Braking Index of Isolated Pulsars

Oliver Hamil

University of Tennessee

Knoxville, TN USA

Definition of Braking Index

- Observational Parameter:

$$n \equiv \frac{\Omega \ddot{\Omega}}{\dot{\Omega}^2}$$

- Highly accurate measurement.
- Attained purely observationally.
- Has well defined values inconsistent with observation, and implications for physics which make it a significant problem worth studying.

Pulsar Spin-Down Mechanism

- General Braking Law: $$\dot{\Omega} = -K\Omega^n$$
- Loss of rotational energy depends on braking torque mechanism.
- Three main types:
  - Relativistic particle wind: n = 1
  - Magnetic dipole radiation: n = 3
  - Quadrupole radiation: n = 5

Observed Braking Index

Pulsar	Frequency (Hz)	n
PSR B0531+21 (Crab)	30.22543701	2.51±0.01
PSR B1509−58	6.633598804	2.837
PSR B0540−69	19.8344965	2.140±0.009
PSR J1119−6127	2.4512027814	2.684
PSR B0833−45 (Vela)	11.2	1.4±0.2
PSR J1846−0258	3.0621185502	2.16±0.13
PSR J1734−3333	0.855182765	0.9±0.2

What is wrong?

- The differential equation predicting braking index makes some assumptions:
  - Stars are essentially non-rotating
  - Parameters defining braking torque are constant
- Is this a good assumption?
  - Moment of inertia?
  - Magnetic field strength?
  - Composition?

Rotation and Braking Torque Mechanism

- Each braking mechanism has the form:

$$\dot{E} = -C\,\Omega^{n+1}$$

  - Where C contains the physics of the associated mechanism, and n is the braking index

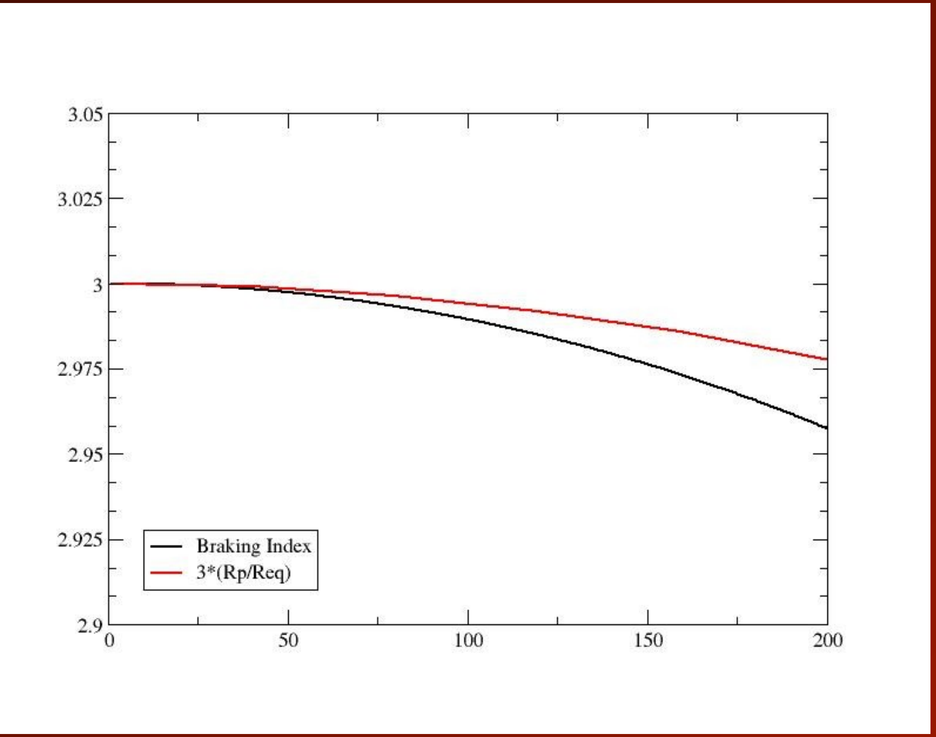
- Introduce rotational energy: $\dot{E} = \frac{d}{dt}\left(\frac{1}{2} I\,\Omega^2\right)$
- Two choices:
  - Moment of Inertia constant: $\dot{\Omega} = -K\,\Omega^n$
    - Where K = C/I
  - Moment of Inertia as function of Ω, which leads to n(Ω)...

Function of Frequency

- Braking index: $$n'(\Omega) = n - \frac{\left(3\Omega I' + \Omega^2 I''\right)}{\left(2I + \Omega I'\right)} + \frac{C'\Omega}{C}$$
- Two sides to the problem:
  - High frequency – moment of inertia significant
  - Low frequency – possible dependence on C?
- For fast pulsars there is an obvious effect from moment of inertia.
- For “slow” pulsars the deviation must come from somewhere in the mechanism.

Frequency

- The plot represents the rotational deformation and braking index of a 2 solar mass pulsar.
- It is clear that even at a frequency of 200Hz the deformation from spherical is less than 1% and the braking index only changes by roughly 1.5%



Mechanisms

- Dipole example: $$C \propto B^2 R^6 \sin^2 \alpha$$
- Goes like magnetic moment and inclination angle – two options:
  - Magnetic field is changing in time?
  - Alignment angle is changing?

*We see that the field strength should increase as the pulsar spins down in the dipole model from the n(Ω) equation.

Magnetic Field

- In what ways can the magnetic field change?
  - Magnetic moment varies with time:
    - Increases, decreases, changes alignment, etc.
  - The torque may vary with frequency in a way that is not a pure dipole:
    - Plasma outflow, currents in magnetosphere.
      - Note: PJ1846-0258 – magnetar outburst followed by reduction of braking index
  - Rotationally driven effects:
    - Phase transition, density profile, superfluidity?

Relativistic Wind

- Particles near the surface of the star are accelerated to relativistic energies and carry away angular momentum.
- The particle wind is also importantly associated with the magnetic field strength as $\sim B^2$
  - Has braking index of $n = 1$ (low end on known observed data)
  - Is related to Dipole by the magnetic field behavior.

Polynomial

- Given that the most accurately measured braking indices are for “slow” rotation, and span a range of values from 1 ~ 2.8, it may be interesting to consider a braking index which may be a combination of more than one mechanism.
- Expand the braking law by a polynomial:

$$\dot{\Omega} = -s(t)\Omega - r(t)\Omega^{3} - g(t)\Omega^{5}$$

  - (Alvarez and Carraminana 2004) where s(t), r(t), and g(t) are functions dependent on the physical mechanisms related to the respective index powers of n = 1, 3, and 5 for wind, MDR, and quadrupole.

...continued

- A few things to note:
  - The polynomial can be used to fit the known braking index values.
  - The roughly 1-3 range in braking index in the region where the torque mechanism should be the governing factor, a combination of Wind (n=1) and Dipole (n=3) could be important.
  - The functions s(t), and r(t) can possibly constrain the time evolution of magnetic field.
  - It should be noted that the quadrupole (n=5) is thought to only be important at high frequencies.

Conclusions

- The problem of braking index can be approached from two ends:
  - Very fast rotation
  - Essentially non-rotation
- Braking index is dominated by the behavior of the torque mechanism at frequencies up to about 200Hz, and is possibly dominated by rotation at much higher frequencies.
- Unknown composition and magnetic field dynamics play crucial role in understanding braking index, and are important to dipole radiation and wind specifically.
  - Magnetic moment evolution
  - Magnetospheric considerations
  - Composition / superfluidity
- Data can be fit to a polynomial to constrain torque mechanism behavior:
  - Can be extended to include physical parameters (data trend?)
  - Can also include fast rotation which increases the importance of higher order terms
- The combination of exhaustive study of these issues together may constrain the braking index issue.

Thank you.