\eta_D = 3/8$ is shown.

Kinetic equation for neutrinos in warm, dense quark matter

$$i\partial_x^\alpha \mathrm{Tr}_\mathrm{D}[\gamma_\alpha G_\nu^<(X,q_2)] = -\mathrm{Tr}[G_\nu^>(X,q_2)\Sigma_\nu^<(X,q_2) - \Sigma_\nu^>(X,q_2)G_\nu^<(X,q_2)],$$

$$iG_\nu^<(t,q_2) = -(\gamma^\beta q_{2,\beta} + \mu_\nu\gamma_0)\frac{\pi}{q_2}\{f_\nu(t,\mathbf{q_2})\delta(p_2^0+\mu_\nu-|\mathbf{q_2}|) - [1-f_{\bar\nu}(t,-\mathbf{q_2})]\delta(q_2^0+\mu_\nu+|\mathbf{q_2}|)\},$$

$$iG_\nu^>(t,q_2) = (\gamma^\beta q_{2,\beta} + \mu_\nu\gamma_0)\frac{\pi}{q_2}\{[1-f_\nu(t,\mathbf{q_2})]\delta(q_2^0+\mu_\nu-|\mathbf{q_2}|) - f_{\bar\nu}(t,-\mathbf{q_2})\delta(q_2^0+\mu_\nu+|\mathbf{q_2}|)\},$$

$$\Sigma_\nu^<(t,q_2) = \frac{G_F^2}{2}\int\frac{d^4q_1}{(2\pi^4)}\gamma^\mu(1-\gamma_5)(\gamma^\alpha q_{1,\alpha}+\mu_e\gamma_0)\gamma^\nu(1-\gamma_5)\Pi_{\mu\nu}^>(q_1-q_2)\frac{\pi}{q_1}f_e(t,\mathbf{q_1})\delta(q_1^0+\mu_e-|\mathbf{q}_1|),$$

$$\Sigma_\nu^>(t,q_2) = \frac{G_F^2}{2}\int\frac{d^4q_1}{(2\pi^4)}\gamma^\mu(1-\gamma_5)(\gamma^\alpha q_{1,\alpha}+\mu_e\gamma_0)\gamma^\nu(1-\gamma_5)\Pi_{\mu\nu}^<(q_1-q_2)\frac{\pi}{q_1}[1-f_e(t,\mathbf{q_1})]\delta(q_1^0+\mu_e-|\mathbf{q_1}|)$$

$$\frac{\partial}{\partial t}f_\nu(t,\mathbf{q_2}) = \frac{G_F^2}{8}\int\frac{d^3\mathbf{q_1}}{(2\pi)^3 p_{F,e}p_{F,\nu}}\mathcal{L}^{\mu\nu}(q_1,q_2)n_F(p_{F,e}-\mu_e)n_B(p_{F,\nu}+\mu_e-p_{F,e})\mathrm{Im}\Pi_{\mu\nu}^R(q)$$

$$\mathcal{L}^{\mu\nu}(q_1,q_2) \equiv \mathrm{Tr}[(\gamma_0 q_1^0 - \vec\gamma\cdot\mathbf{q_1})\gamma^\mu(1-\gamma^5)(\gamma_0 q_2^0 - \vec\gamma\cdot\mathbf{q_2})\gamma^\nu(1-\gamma^5)]$$

$$\frac{\partial}{\partial t}f_\nu(t,\mathbf{q_2}) = -\frac{G_F^2\pi}{8}\cos^2\theta_c\int\frac{d^3\mathbf{q_1}}{(2\pi)^3 p_{F,e}p_{F,\nu}}\int\frac{d^3\mathbf{p}}{(2\pi)^3}n_F(p_{F,e}-\mu_e)n_B(p_{F,\nu}+\mu_e-p_{F,e})$$
$$\times\sum_{r=1,3,5}[2A_r^*(E_p,E_k)\mathcal{L}^{\mu\nu}(q_1,q_2)\underbrace{\mathcal{H}_{\mu\nu}^{(n)}(\hat p,\hat k)}_{\text{Focus on the normal part}} - \Delta^2 B_r^*(E_p,E_k)\mathcal{L}^{\mu\nu}(q_1,q_2)\mathcal{H}_{\mu\nu}^{(a)}(\hat p,\hat k)].$$

Kinetic equation for neutrinos in warm, dense quark matter

$$\frac{\partial}{\partial t} f_\nu(t,\mathbf{q_2}) \simeq -\frac{64}{3}\alpha_s G_F^2 \cos^2\theta_c \mu_e\mu_u\mu_d \int \frac{dk d\Omega_k}{(2\pi)^3}\int \frac{dp d\Omega_p}{(2\pi)^3}(1-\cos\theta_{\nu d})\delta(\cos\theta_{ud}-\cos\theta_0) \times \sum_{r=1,3,5}\sum_{e_1,e_2=\pm} B_p^{e_1}B_k^{e_2} n_F(-e_1\xi_{p,r})n_F(e_2\xi_{k,r})n_F(p_{F,\nu}-\xi^-_{p,r}+\xi^-_{k,r}),$$

abbreviations $x \equiv \frac{k-\mu_d}{T}$, $y \equiv \frac{p-\mu_u}{T}$, $z \equiv \frac{q_2}{T}$ and neglect $B^{e_2}_{\mathbf{k},r,d} = \frac{1}{2} - \frac{e_2 x}{2\sqrt{x^2+\lambda_{\mathbf{k},r}\Delta_d^2}}$, $B^{e_2}_{\mathbf{p},r,u} = \frac{1}{2} - \frac{e_1 y}{2\sqrt{y^2+\lambda_{\mathbf{p},r}\Delta_u^2}}$,

$$\frac{\partial}{\partial t} f_\nu(t,\mathbf{q_2}) \simeq -\frac{64}{3}\alpha_s G_F^2 \cos^2\theta_c \mu_e\mu_u\mu_d T^2 \sum_{r=1,3,5}\int \frac{d\Omega_k}{(2\pi)^3}\int \frac{d\Omega_p}{(2\pi)^3}(1-\cos\theta_{\nu d})\delta(\cos\theta_{ud}-\cos\theta_0)\mathcal{F}_r(z),$$

$$\mathcal{F}_r(z) = \sum_{e_1,e_2=\pm}\int_0^\infty\int_0^\infty dxdy\left(e^{-e_1\sqrt{y^2+a_{u,r}\Delta_u^2}}+1\right)^{-1}\left(e^{e_2\sqrt{x^2+a_{d,r}\Delta_d^2}}+1\right)^{-1}\left(e^{z+e_1\sqrt{y^2+a_{u,r}\Delta_u^2}-e_2\sqrt{x^2+a_{d,r}\Delta_d^2}}+1\right)^{-1}.$$

Neutrino emissivity: $$\varepsilon_\nu \equiv -\frac{\partial}{\partial t}\int \frac{d^3\mathbf{q_2}}{(2\pi)^3}|\mathbf{q_2}|[f_\nu(t,\mathbf{q_2})+f_{\bar\nu}(t,\mathbf{q_2})] = -2\frac{\partial}{\partial t}\int\frac{d^3\mathbf{q_2}}{(2\pi)^3}p_{F,\nu}f_\nu(t,\mathbf{q_2}).$$

$$\sum_{e_1,e_2=\pm}\int_0^\infty dz z^3\int_0^\infty dx\int_0^\infty dy(e^{-e_1 y}+1)^{-1}(e^{e_2 x}+1)^{-1}(e^{z+e_1 y-e_2 x}+1)^{-1} = \frac{457}{5040}\pi^6$$

Direct Urca (gapless) emissivity [Iwamoto (1982)]: $$\epsilon_0 = \frac{457}{630}\alpha_s G_F^2\mu_e\mu_u\mu_d T^6$$

Emissivity for direct URCA process in CSL and 2SC+X quark matter

Emissivity for gapped phases

$$\epsilon_{\text{Urca}} = \epsilon_0 G_3(\Delta_u, \Delta_d),$$

$$G_n(\Delta_u, \Delta_d) = \frac{5040}{1371\pi^6} \int_0^\infty dz z^n [\mathcal{F}_1(z) + \mathcal{F}_3(z) + \mathcal{F}_5(z)],$$

$$\Delta(T) = \Delta_0 \sqrt{1 - (T/T_c)^\beta},$$

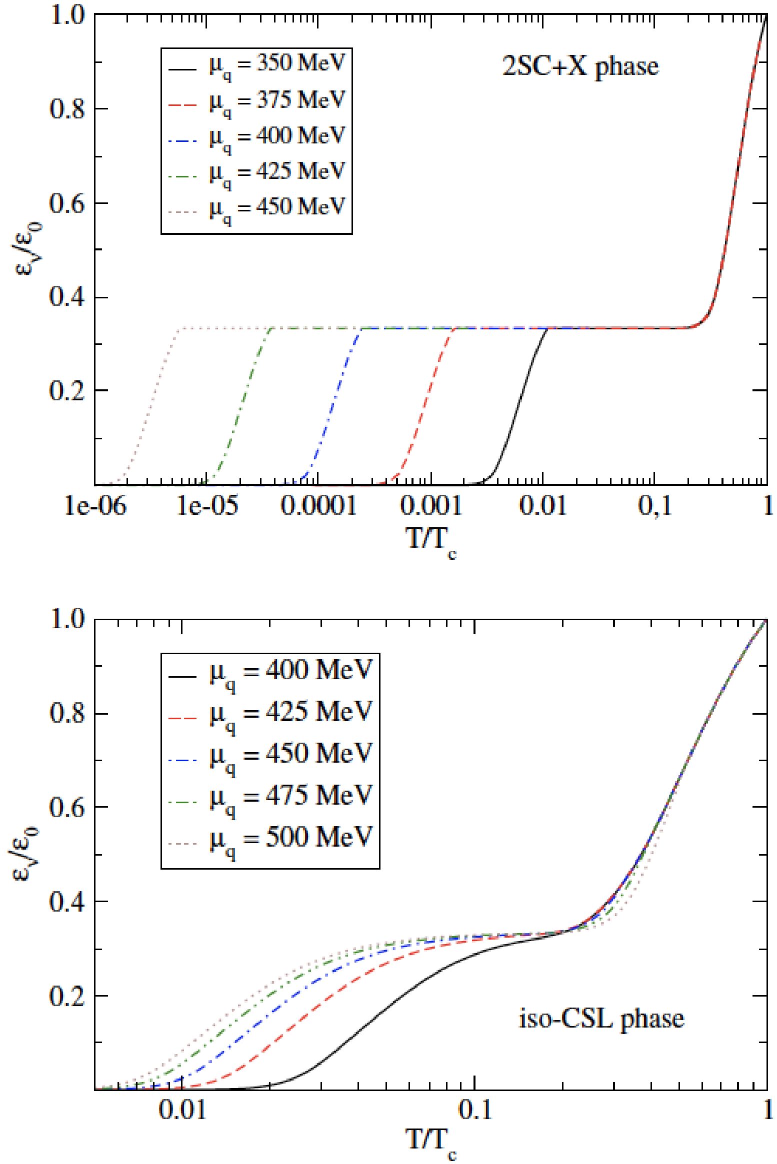


FIG. 3. Neutrino emissivities due to direct Urca processes in the 2SC + X phase (upper panel) and in the iso-CSL phase (lower panel).

Evaluation of Feynman diagram for direct URCA process