Equation of State in a Generalized Relativistic Density Functional Approach

Stefan Typel

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in collaboration with: Sofija Antić (GSI Darmstadt), David Blaschke (IFT, U Wrocław), Jaroslava Hrtánková (CTU Praha), Thomas Klähn (IFT, U Wrocław), Gevorg Poghosyan (KIT Karlsruhe), Gerd Röpke (U Rostock), Maria Voskresenskaya (TU Darmstadt), Hermann Wolter (LMU München)

Workshop "Compact Stars in the QCD Phase Diagram IV"

Outline

- **Motivation**
  Astrophysics and EoS, Objectives, Dense Matter in Nature, Nuclear and Stellar Matter, Correlations
- **Generalized Relativistic Density Functional**
  Details of gRDF Model, Effective Interaction, Mass Shifts, Chemical Composition of Matter
- **Symmetry Energy and Neutron Skins**
  Density Dependence, Neutron Skins with α-Cluster Correlations
- **Outlook**
  Extension with Quark Degrees of Freedom
- **Conclusions**

Details:

S. Typel, G. Röpke, T. Klähn, D. Blaschke, H.H. Wolter, Phys. Rev. C 81 (2010) 015803
M.D. Voskresenskaya, S. Typel, Nucl. Phys. A 887 (2012) 42
G. Röpke, N.-U. Bastian, D. Blaschke, T. Klähn, S. Typel, H.H. Wolter, Nucl. Phys. A 897 (2013) 70
S. Typel, H.H. Wolter, G. Röpke, D. Blaschke, Eur. Phys. J. A 50 (2014) 17
S. Typel, Phys. Rev. C 89 (2014) 064321

Motivation

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- essential ingredient in astrophysical model calculations:
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  $\Rightarrow$ dynamical evolution of supernovae
  $\Rightarrow$ static properties of neutron stars
  $\Rightarrow$ conditions for nucleosynthesis
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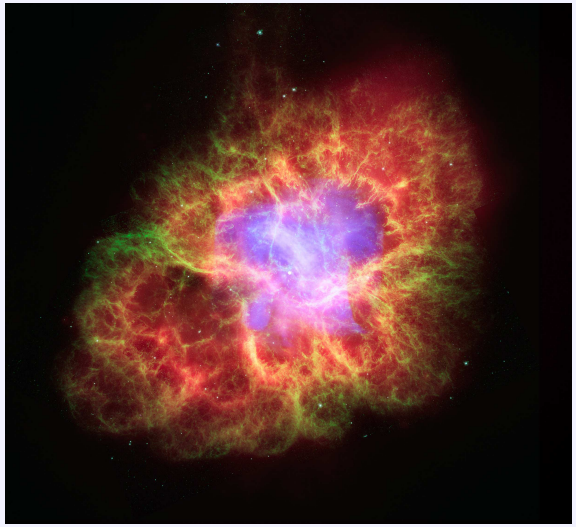

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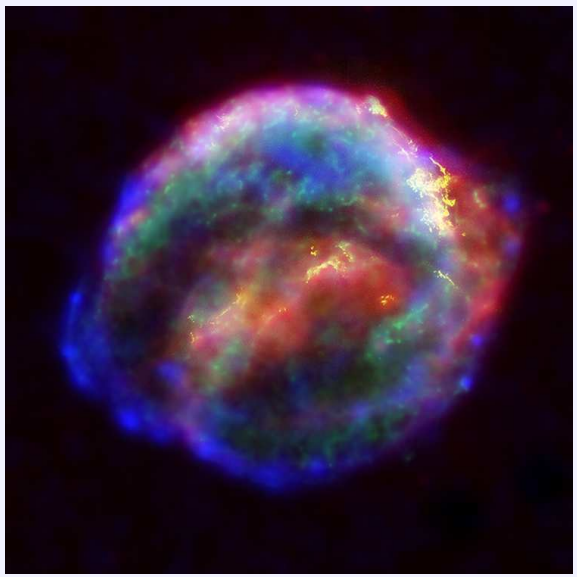

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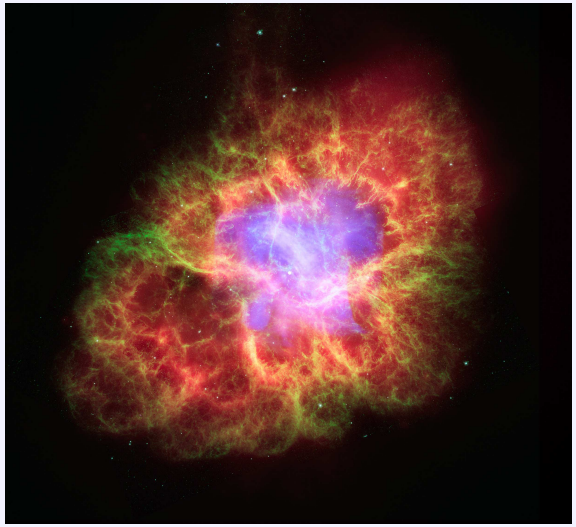

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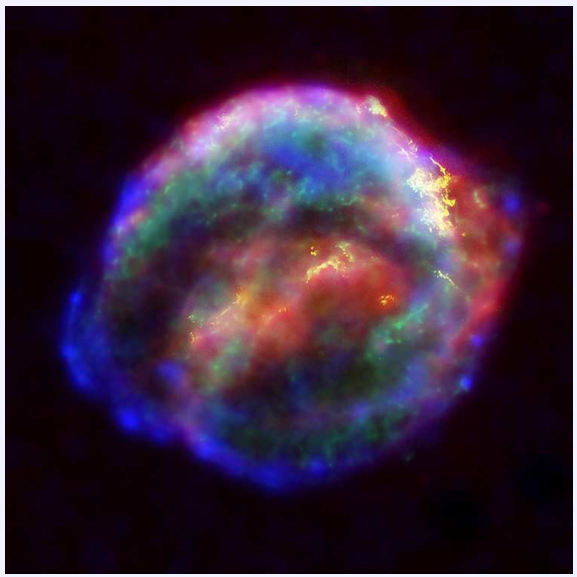

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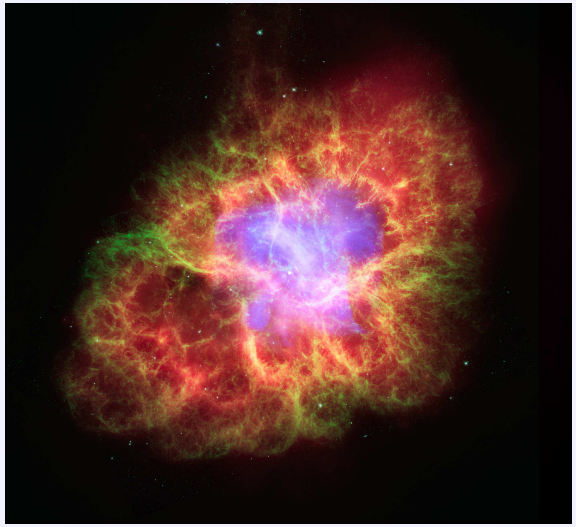
- **wide range of thermodynamic variables** (temperature, density, isospin asymmetry)
  $\Rightarrow$ global EoS required



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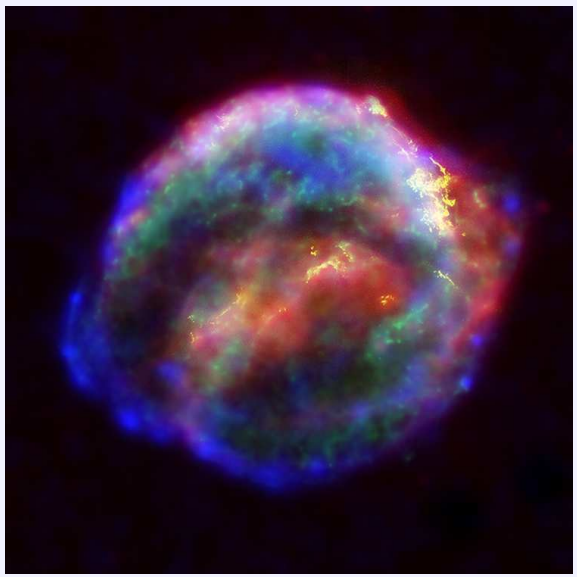

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challenge: covering of full range of thermodynamic variables in a unified model

Nuclear Matter & Stellar Matter

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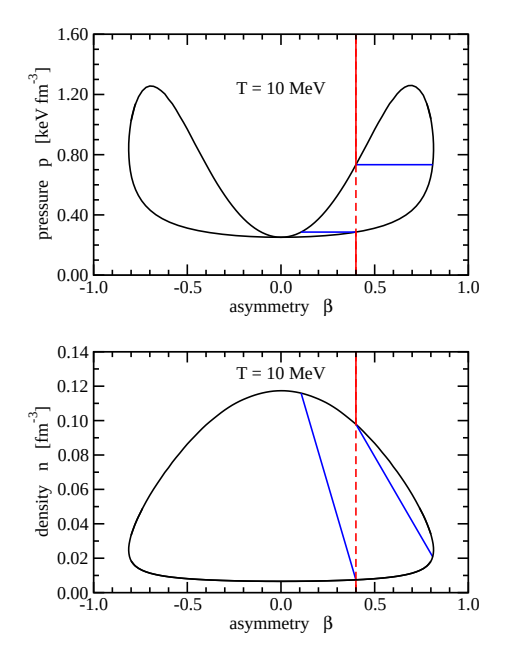
- no electromagnetic interaction, no charge neutrality
- densities below nuclear saturation density
  $\Rightarrow$ 'non-congruent' liquid-gas phase transition: coexistence of low-density and high-density phases with different isospin asymmetries



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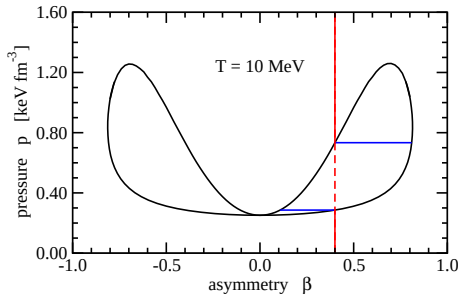
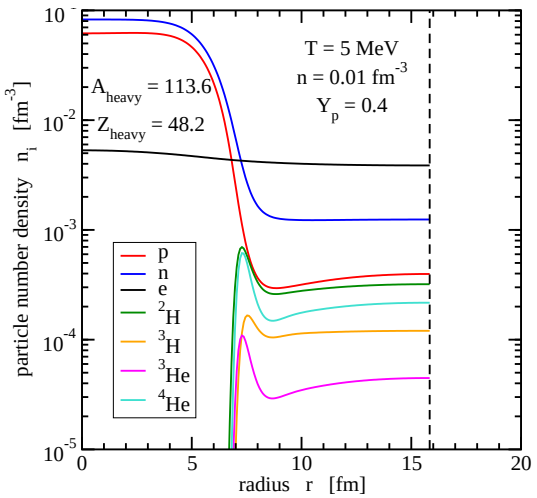
stellar matter

- hadrons and leptons
- strong and electromagnetic interaction
- specific condition: charge neutrality
- formation of inhomogeneous matter
  ⇒ new particle species (nuclei)
  ⇒ 'pasta phases'
- lattice formation at low temperatures
  ⇒ phase transition: liquid/gas ↔ solid



gRDF, spherical Wigner-Seitz cell



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⇒ construction of generalized relativistic density functional
with correct limits and explicit cluster degrees of freedom

Generalized Relativistic Density Functional

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  - stellar matter
    - charge neutrality condition
    - Coulomb correlations with correct limits $\Rightarrow$ phase transition to crystal

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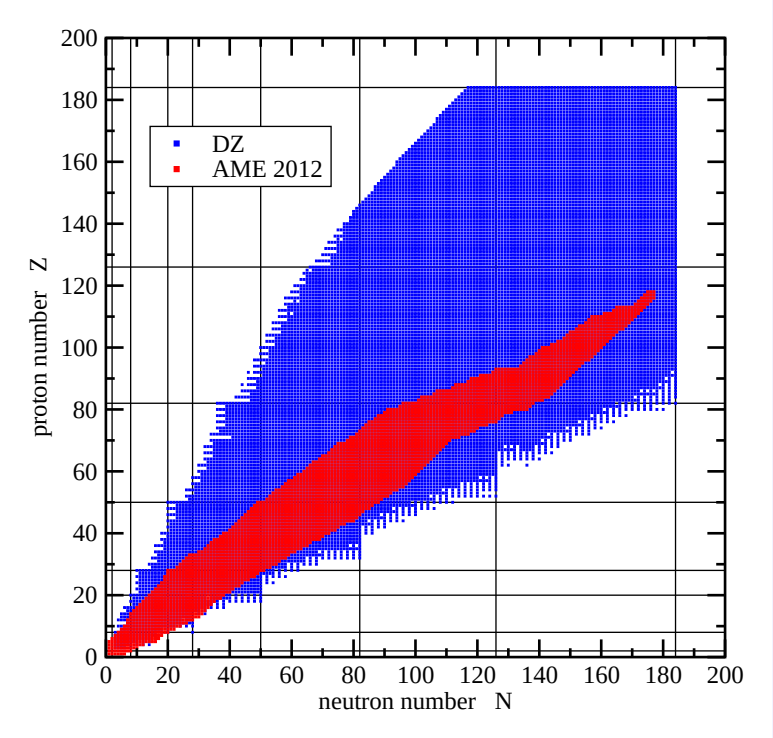

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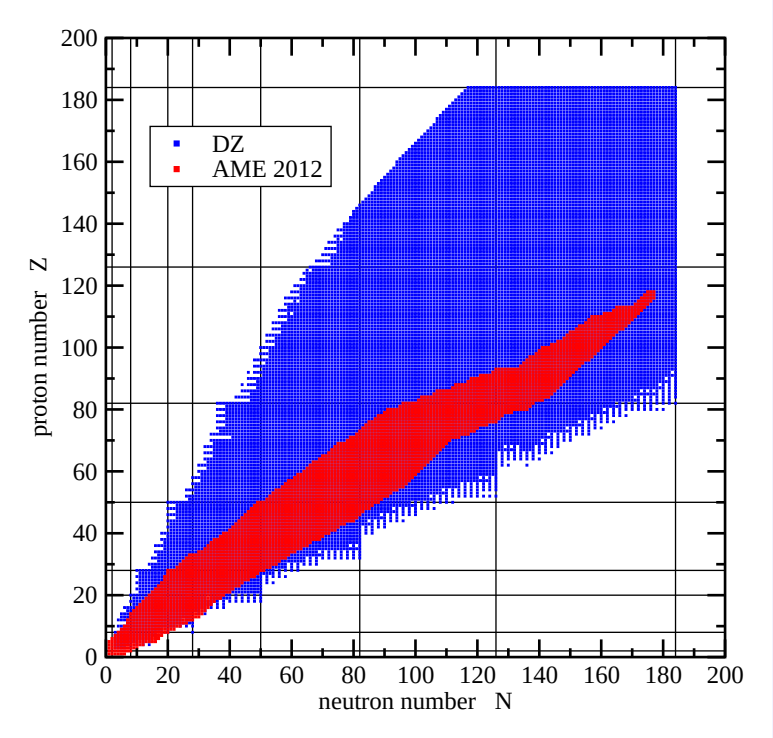

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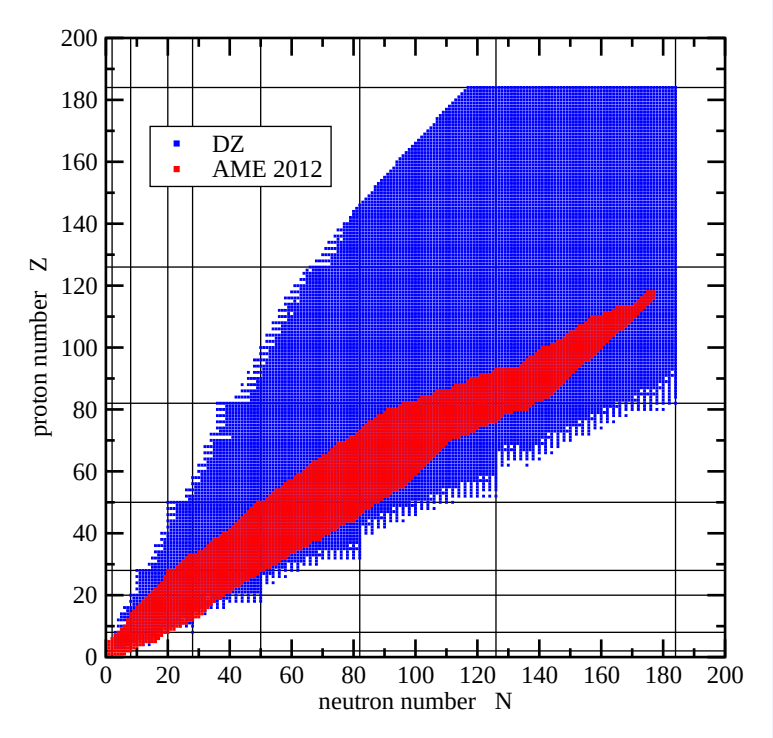

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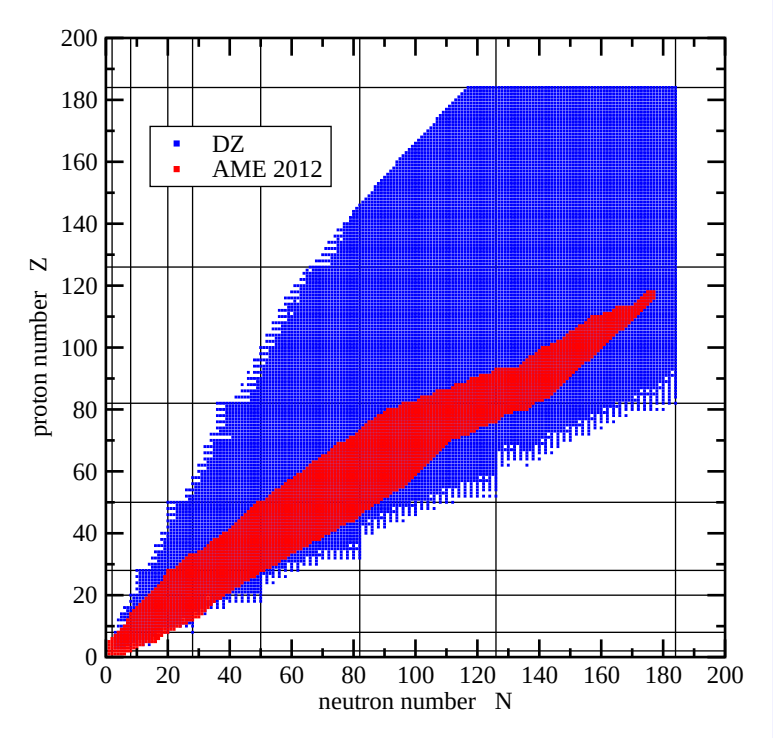

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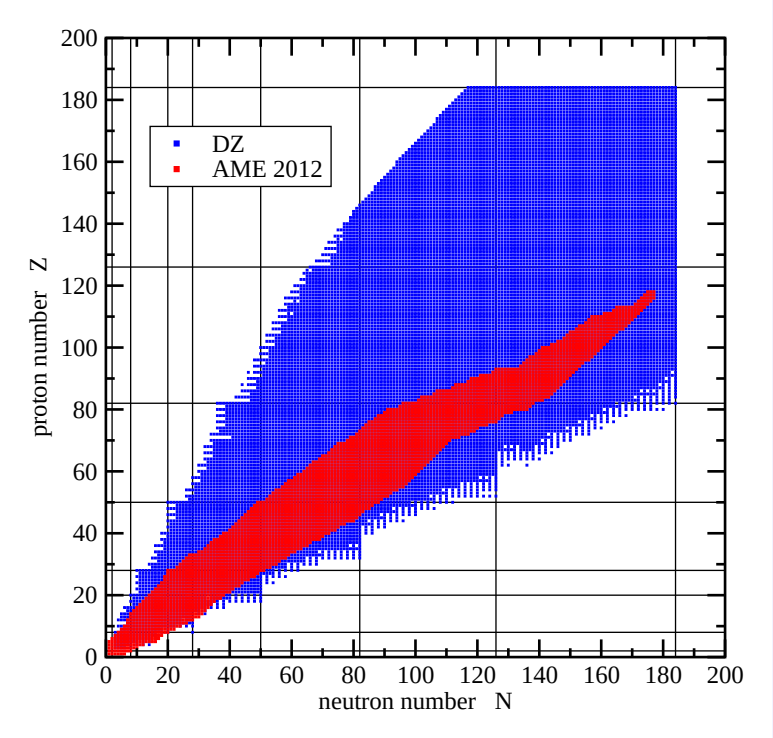

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  - represented by (classical) fields A_m with mass m_m
  - coupling to constituents: $\Gamma_{im} = g_{im}\Gamma_m$
    - scaling factors g_{im}

      e.g. $g_{i\omega} = g_{i\sigma} = N_i + Z_i$, $g_{i\rho} = N_i - Z_i$
    - density dependent $\Gamma_m = \Gamma_m(\varrho)$

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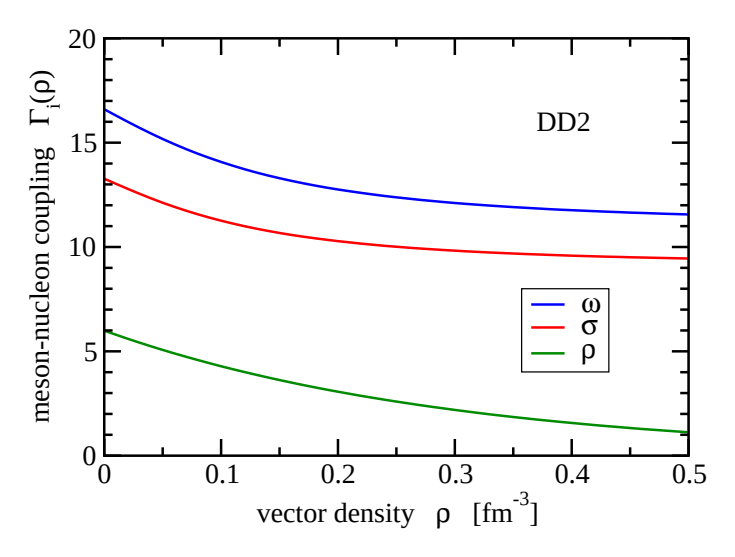
      (S. Typel et al., Phys. Rev. C 81 (2010) 015803)



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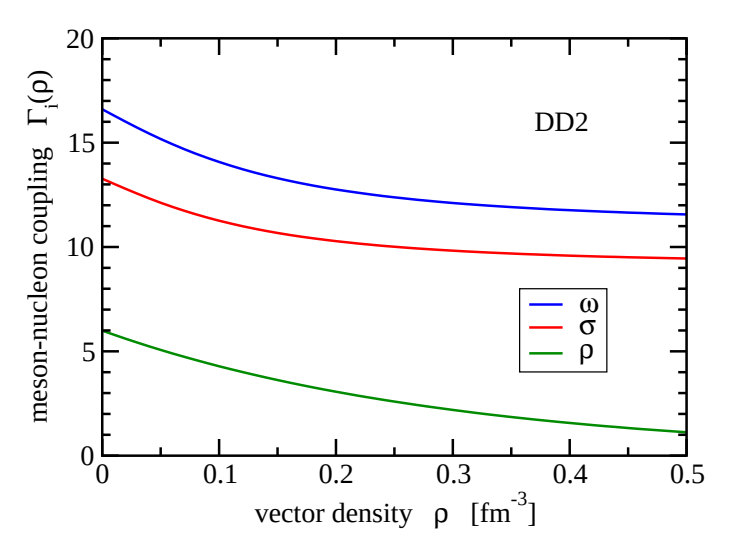
◦ scalar potential $S_i = \sum_{m \in \mathcal{S}} \Gamma_{im} A_m - \Delta m_i$

with medium-dependent mass shift $\Delta m_i(T, n_j)$



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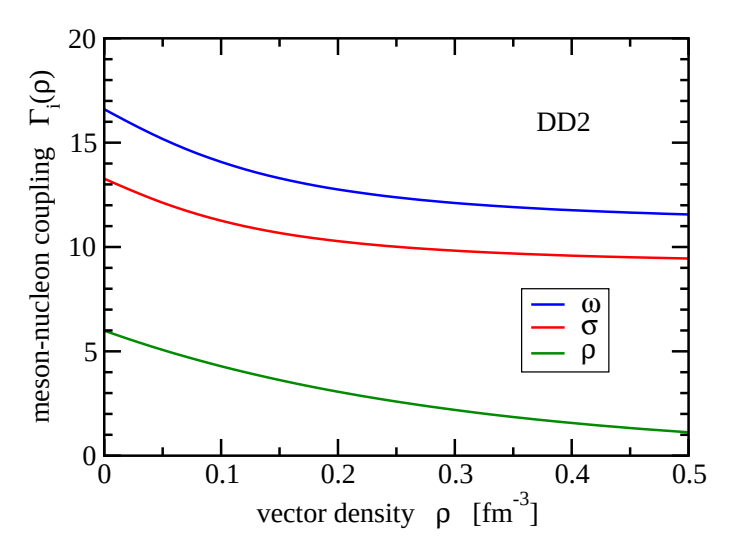

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  with medium-dependent mass shift $\Delta m_i(T, n_j)$
- vector potential $V_i = \sum_{m\in\mathcal{V}} \Gamma_{im} A_m + V_i^{(\mathrm{r})} + V_i^{(\mathrm{em})}$

  with "rearrangement" contribution $V_i^{(r)}$ (density dependence of couplings)

  and electromagnetic contribution $V_i^{(\mathrm{em})}$ (Coulomb correlations in uniform stellar matter!)

Effective Interaction

exchange of

- Lorentz scalar mesons $m \in \mathcal{S} = \{\sigma, \delta, \sigma_*, \ldots\}$
- Lorentz vector mesons $m \in \mathcal{V} = \{\omega, \rho, \phi, \ldots\}$

◦ represented by (classical) fields A_m with mass m_m

◦ coupling to constituents: $\Gamma_{im} = g_{im}\Gamma_m$

– scaling factors g_{im}

e.g. $g_{i\omega} = g_{i\sigma} = N_i + Z_i$, $g_{i\rho} = N_i - Z_i$

– density dependent $\Gamma_m = \Gamma_m(\varrho)$

$\varrho = \sum_i (N_i + Z_i) n_i$ with parametrization DD2

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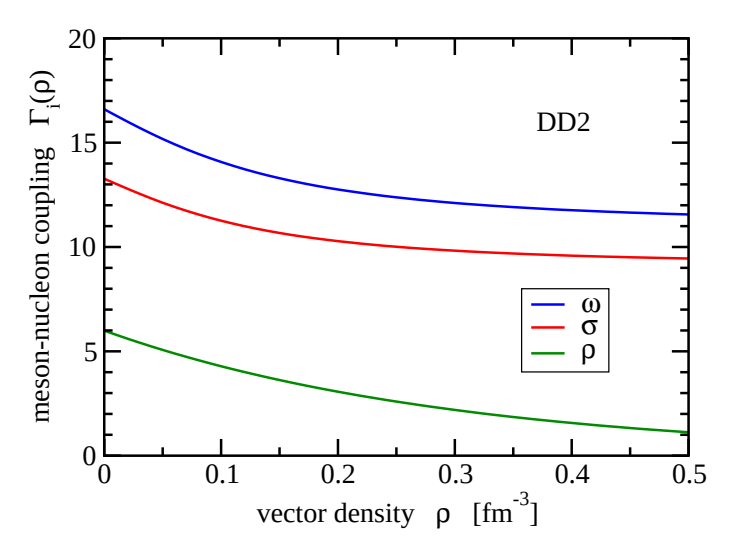

nuclear matter parameters

$n_{\mathrm{sat}} = 0.149$ fm^{-3}

$a_V = 16.02$ MeV

$K = 242.7$ MeV

$J = 31.67$ MeV

$L = 55.04$ MeV

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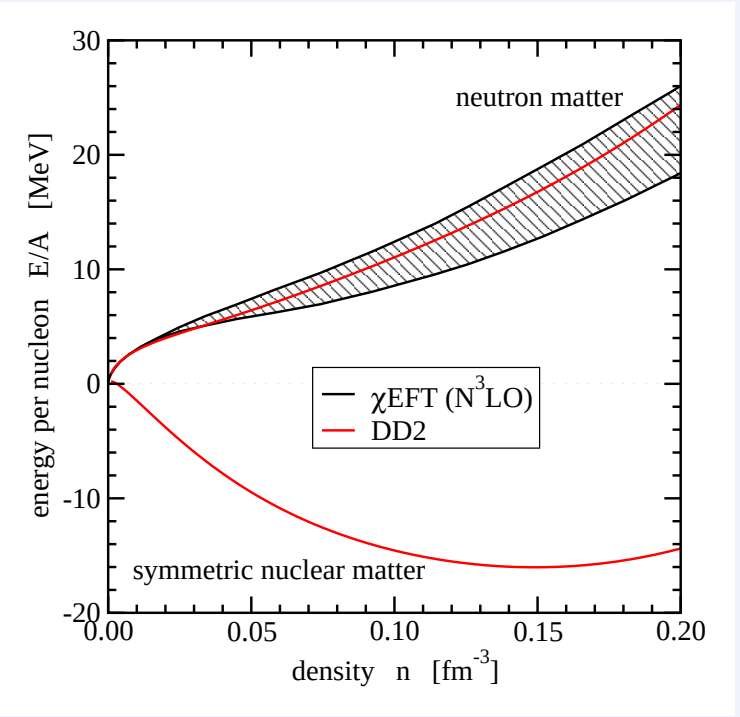

χEFT(N^{3}LO):

I. Tews et al., Phys. Rev. Lett 110 (2013) 032504

T. Krüger et al., Phys. Rev. C 88 (2013) 025802

Mass Shifts I

- concept applies to composite particles: **clusters**
  - light and heavy nuclei
  - nucleon-nucleon continuum correlations

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    (replaces traditional approach: excluded-volume mechanism)
- **electromagnetic shift** $\Delta E_i^{(\mathrm{Coul})}$ (in stellar matter)
  - electron screening of Coulomb field
    $\Rightarrow$ increase of binding energies

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light nuclei

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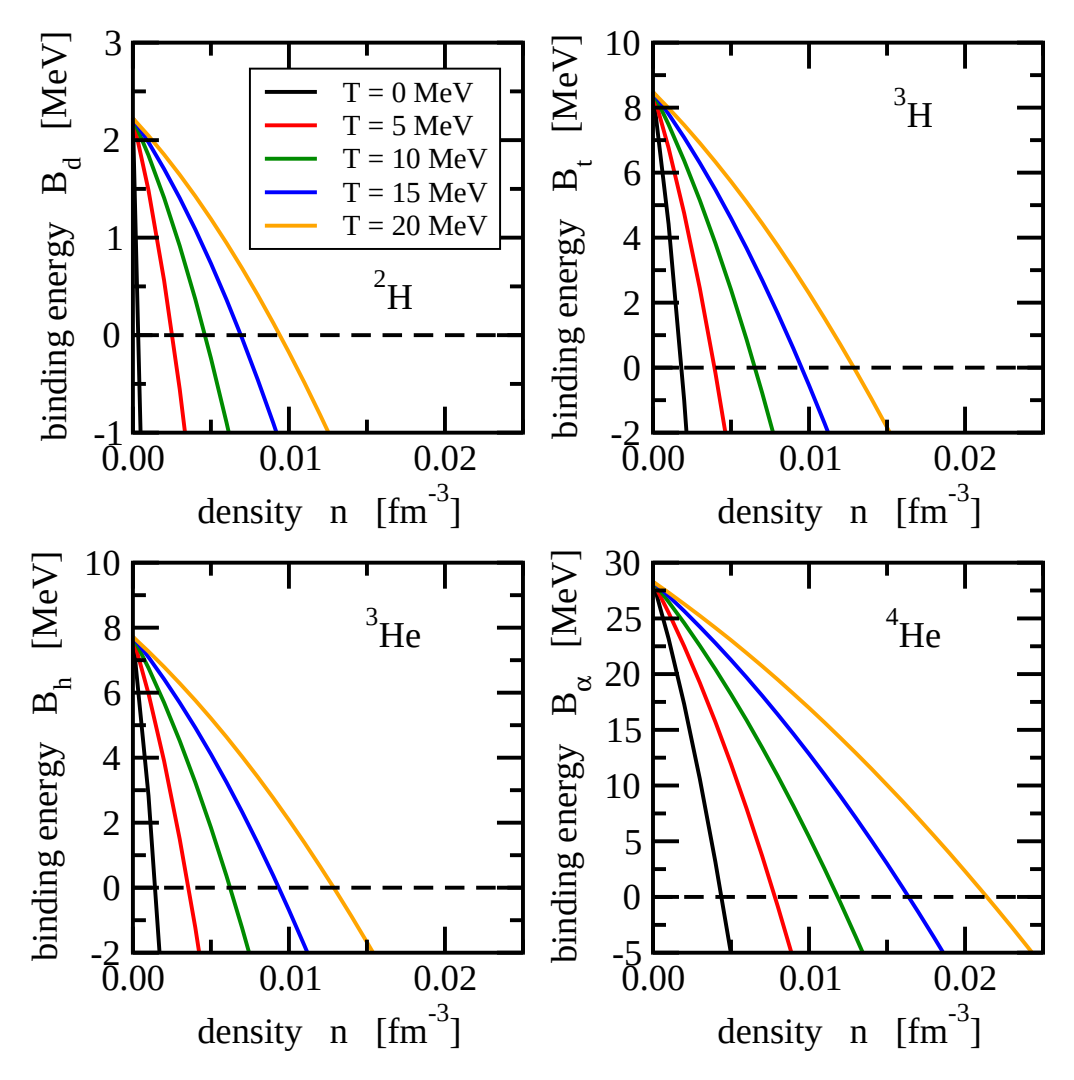
- example: symmetric nuclear matter, nuclei at rest in medium
- nuclei become unbound ($B_i < 0$) with increasing density of medium

  $\Rightarrow$ dissolution of nuclei



Mass Shifts III

heavy nuclei $(A > 4)$

- spherical Wigner-Seitz cell calculation
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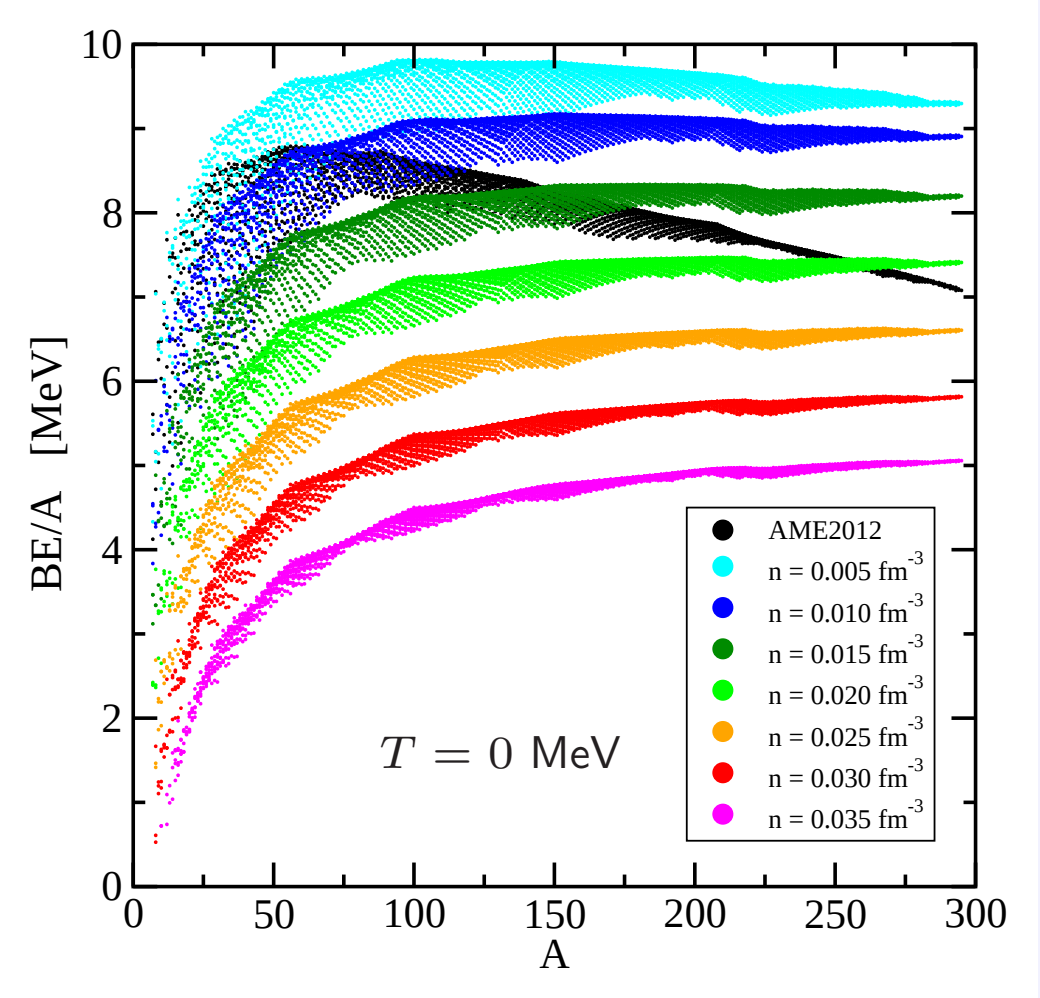

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- comparison with uniform matter
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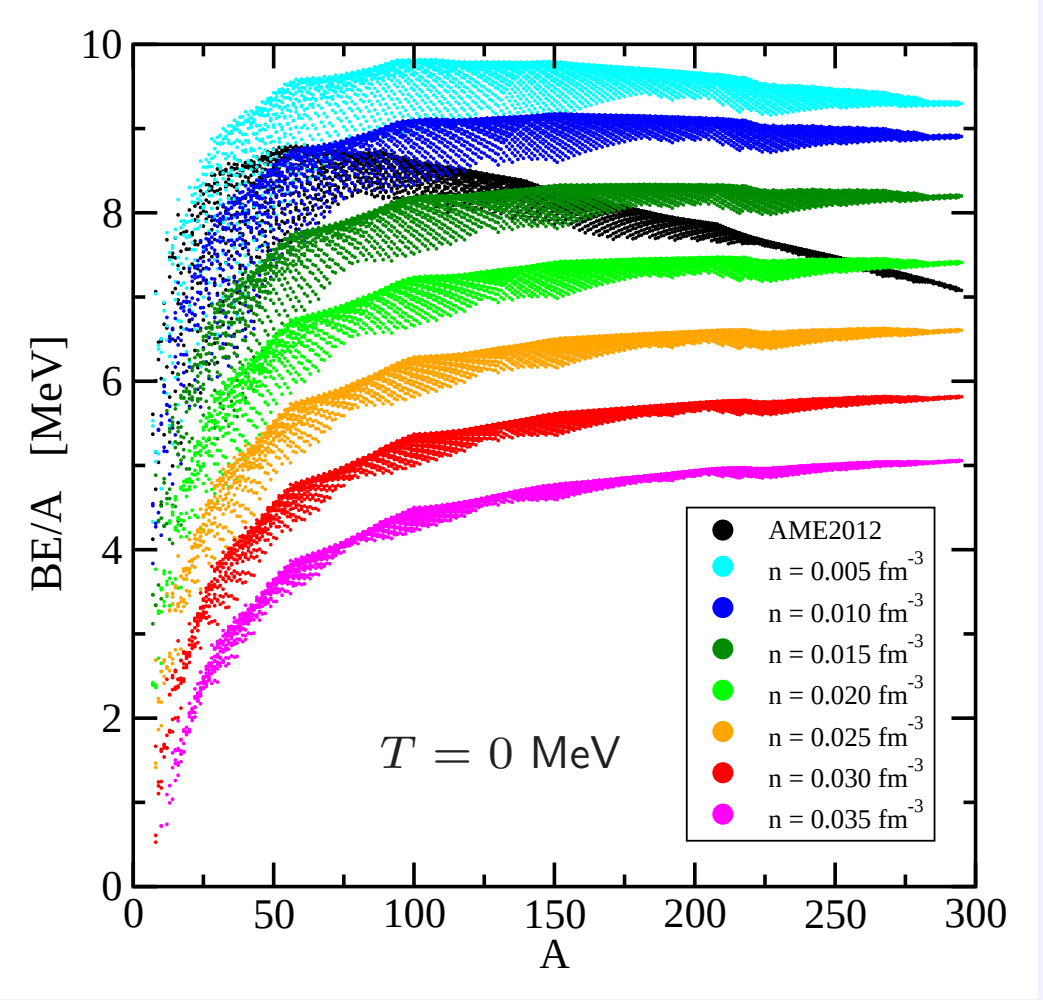
- preliminary parametrization
  $\Rightarrow$ improvement with more systematic calculations



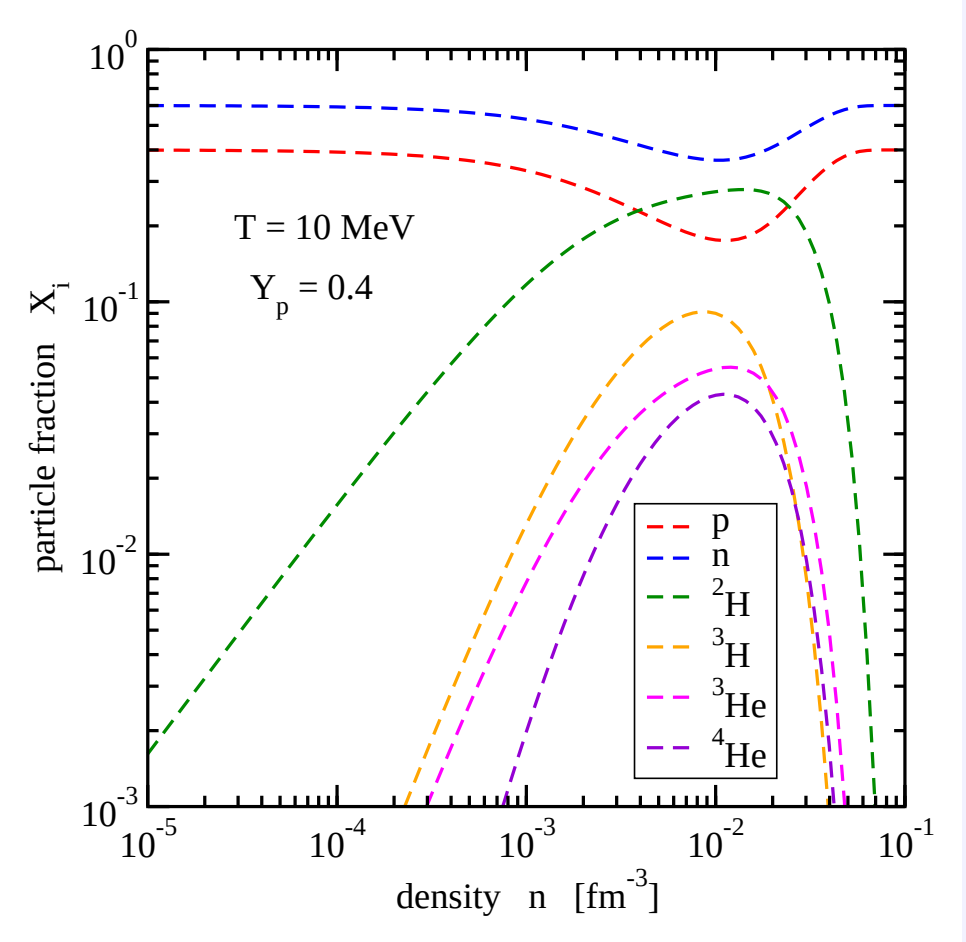
Chemical Composition of Nuclear Matter

- mass fractions

$$X_i = A_i \frac{n_i}{n_B} \qquad n_B = \sum_i A_i n_i$$

- low densities:
  two-body correlations most important
- high densities:
  dissolution of clusters
  $\Rightarrow$ Mott effect

generalized relativistic density functional



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- low densities:
  two-body correlations most important
- high densities:
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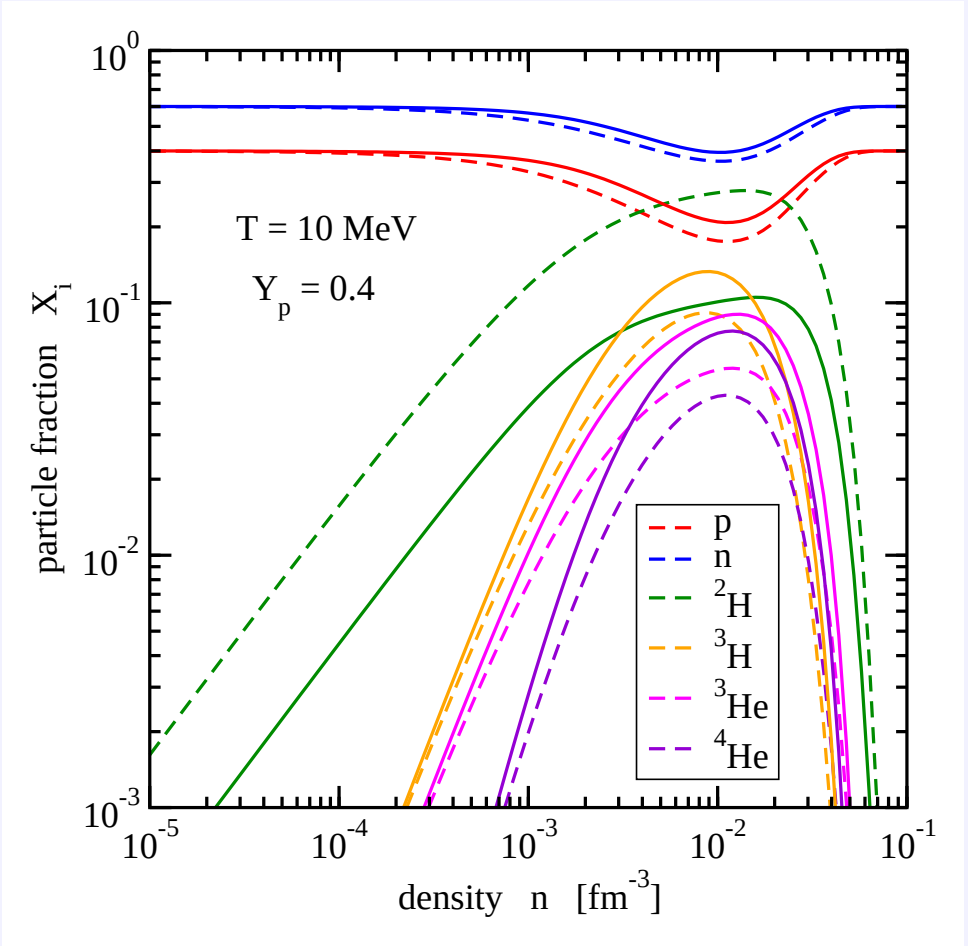
  $\Rightarrow$ Mott effect
- effect of NN continuum correlations
  - dashed lines: without continuum
  - solid lines: with continuum
  $\Rightarrow$ reduction of deuteron fraction,
  redistribution of other particles
  - essential for correct low-density limit

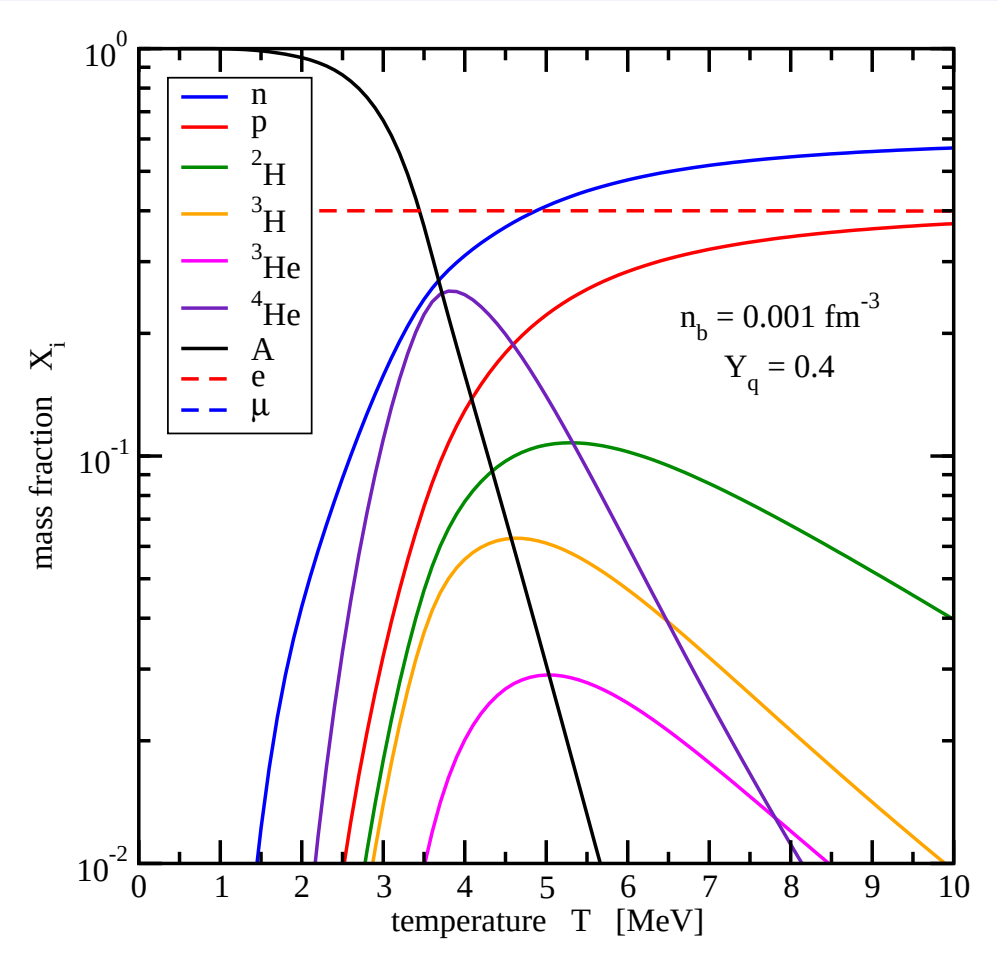
generalized relativistic density functional

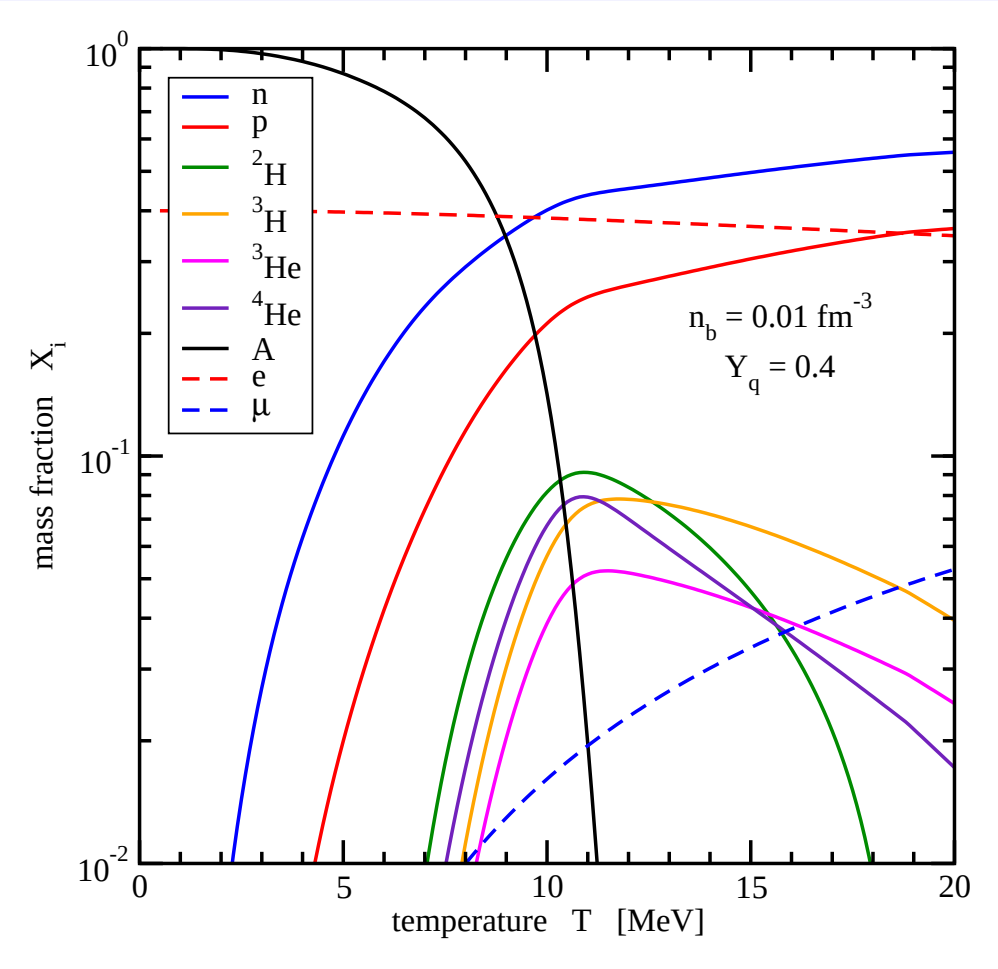


(without heavy clusters)

Chemical Composition of Stellar Matter I

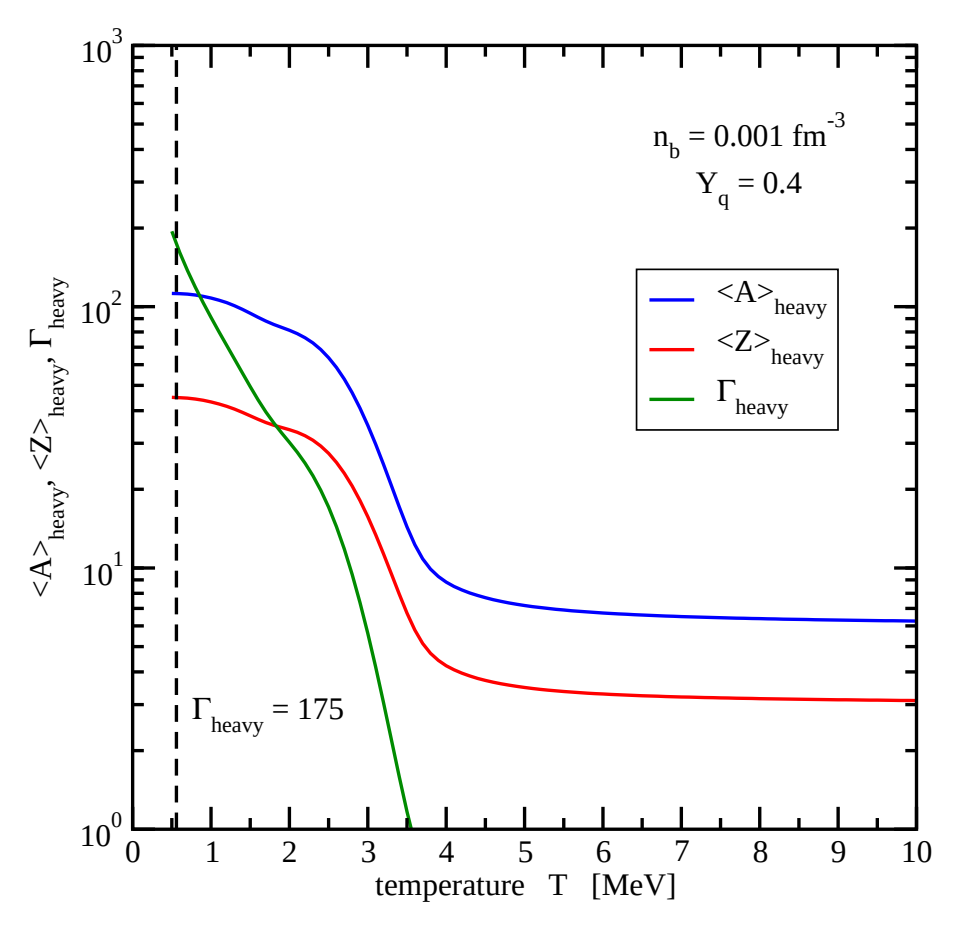
- full calculation in gRDF approach
- mass fractions of nucleons, light and heavy nuclei, electrons, muons

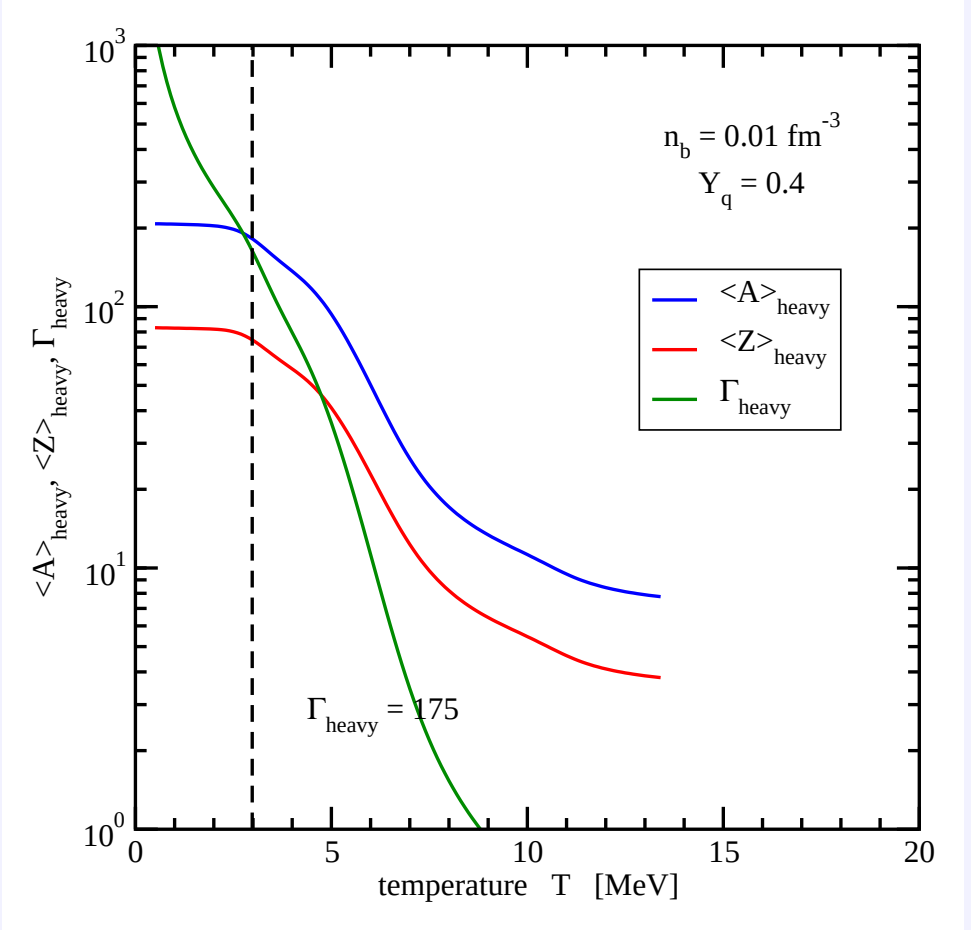




Chemical Composition of Stellar Matter II

- average mass number of heavy nuclei $\langle A \rangle_{\rm heavy} = \sum_{i,A_i>4} A_i n_i / \sum_{i,A_i>4} n_i$
- average charge number of heavy nuclei $\langle Z \rangle_{\rm heavy} = \sum_{i,A_i>4} Z_i n_i / \sum_{i,A_i>4} n_i$
- plasma parameter $\Gamma_{\rm heavy} = \langle Z \rangle^{5/3}_{\rm heavy} e^2/(a_q T)$ $\quad a_q = [3/(4\pi Y_q n_b)]^{1/3}$

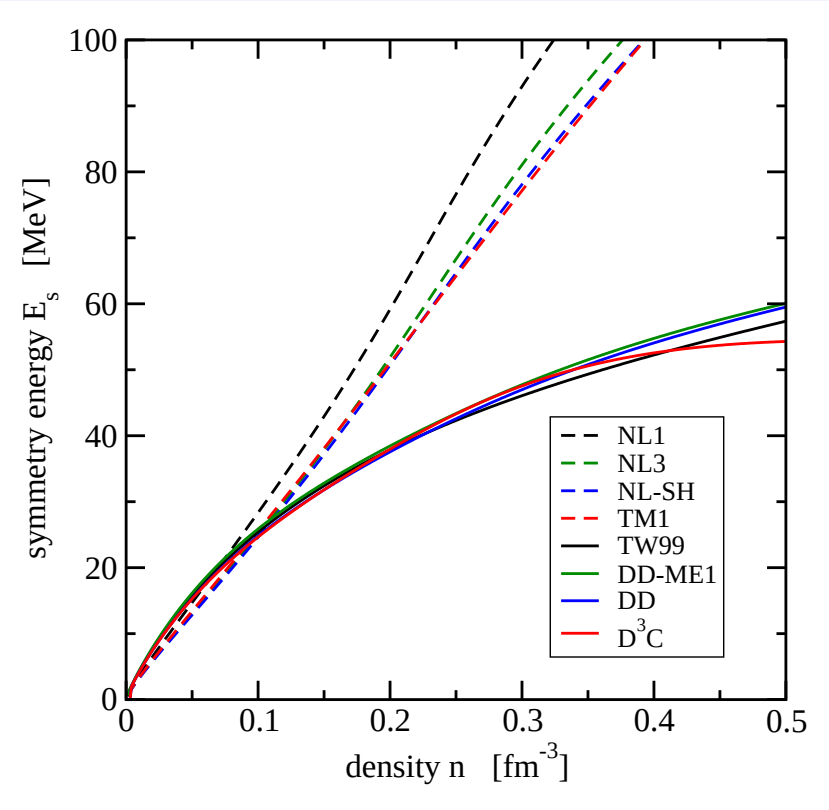




Symmetry Energy and Neutron Skins

Density Dependence of the Symmetry Energy

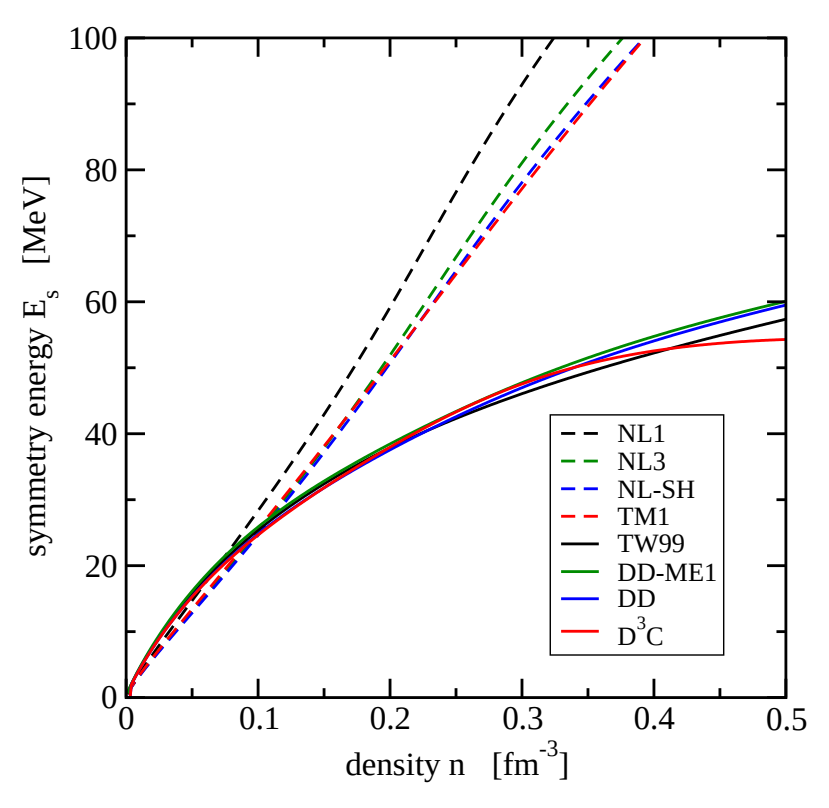
$$\frac{E}{A}(n,\beta) = E_0(n) + E_s(n)\beta^2 + \ldots \qquad n = n_n + n_p \qquad \beta = (n_n - n_p)/n$$

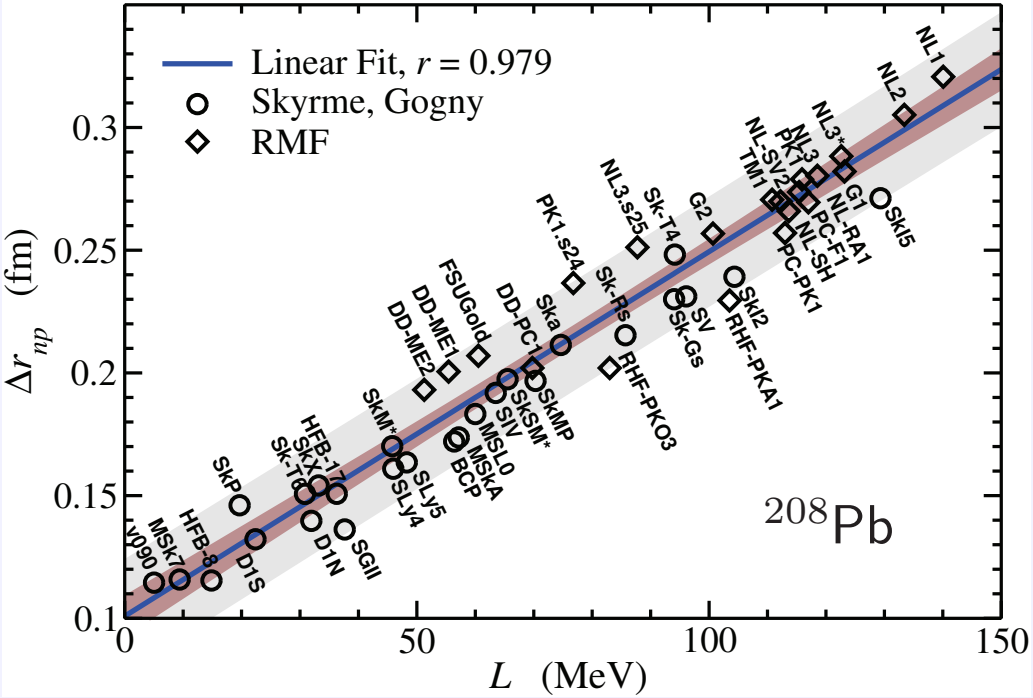


Density Dependence of the Symmetry Energy

$$\frac{E}{A}(n,\beta) = E_0(n) + E_s(n)\beta^2 + \ldots \qquad J = E_s(n_{\rm sat}) \qquad L = 3n\frac{d}{dn}E_s\Big|_{n=n_{\rm sat}}$$

- correlation: neutron skin thickness $\Delta r_{np} \Leftrightarrow$ stiffness of neutron matter EoS $\Leftrightarrow$ slope parameter L of symmetry energy E_s





(X. Viñas et al., Eur. Phys. J. A50 (2014) 27)

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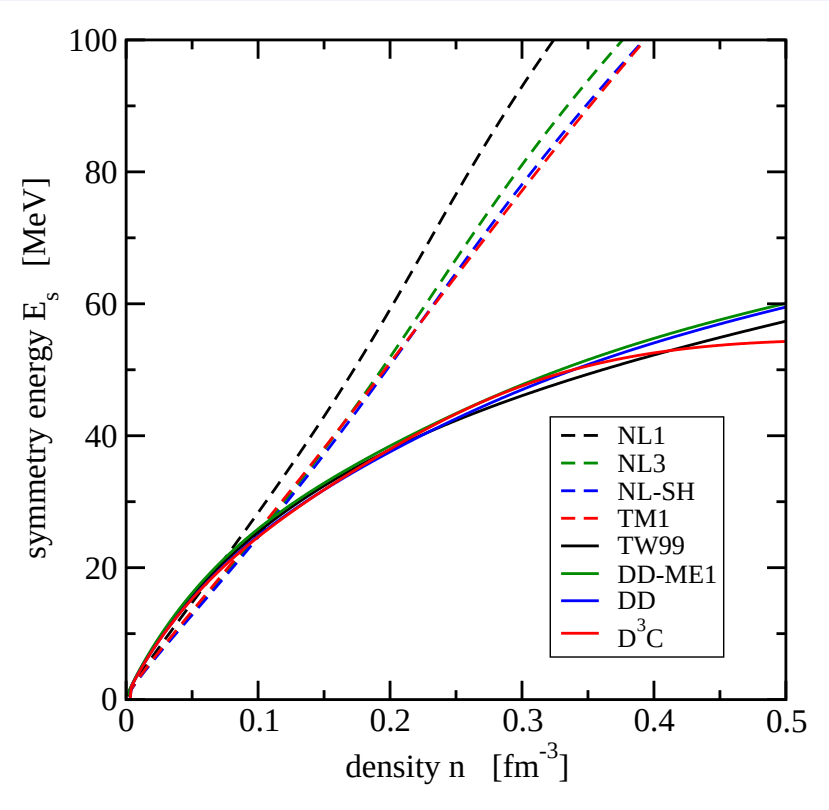
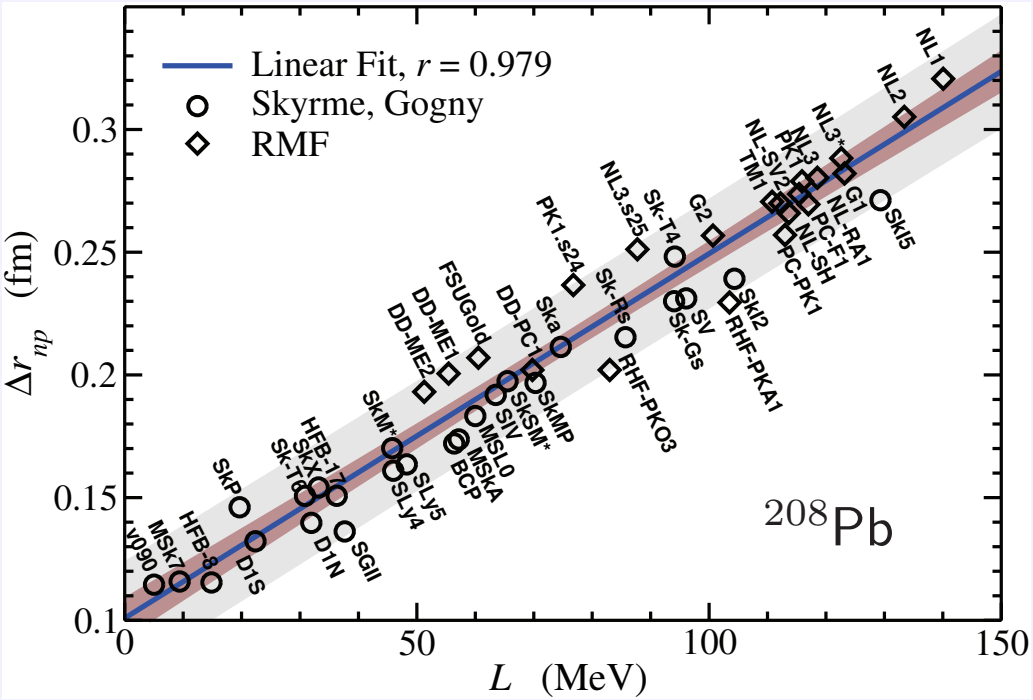
  based on mean-field calculations without cluster correlations





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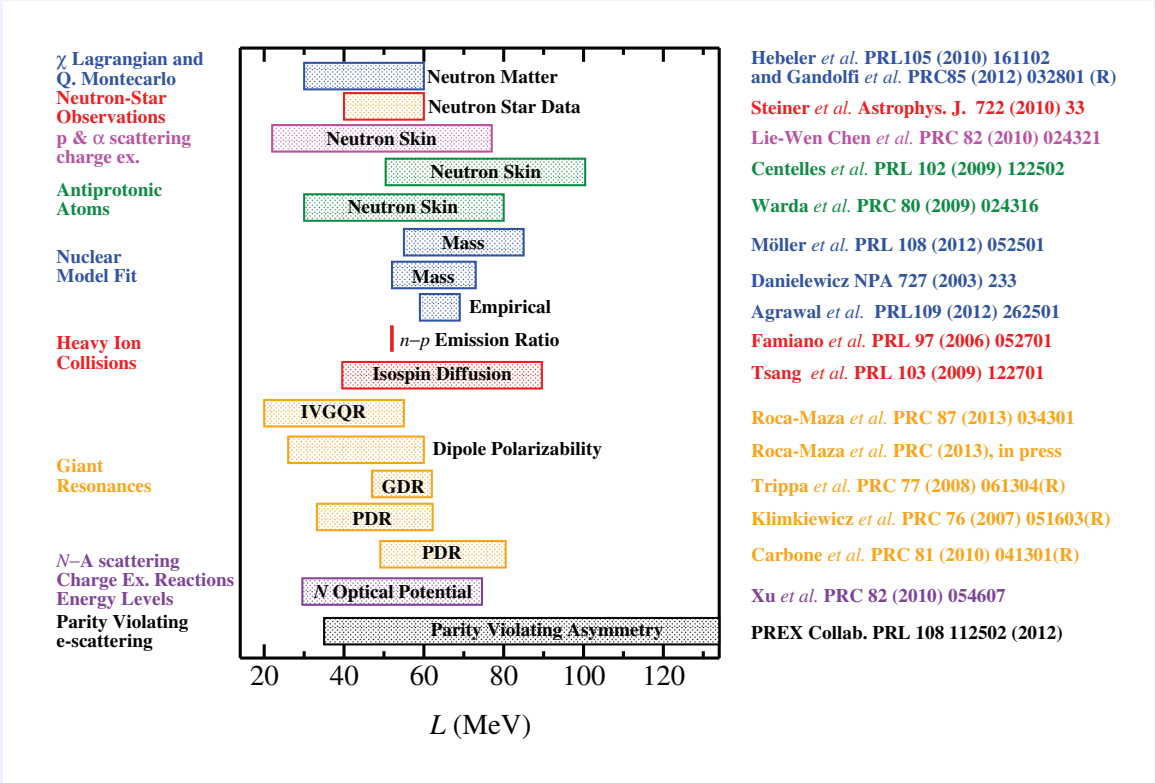
Symmetry Energy Parameters

- many attempts to determine $J = S_0 = S_v$ and L experimentally



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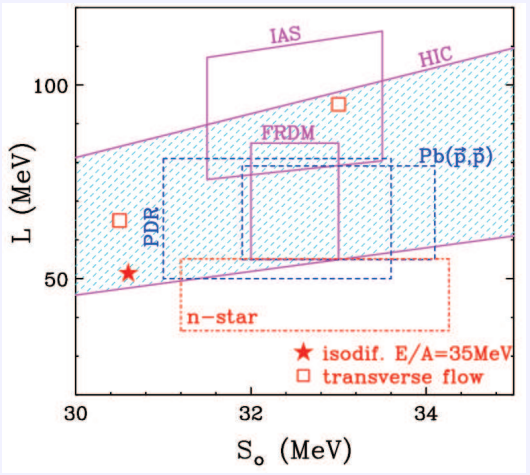
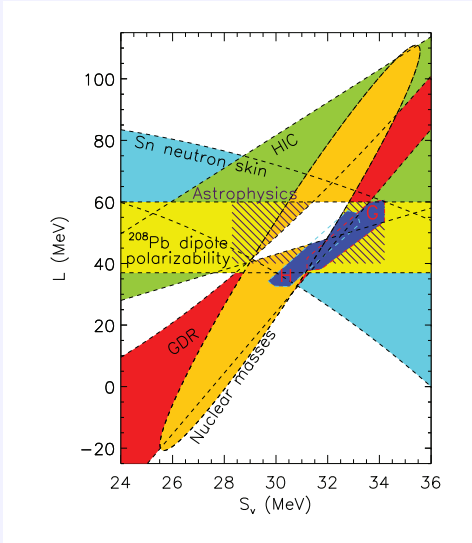


(M.B. Tsang et al., arXiv:1204.0466 [nucl-ex])

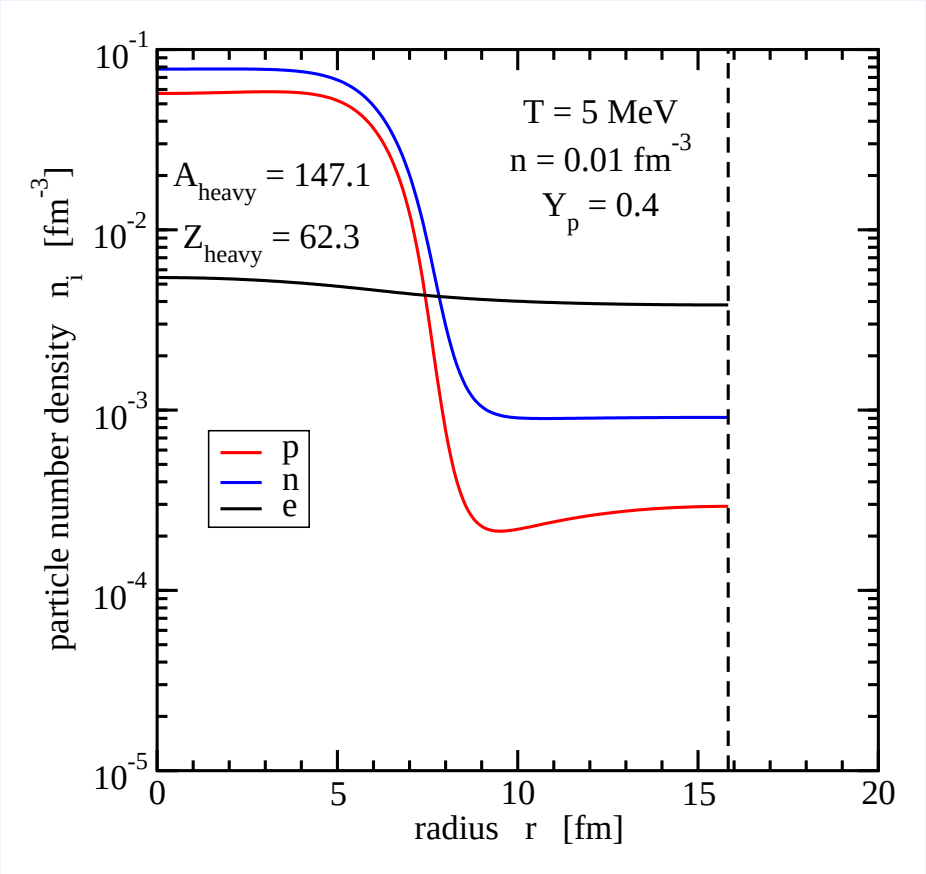


(J.M. Lattimer, Y. Lim, ApJ. 771 (2013) 51)

- measurement of neutron skin thickness (e.g. PREX), correlation with L from mean-field calculations: effects of clustering on surface of nuclei?

Neutron Skins with Cluster Correlations

- finite temperature gRDF calculations in spherical Wigner-Seitz cell, extended Thomas-Fermi approximation



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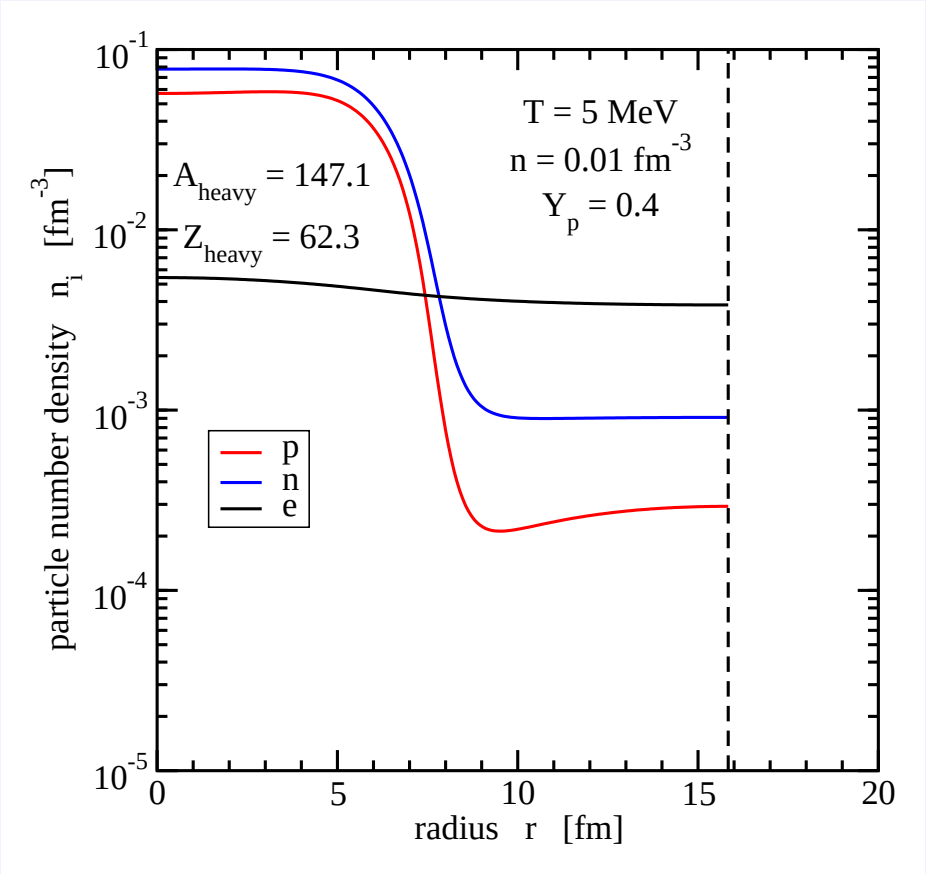

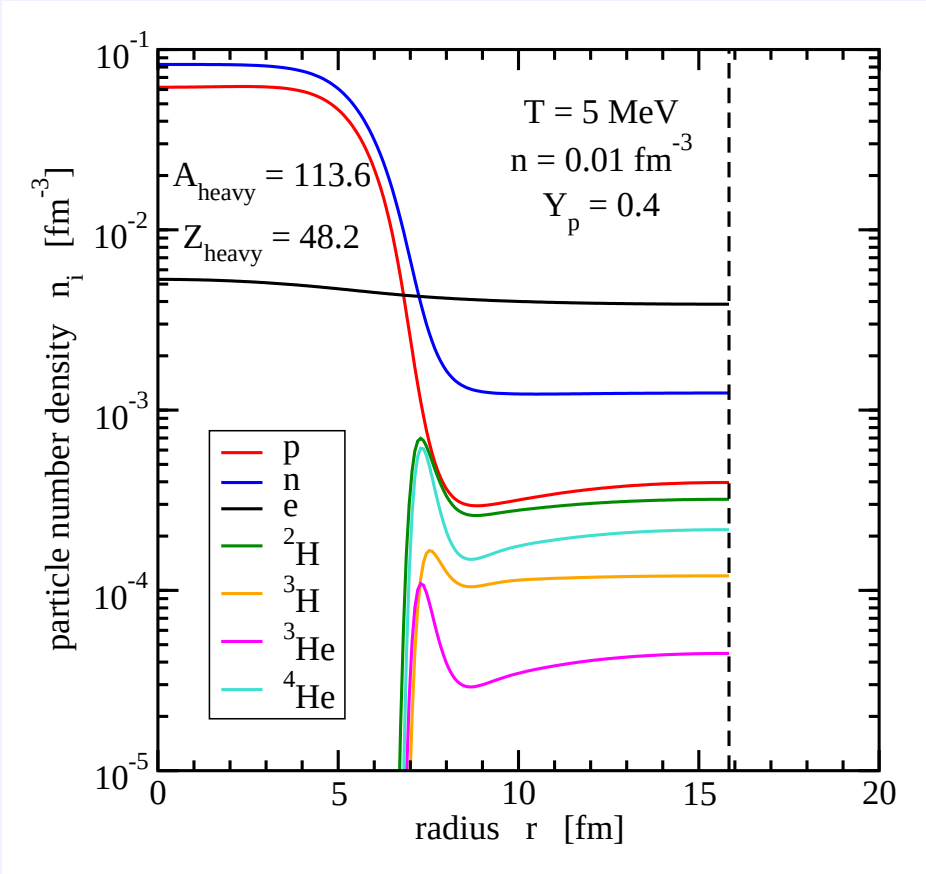

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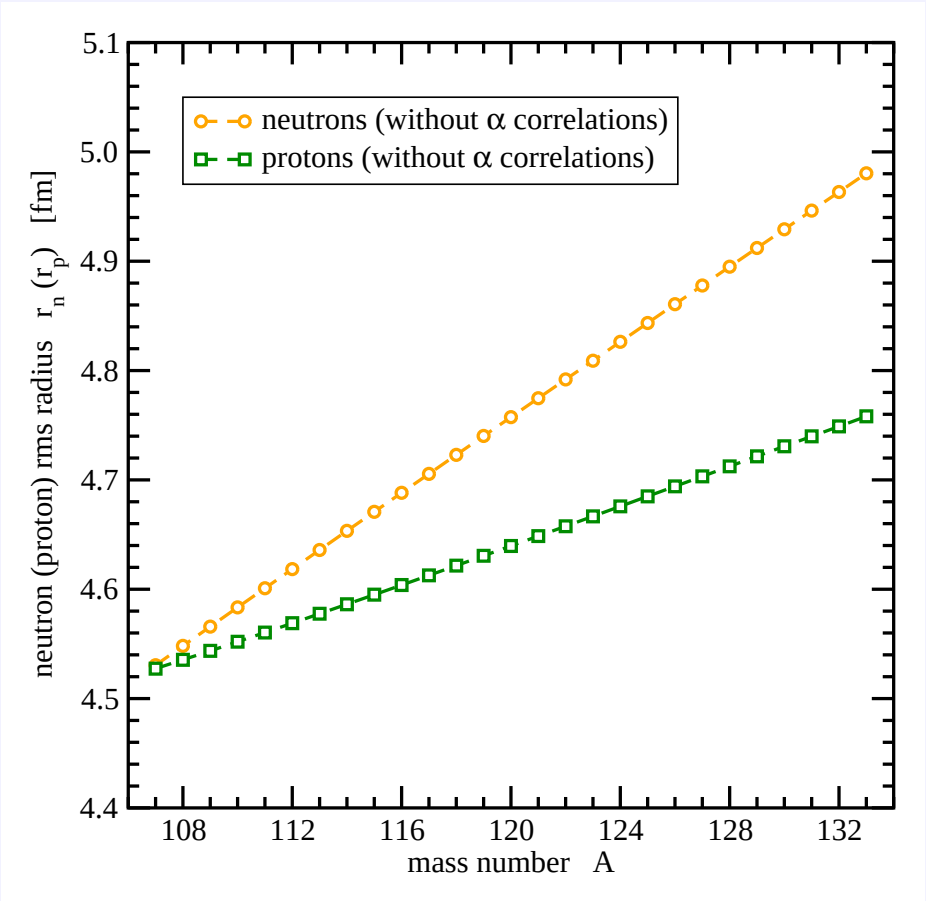
- variation of isovector interaction $\Rightarrow$ modified parametrizations, ^{208}Pb nucleus

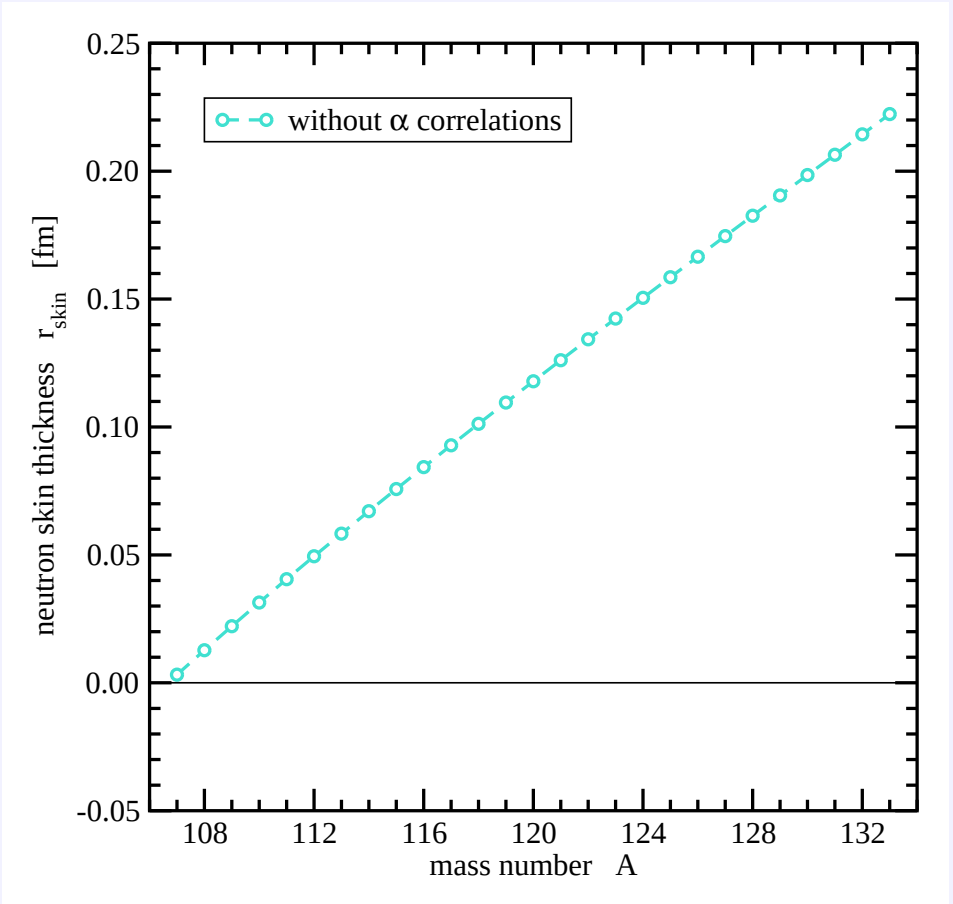
parametrization	symmetry energy J [MeV]	slope coefficient L [MeV]	ρ-meson coupling $\Gamma_\rho(n_{\rm ref})$	ρ-meson parameter a_ρ
DD2^{+++}	35.34	100.00	4.109251	0.063577
DD2^{++}	34.12	85.00	3.966652	0.193151
DD2^{+}	32.98	70.00	3.806504	0.342181
DD2	31.67	55.04	3.626940	0.518903
DD2^{-}	30.09	40.00	3.398486	0.742082
DD2^{--}	28.22	25.00	3.105994	1.053251

$$\Gamma_\rho(n) = \Gamma_\rho(n_{\rm ref}) \exp\left[-a_\rho\left(\frac{n}{n_{\rm ref}} - 1\right)\right]$$

Neutron Skin of Sn Nuclei

- neutron and protons rms radii r_n and r_p
- neutron skin thickness $r_{\mathrm{skin}} = r_n - r_p$





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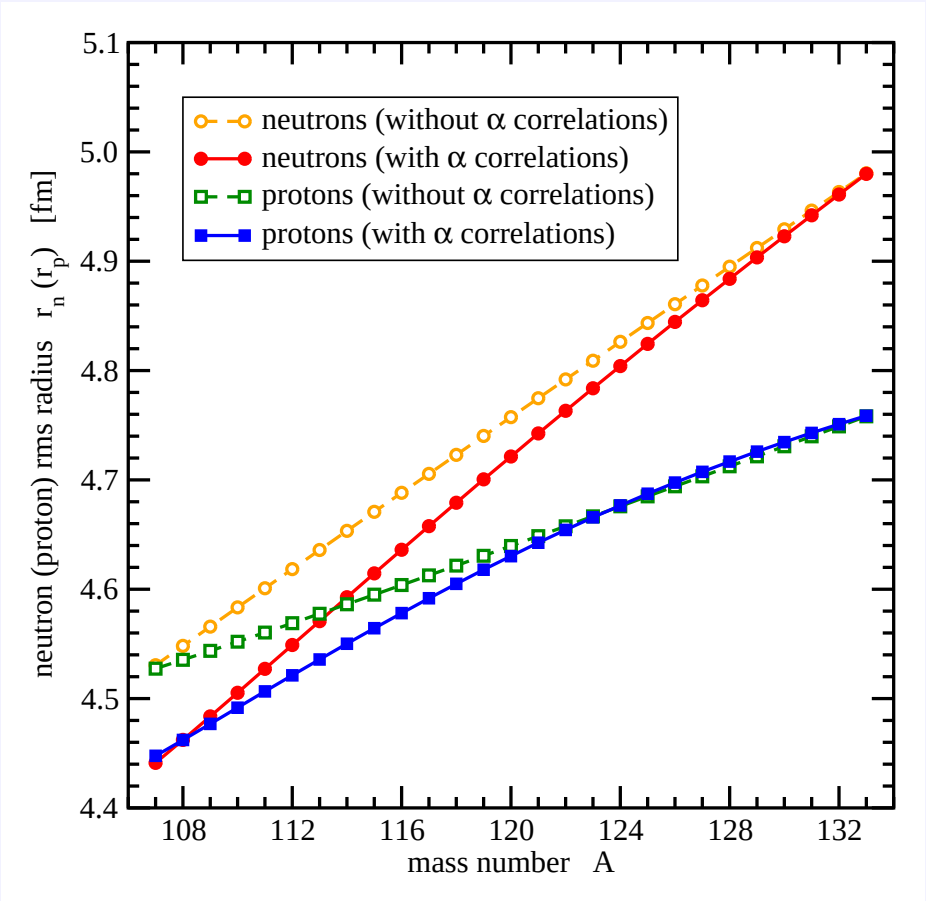
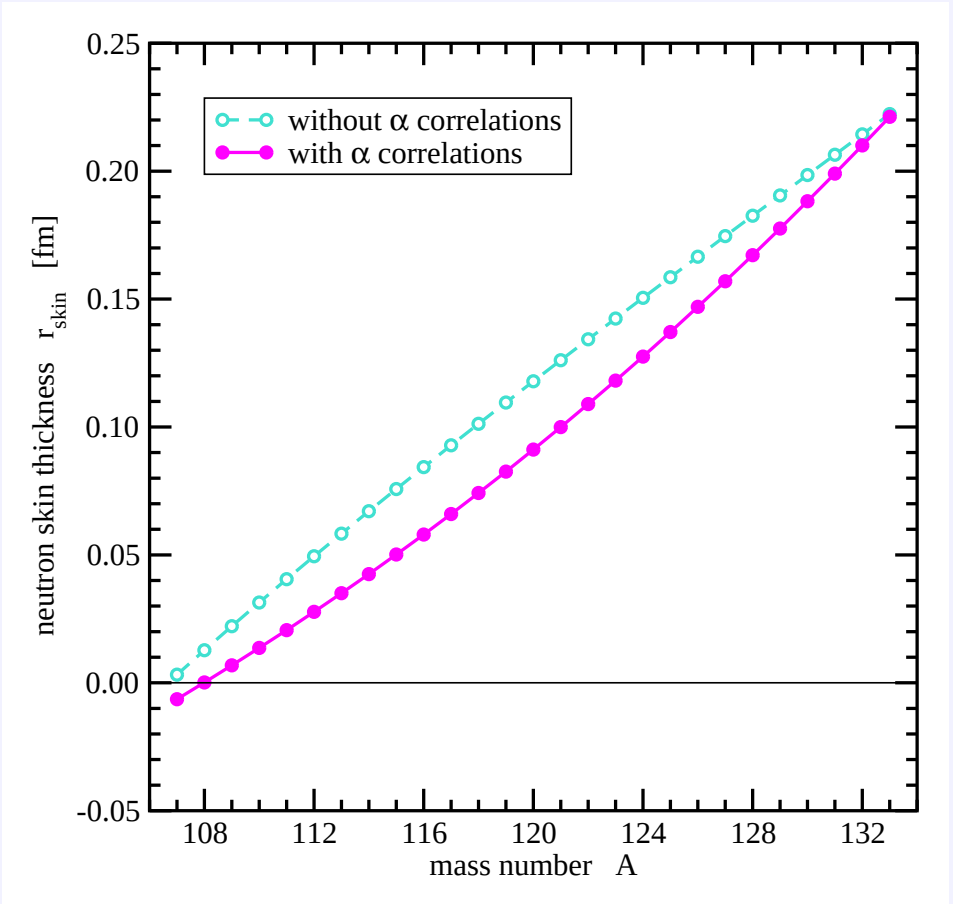
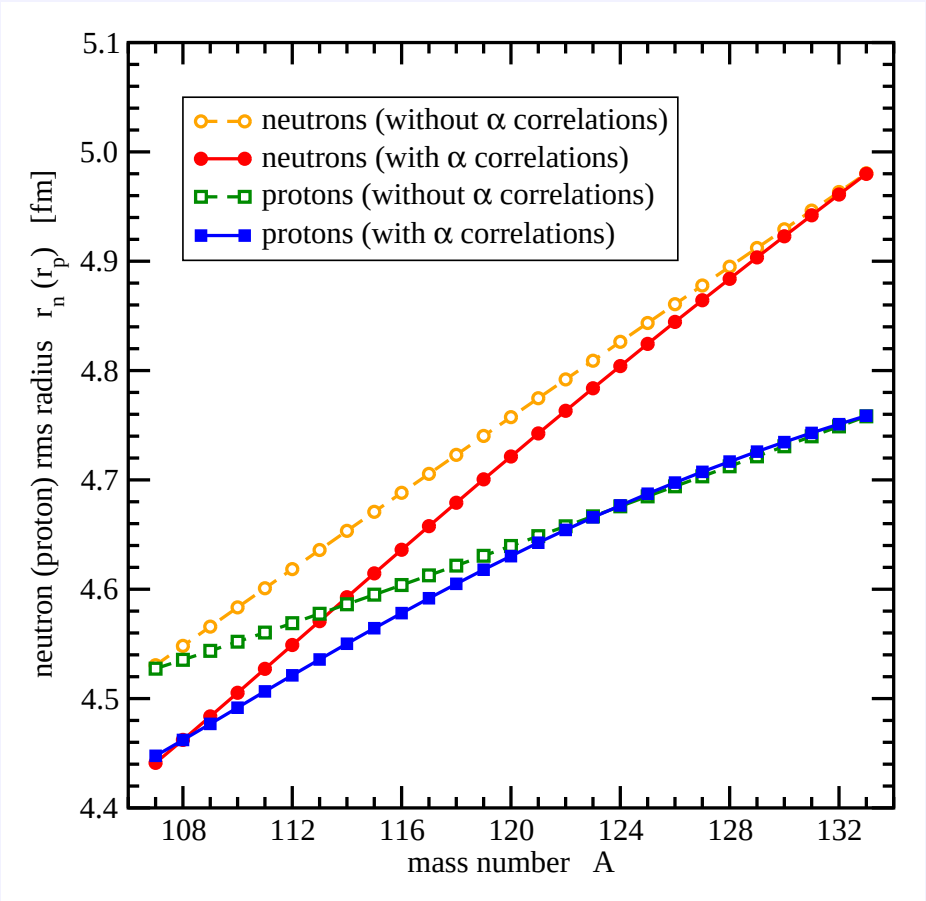


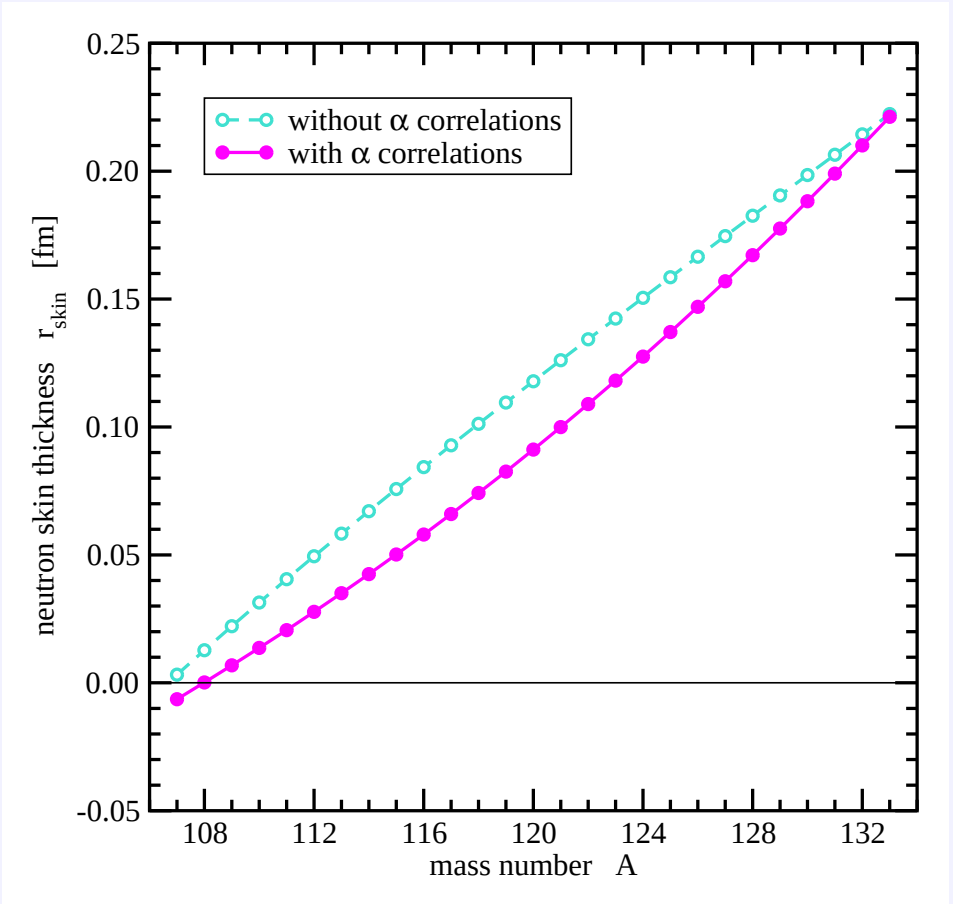


Neutron Skin of Sn Nuclei

- neutron and protons rms radii r_n and r_p
- neutron skin thickness $r_{\mathrm{skin}} = r_n - r_p$

$\Rightarrow$ experimental study: quasifree (p,pα) reactions (RCNP Osaka)

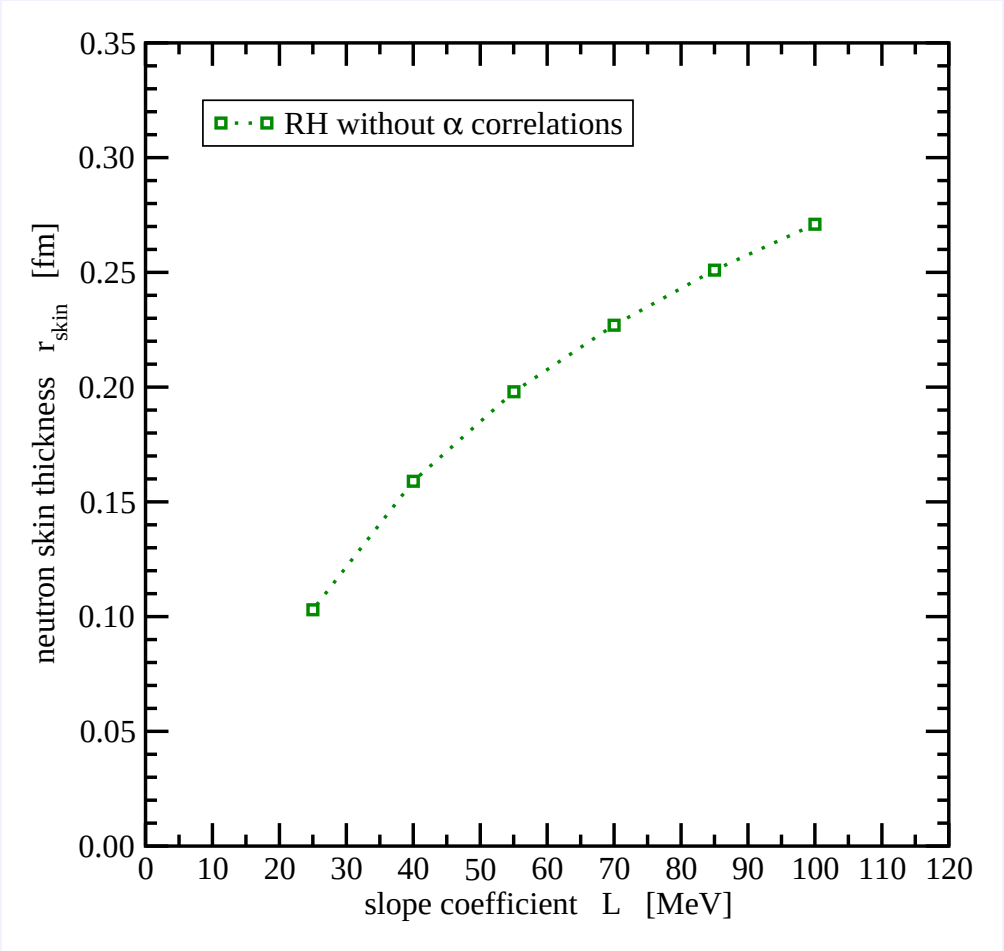




Neutron Skin of ^{208}Pb

- dependence on symmetry energy slope coefficient L
  $\Rightarrow$ use parametrizations
  DD2^{+++}, . . . , DD2^{--}
- relativistic Hartree (RH) calculation used in original fit of model parameters

  (correlation $r_{\mathrm{skin}} \leftrightarrow L$ not linear because no complete refit of model parameters, only of effective isovector interaction)



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- dependence on symmetry energy slope coefficient L
  $\Rightarrow$ use parametrizations DD2^{+++}, . . . , DD2^{--}
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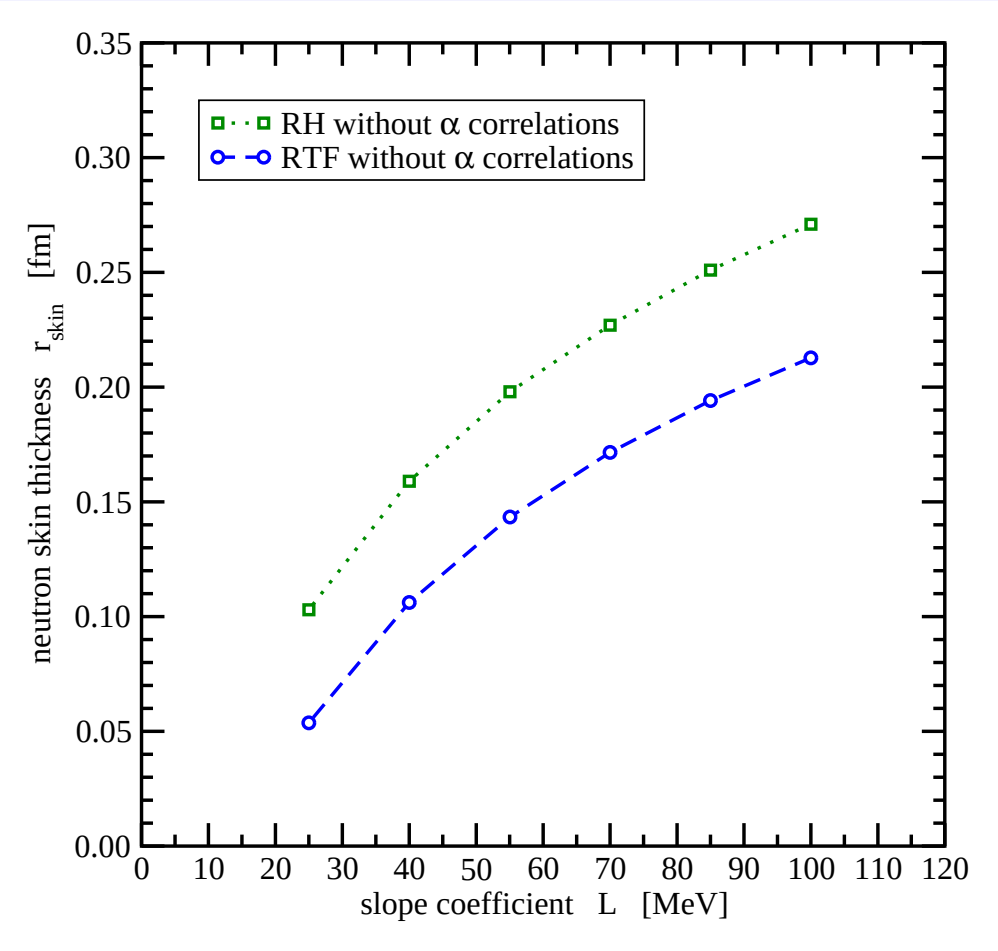

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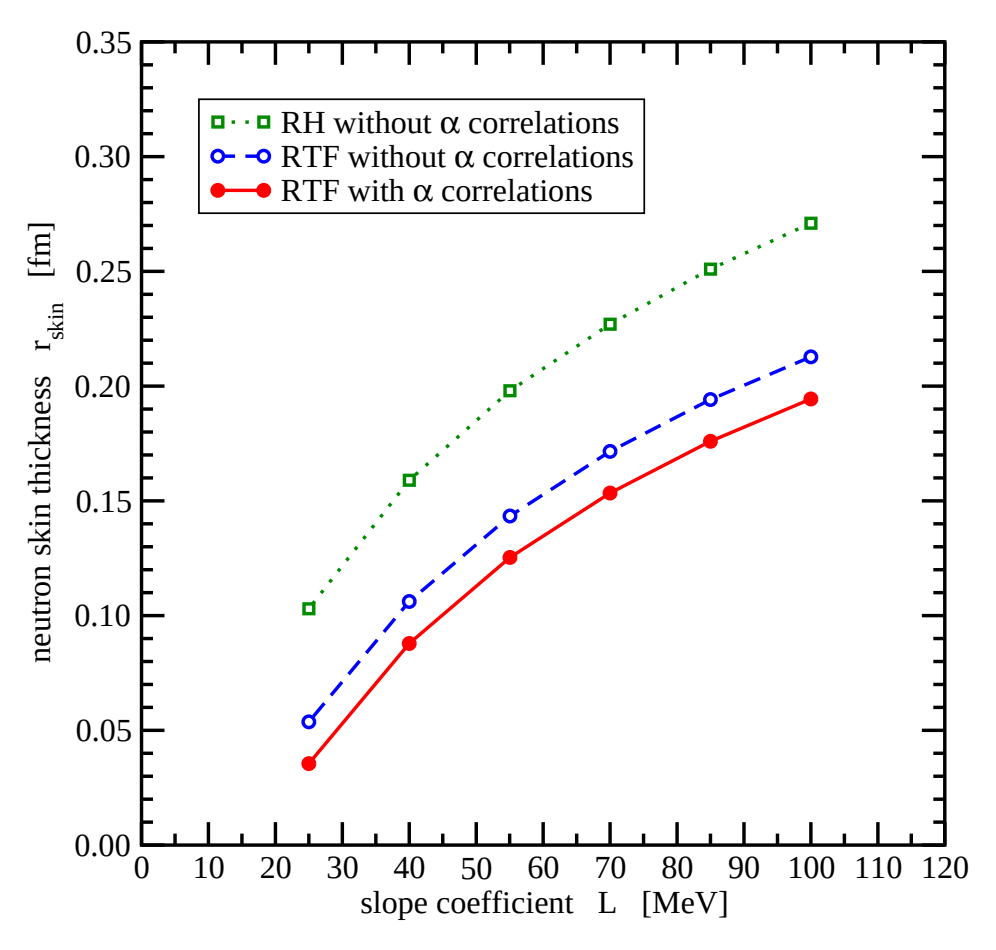
- with α-particles at surface
  $\Rightarrow$ reduction of neutron skin
  $\Rightarrow$ consequences for determination of L from r_{skin} measurements?



Outlook

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  $\Rightarrow$ generalization of **"excluded-volume" mechanism**
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  - thermodynamic consistency
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  - choose dependence $\Phi_i(\{n_j^{(s)}\}) \Rightarrow$ mass shifts S_i
  - $\Phi_i = 0 \Rightarrow$ particle i does not appear

Conclusions

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- nuclear/stellar matter: correlations in many-body system essential
  ⇒ clustering, phase transitions
  - modification of chemical composition and thermodynamic properties
- generalized relativistic density functional for dense matter
  - density-dependent couplings, well-constrained parameters
  - extended set of constituents: explicit cluster degrees of freedom, quasiparticle description
  - medium-dependent properties (mass shifts!) of composite particles
    ⇒ formation and dissolution of clusters, correct limits
  - thermodynamic consistency ⇒ rearrangement contributions
- applications:
  - equation of state of stellar matter ⇒ astrophysical simulations
  - nuclear structure ⇒ reduction of neutron skin
- future:
  - minor improvements of model ⇒ preparation of global EoS table
  - experimental study of α-particle correlations on surface of Sn nuclei
    ⇒ experiment with quasifree (p,pα) reactions at RCNP Osaka
  - extension with quarks ⇒ hadron-quark phase transition