Computer simulations of fluid dynamics

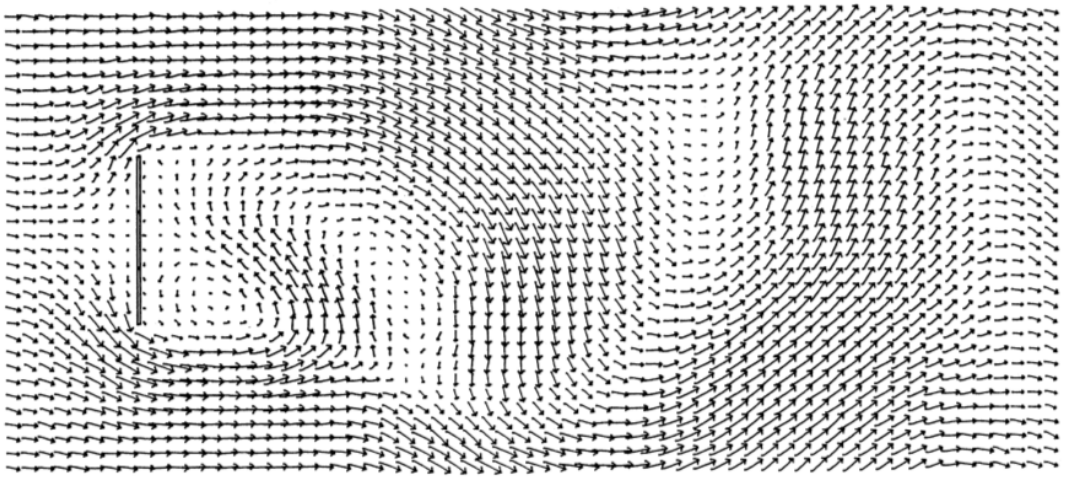
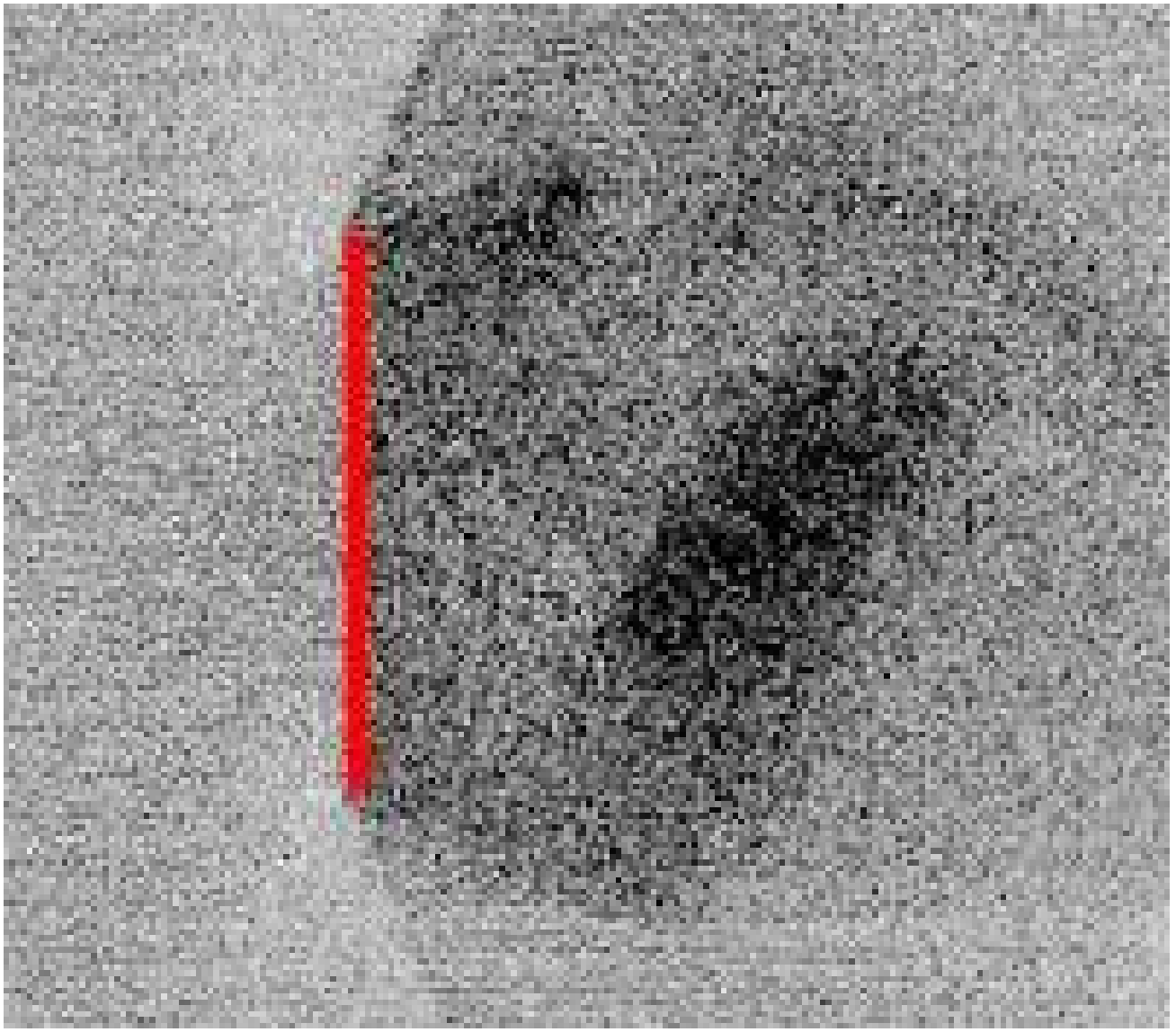


FIG. 3. Two-dimensional flow past a flat plate, simulated using the FHP lattice gas (d'Humières, Pomeau, and Lallemand, 1985). The Reynolds number is approximately 70.

Lecture 3: FHP Models (details, review)

Maciej Matyka

FHP (CUDA)



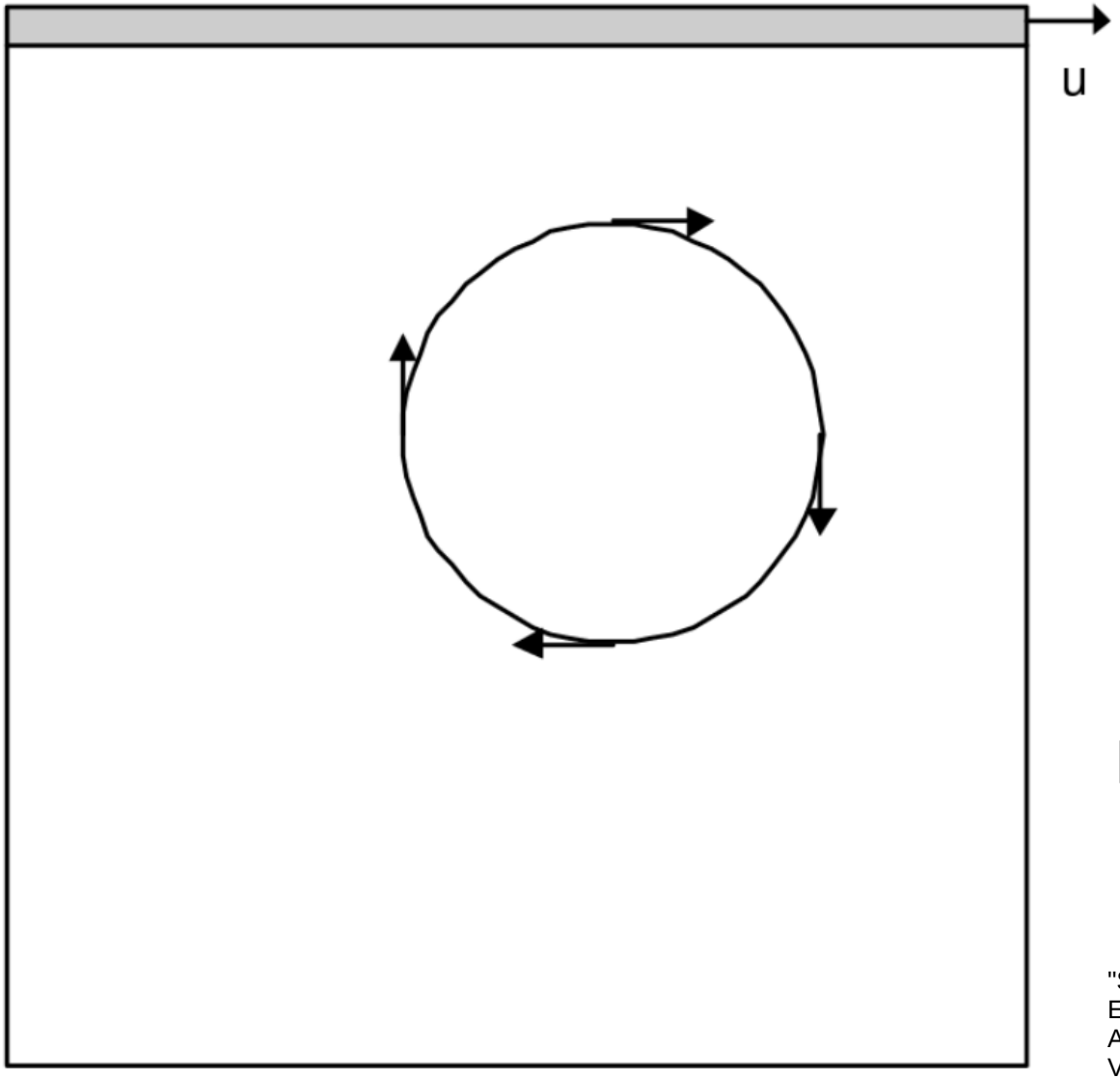
LatticeGasCUDA.mp4

By Lucas Campos

http://youtu.be/0xlRPnQGSXg

Challenge
Challenge

Driven Cavity

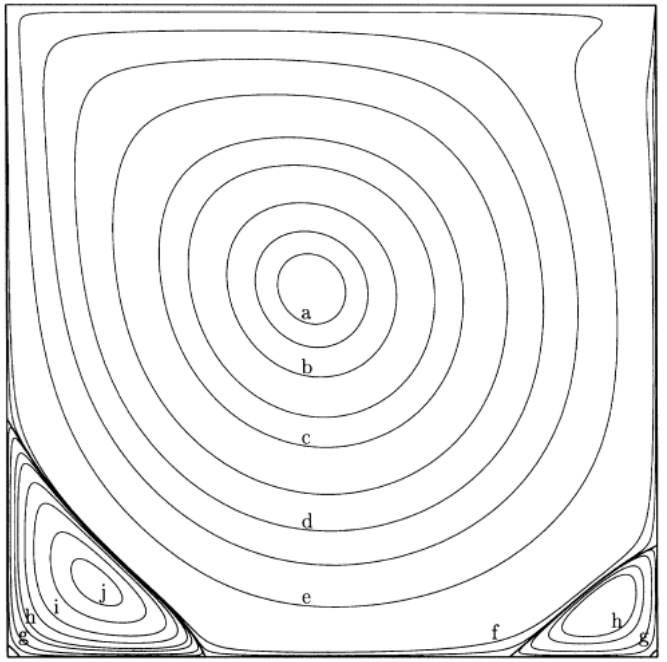


Project taken:
Wiktoria Saramak

"Solution to two-dimensional Incompressible Navier-Stokes Equations with SIMPLE, SIMPLER and Vorticity-Stream Function Approaches. Driven-Lid Cavity Problem: Solution and Visualization.", Maciej Matyka , University of Linkoping, CFD Project Report #3

Project idea: Lid-driven Cavity

- Implement FHP in another geometry
- **Lid-driven cavity** problem
- Capture vortices at various lid velocities
- Render movies with particles



Computers & Fluids Vol. 27, No. 4, pp. 421±433, 1998

10.2.2.1 Collision algorithms with logical operators

When the number of channels per node is relatively small, and if the collision rule is sufficiently simple, it may be efficient to design a combination of logical operators that transforms the pre-collision state into the post-collision state. With this combination, one can handle random Boolean variables for non-deterministic models. The solution is equivalent to a transcription with logical operators of the polynomial collision term of the microdynamic equation (see for example Equation (3.6) for the FHP-1 model). This transcription simply amounts to changing the multiplications and the additions by AND and OR operations, respectively. (The OR substitution is safely possible because in the microdynamic equations only one term in each sum can be non-zero.) Except for simple models such as HPP, FHP-1 and FHP-2, this method is prohibitively slow.

10.2.2.2 Collision algorithms with look-up tables

This solution is memory-consuming but rather fast, especially on machines where indirect addressing is efficient. The principle is simple: take the pre-collision state as an address in a table, and the information stored at this address gives the post-collision state. The table must have 2^b entries for a model with b channels per node. This rapidly becomes cumbersome for models such as FCHC which require up to 24 or even 26 channels per node. On a 64-bit machine, a look-up table for FCHC-8 requires 512 megabytes of random access memory, just to hold the table. Of course, a single table makes the model deterministic. This is often incompatible with the condition of G-invariance required for correct macrodynamics. Three solutions exist: (i) use several tables with a random swap between each table, (ii) keep a single table designed to incorporate some

2 ways

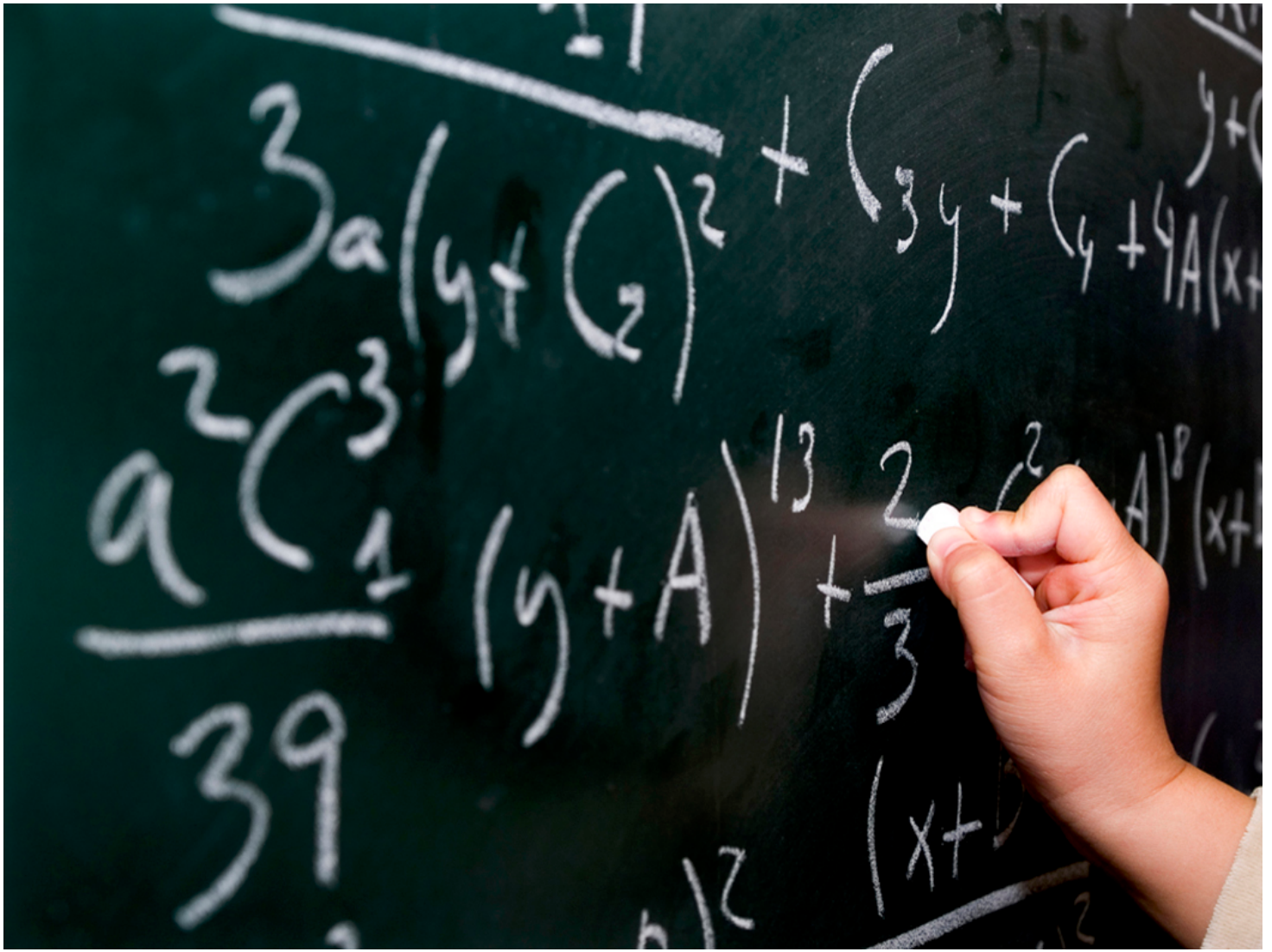
J.-P. Rivet,J. P. Boon, *Lattice Gas Hydrodynamics,* Cambridge Univ. Press 2005

Implementation details

- Collision tables
  - (MSc **S. Szczecina**)
  - (MSc **S. Szkoda**)

- Equation for site occupation evolution notes
  (B. Chopard, M. Droz)

Microdynamics of FHP model



(B. Chopard, M. Droz - Derivation)

Wielkości makroskopowe

- Prawdopodobieństwo, że cząsteczka zmierza do węzła w pozycji **r** w czasie t

$$N_i = \langle n_i(\vec{r}, t) \rangle$$

- Średnia po konfiguracjach
- Można też uśrednić w przestrzeni (wygładzenie pól)

Macroscopic variables

- Density

$$\varrho(\vec{r},t) = \sum_{i=1}^{6} N_i$$

- Momentum

$$\varrho(\vec{r},t)\vec{u}(\vec{r},t) = \sum_{i=1}^{6} \vec{v}_i N_i$$

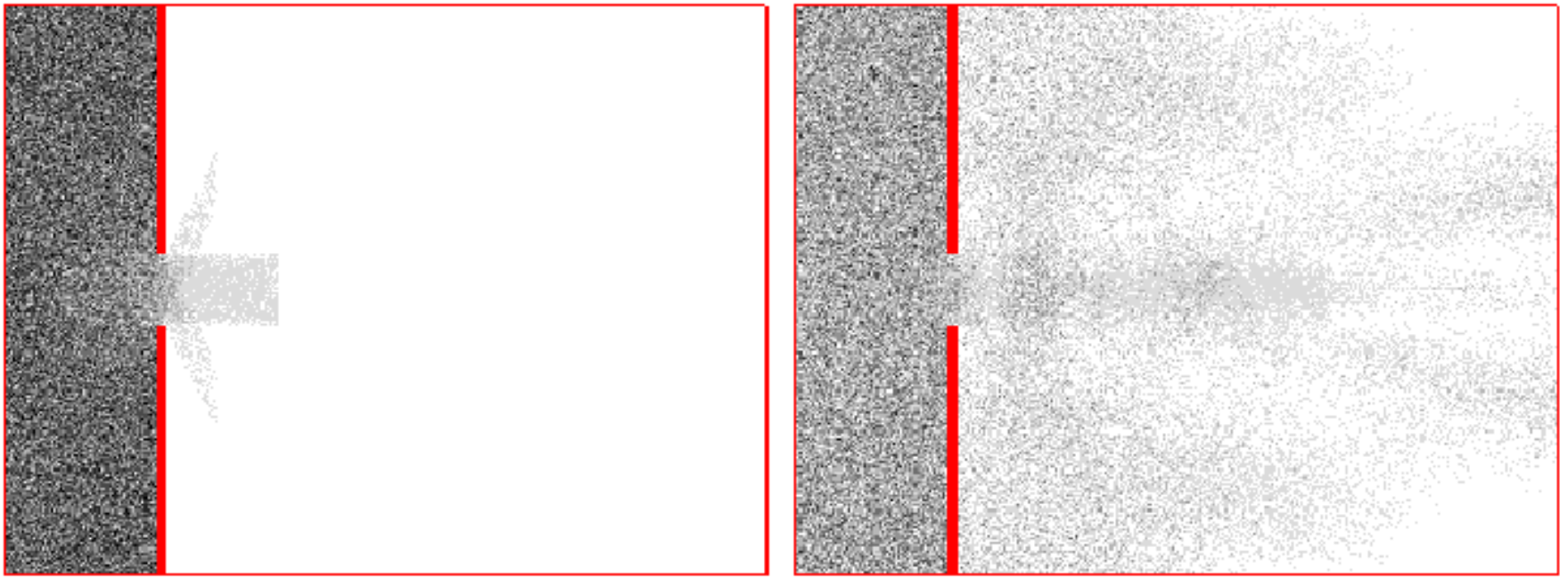
Practical questions

- **Lattice size**
  - Keep it resonable & adopt to problem
  - 300 x 60 for Poiseuille is OK at start
  - Good practice: adjust & test various sizes

Practical questions

- **Visualization: produce macroscopic u,v,rho fields**
  1) Export velocity profile
  2) Export velocity to .vtk file (lecture?) → Paraview
  3) Visualize realtime
     - Use Open Frameworks (http://openframeworks.cc/)

Model FHP5



(animation fhp5-diffusion2.avi)

Determinism?

- Cellular automata – **deterministic by definition**
- FHP III consists of **stochastic rules** – non deterministic
- It may be implemented in **deterministic** way

  (B. Chopard, M. Droz)

Field-programmable gate array

Journal of VLSI Signal Processing 12, 51–66 (1996)
© 1996 Kluwer Academic Publishers. Manufactured in The Netherlands.

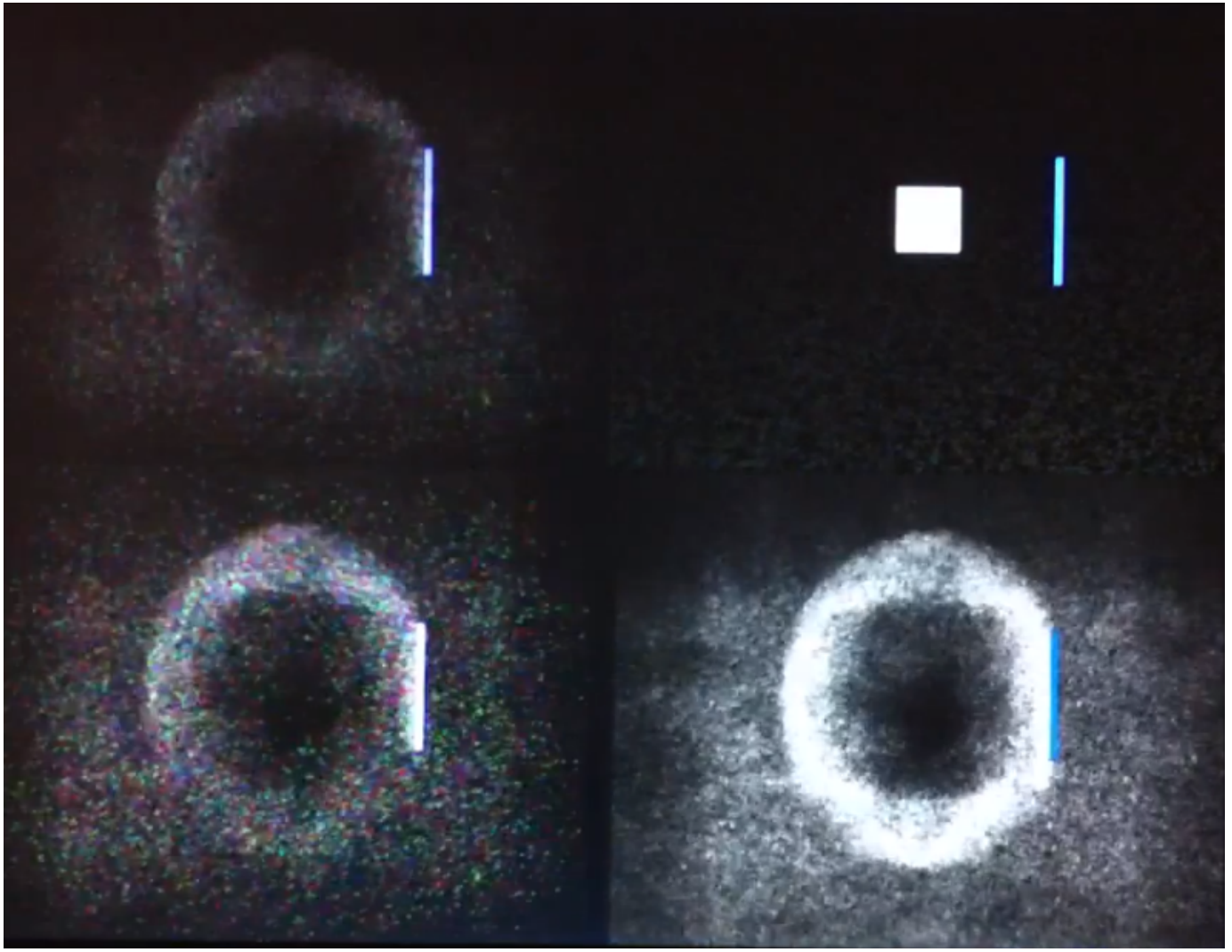
Implementation of Lattice Gases Using FPGAs

PAUL SHAW, PAUL COCKSHOTT AND PETER BARRIE
Department of Computer Science, University of Strathclyde, Glasgow

Received November 25, 1994; Revised July 14, 1995

Abstract. Lattice gas models have been widely studied over the last decade due to their simplicity and scope for parallelism. Standard parallel computers based on the stored-program paradigm can run such models quickly but are expensive. We report here a new approach based on reconfigurable logic circuits. A circuit is constructed to realize the behaviour of the model. The suitability of this method is demonstrated by modelling sound propagation in a lattice gas. For this application it is shown that supercomputer performance can be achieved at a fraction of supercomputer cost.

Exemplary results (FHPII on FPGA)



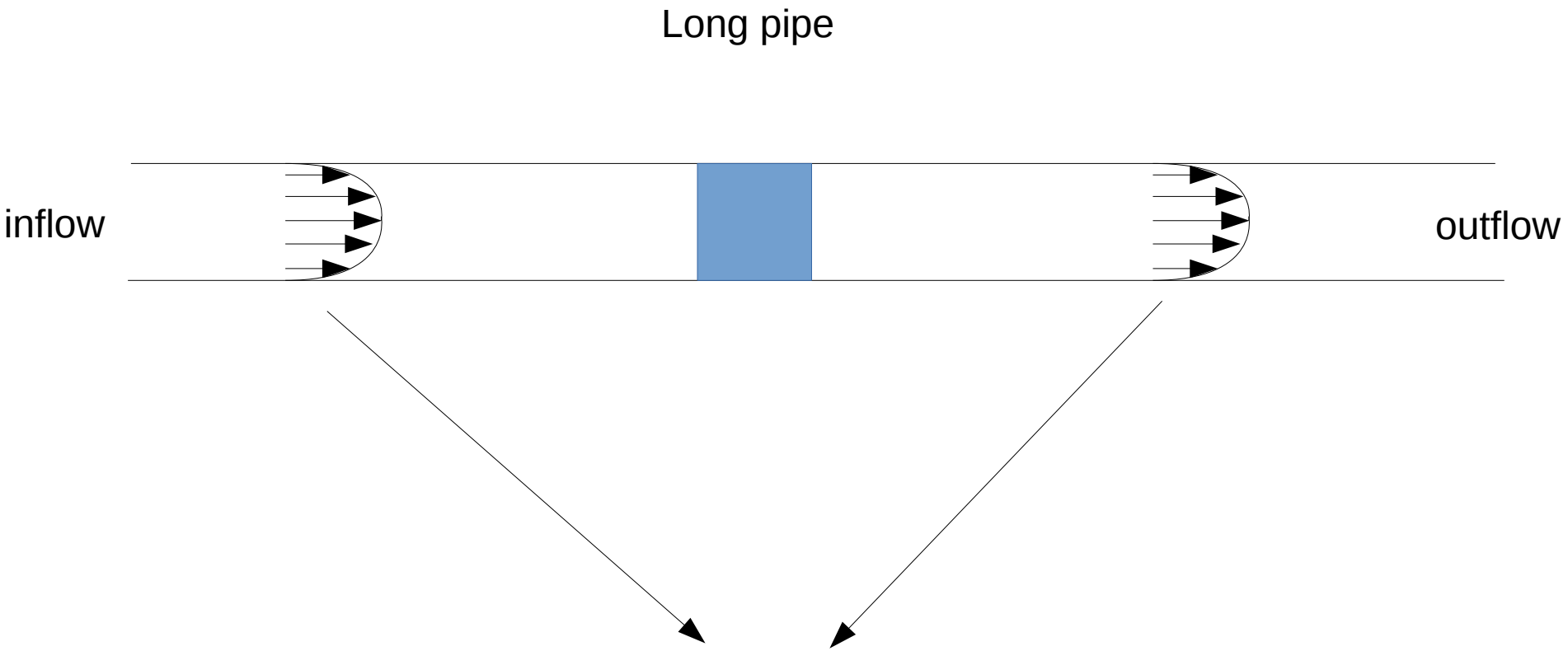
Lattice Gas Automaton Simulation (FHP-II) with FPGA.mp4

http://youtu.be/8m9cTYY3EGA

Microchannel flow project

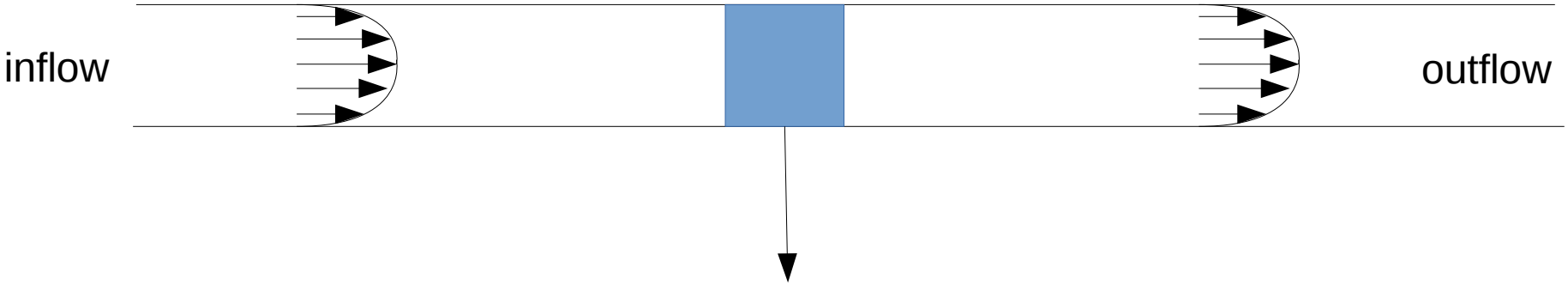
- important applications (chemistry, cpu cooling)
- provide good data for CFD benchmarking
- lot of troubles with flow visualization (experiments)
- micro PIV technique (microscope + PIV)

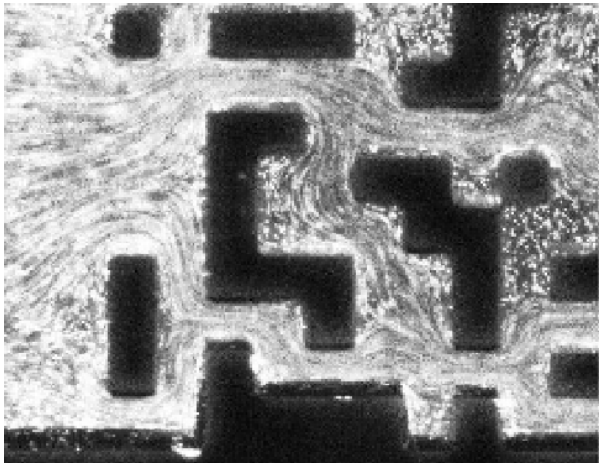
Microchannel flow



Poiseuille profiles develops

Comparison to experiment (Bremen, PIV experiment)

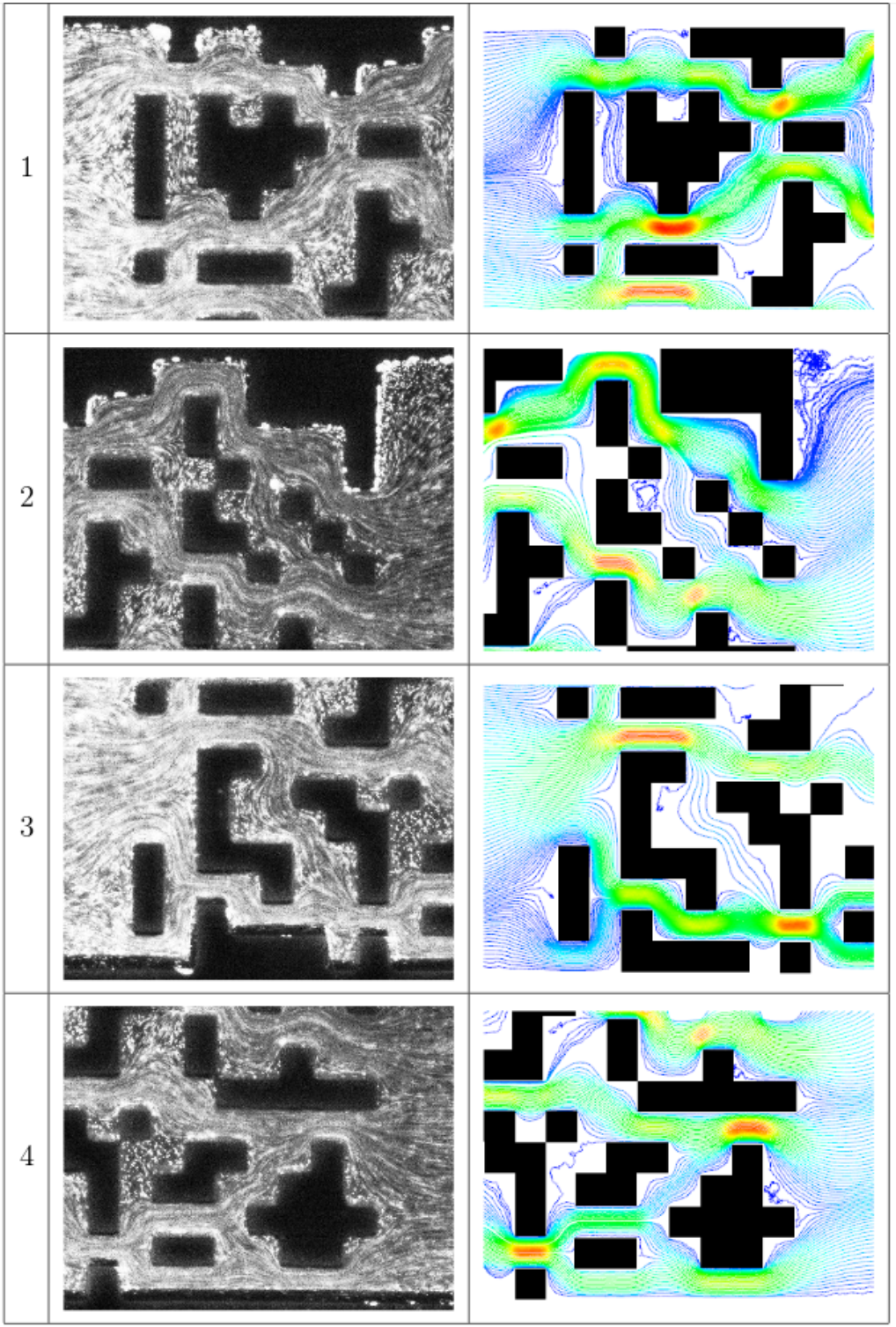




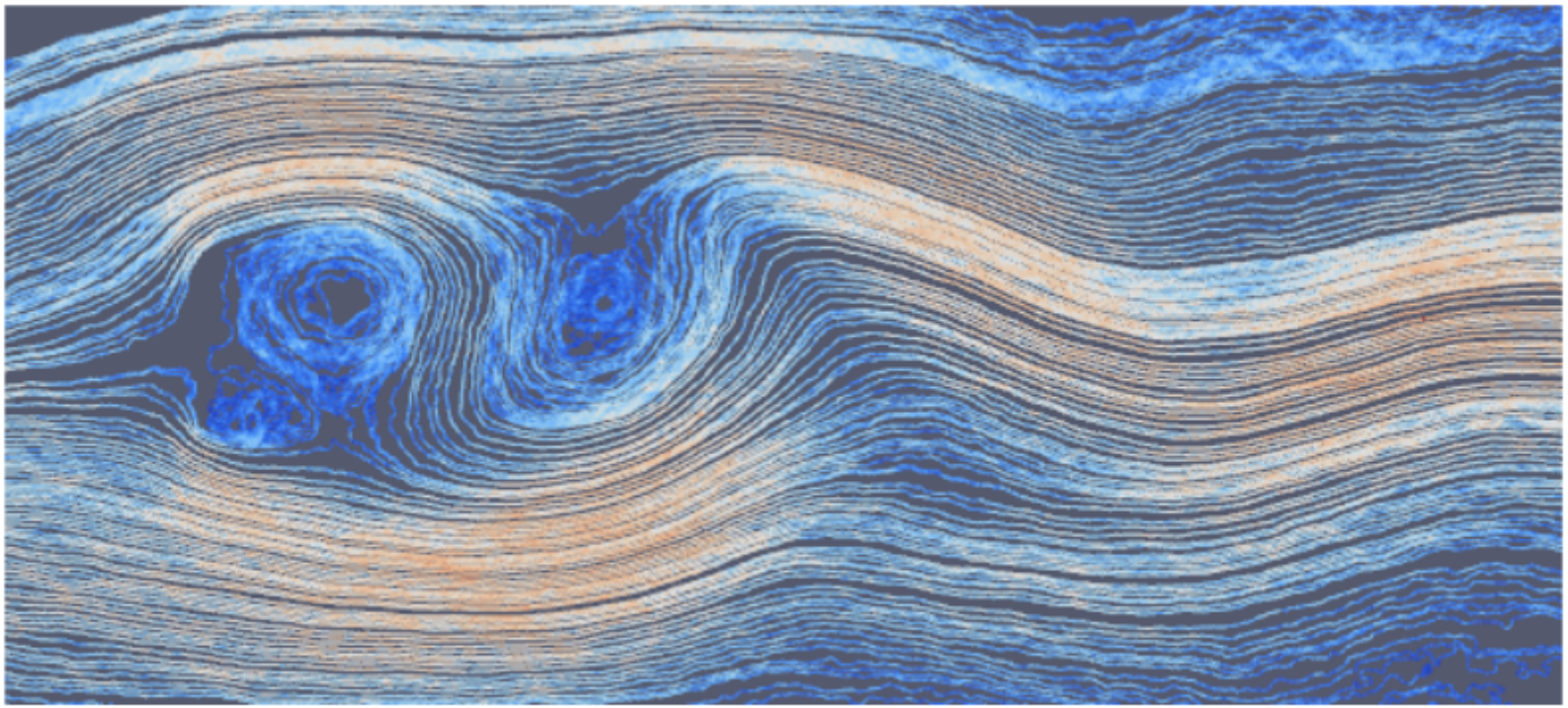
sample (2D)

FHP3

Comparison to experiment (Bremen, **mPIV** experiment)

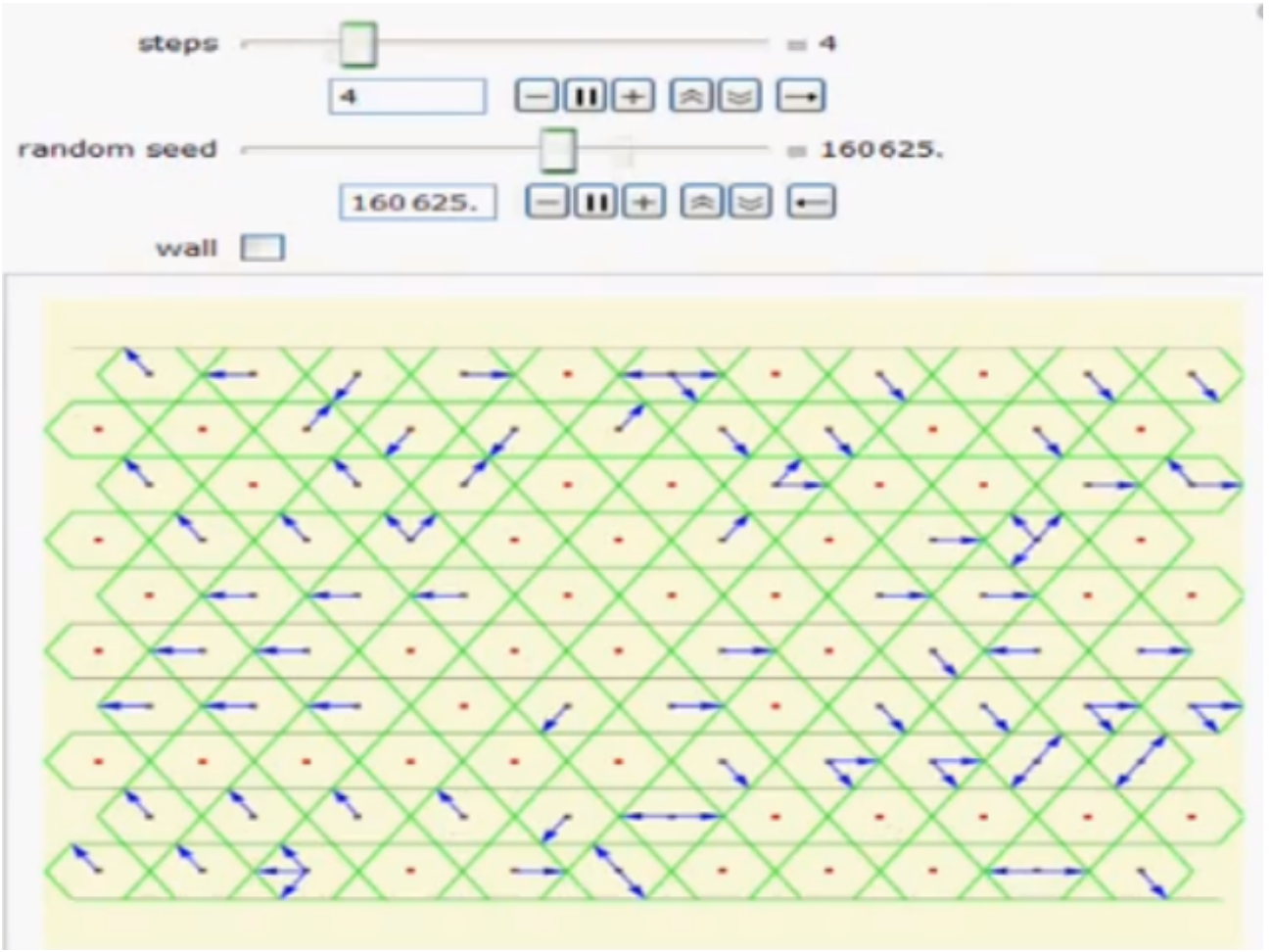


Sebastian Szkoda,
Implementacja modelu FHP w technologii NVIDIA CUDA
Msc, 2010



- Sebastian Szkoda, *Implementacja modelu FHP w technologii NVIDIA CUDA,* Praca Magisterska 2010

FHP in action



[Modeling Fluid using Cellular Automata.mp4]

CS7030 Numerical Methods and Advanced Mathematical Modeling.

Mathematica demonstration of how cellular automata can model fluids

http://youtu.be/8zsAZ1i6JtI

FHP in Science

- Review
- Overview of FHP use in physics
- Extensions
- FHP in CFD

PHYSICAL REVIEW E VOLUME 48, NUMBER 2 AUGUST 1993

Cellular-automata studies of circular Couette flows and chaotic mixing

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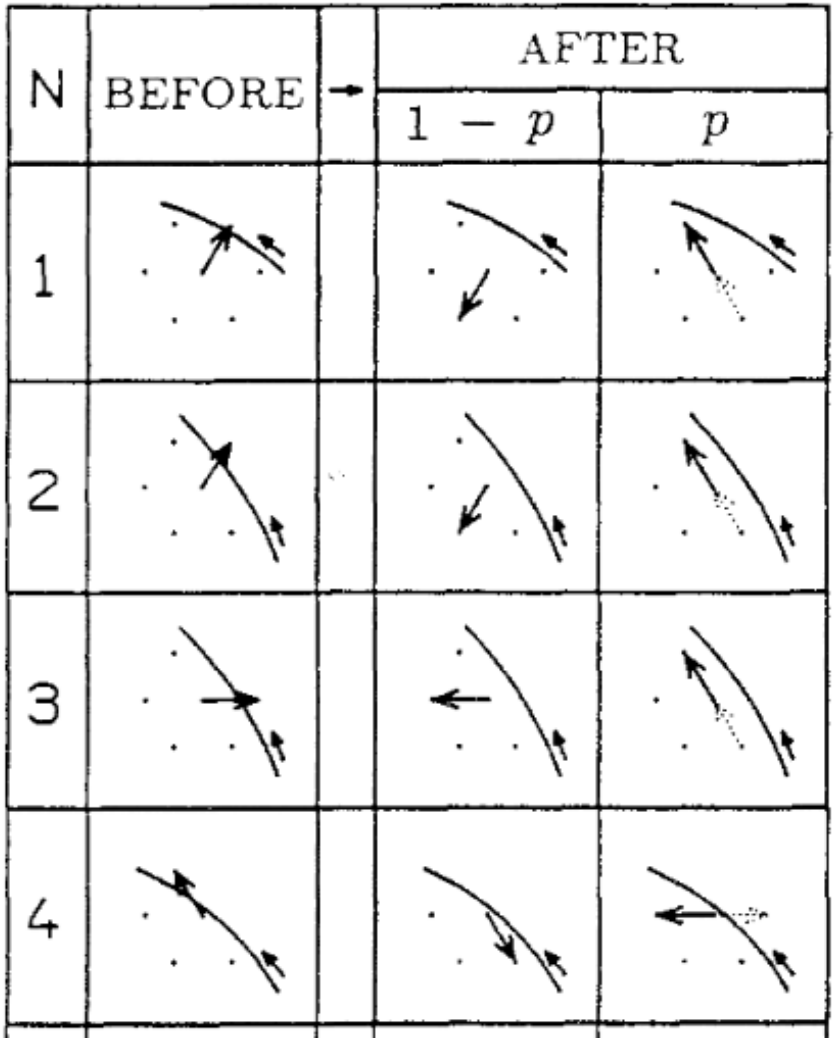
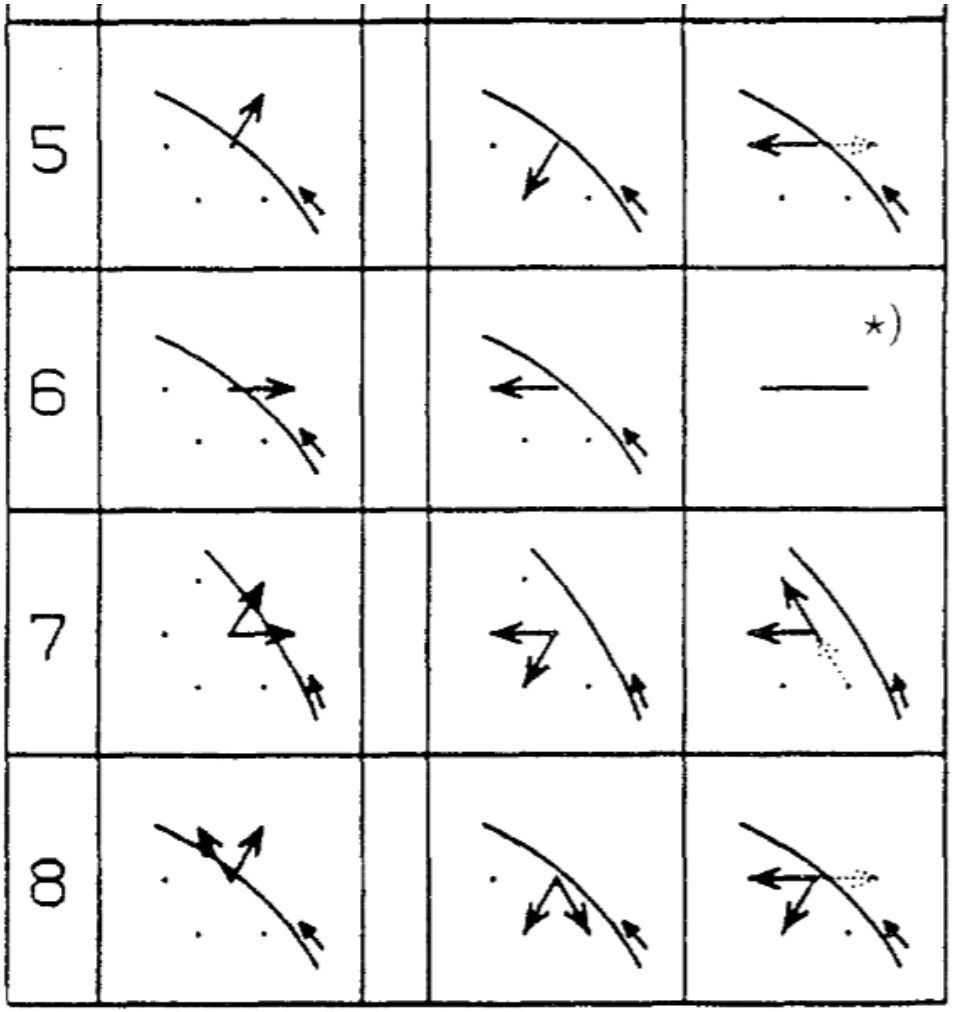
(Received 26 October 1992)

The results of numerical calculations of the velocity profiles and the viscosities of Boolean and Boltzmann cellular automata for circular Couette flows in two dimensions are presented. The necessary forcing rules to induce such flows are constructed. It is shown that the Boltzmann cellular automata offer a more natural way of simulating flows around curved boundaries. A comparison to the results for Poiseuille and planar Couette flows is made. The Boltzmann automata are used to study chaotic mixing in time-periodic flows between two eccentric circles. The Poincaré sections obtained are similar to those found in experiments.

PACS number(s): 05.45.+b, 66.10.Cb

Curved boundaries





Phys. Rev. E 48, 757 (1993)

Flow field

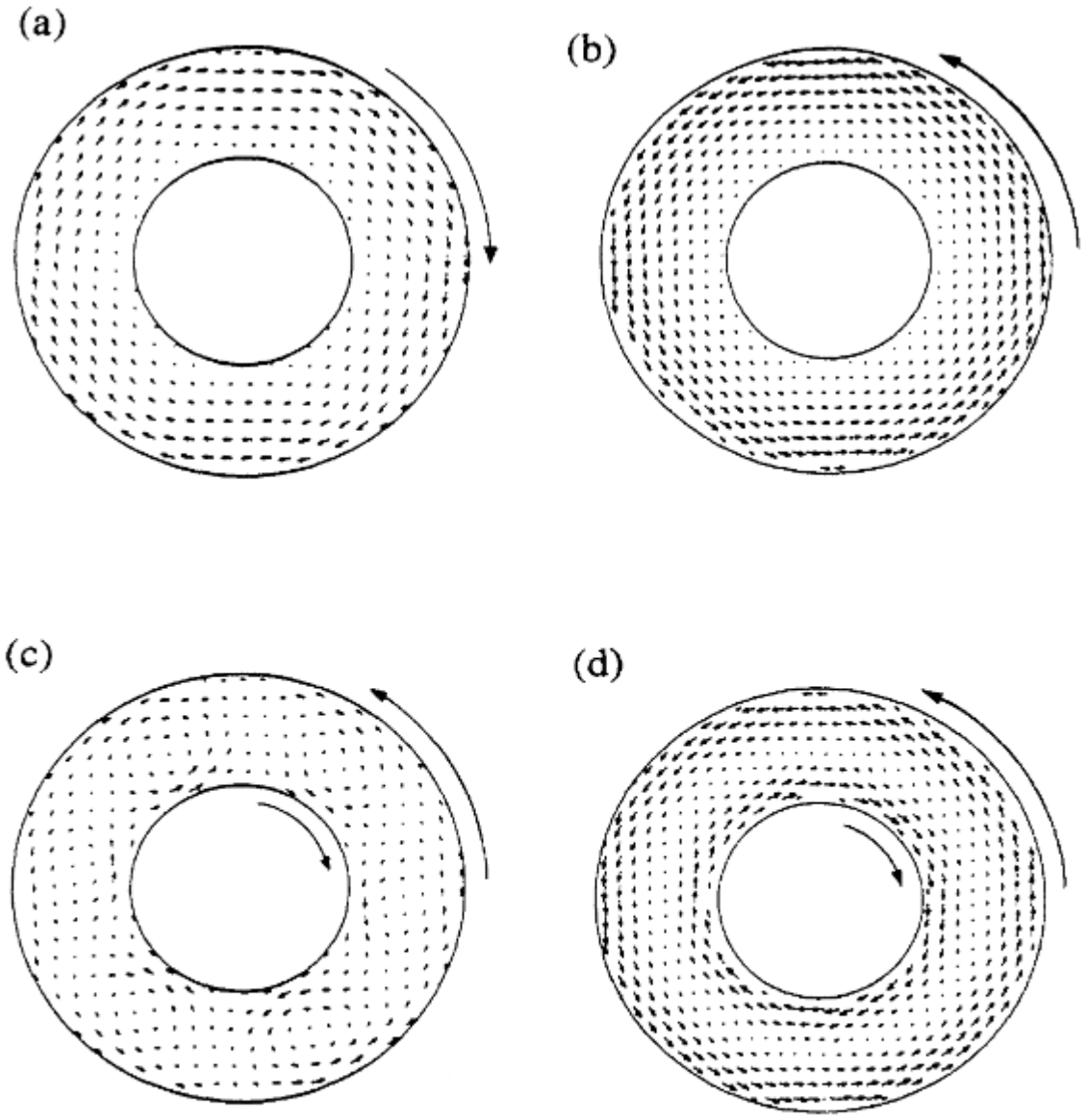


FIG. 3. Flow fields for the circular Couette flow: in the top two figures the inner circle stays at rest and the outer rotates, and in the two bottom figures both circles rotate in opposite directions as indicated by the big arrows. (a) and (c) were ob-

Phys. Rev. E 48, 757 (1993)

Anyone?

Project idea

- Re-implement FHP with curved boundary
- Investigate another geometry
  - (i.e. external flow over cylinder)
- Compare to standard FHP boundary rules
- Render movies with particles

Lattice-gas models of phase separation: interfaces, phase transitions, and multiphase flow

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http://rothmangroup.mit.edu

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Stéphane Zaleski

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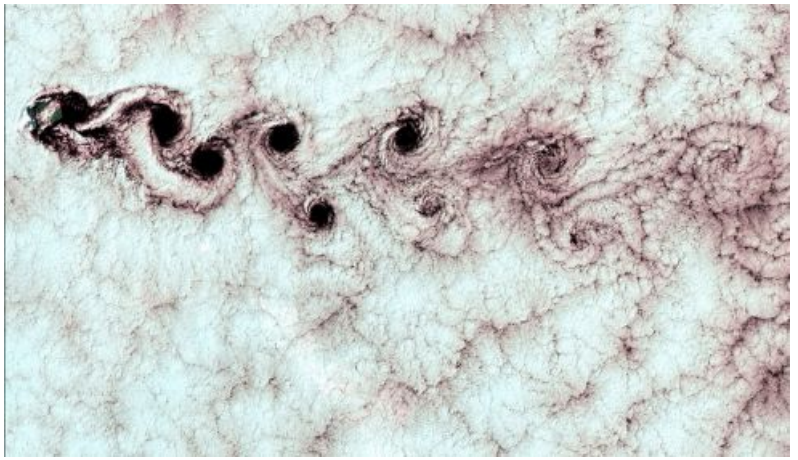
Momentum-conserving lattice gases are simple, discrete, microscopic models of fluids. This review describes their hydrodynamics, with particular attention given to the derivation of macroscopic constitutive equations from microscopic dynamics. Lattice-gas models of phase separation receive special emphasis. The current understanding of phase transitions in these momentum-conserving models is reviewed; included in this discussion is a summary of the dynamical properties of interfaces. Because the phase-separation models are microscopically time irreversible, interesting questions are raised about their relationship to real fluid mixtures. Simulation of certain complex-fluid problems, such as multiphase flow through porous media and the interaction of phase transitions with hydrodynamics, is illustrated.

Rev. Mod. Phys., Vol. 66, No. 4 (1994)

Vortex Karman street



http://www.strudel.org.uk/blog/astro/000826.shtml

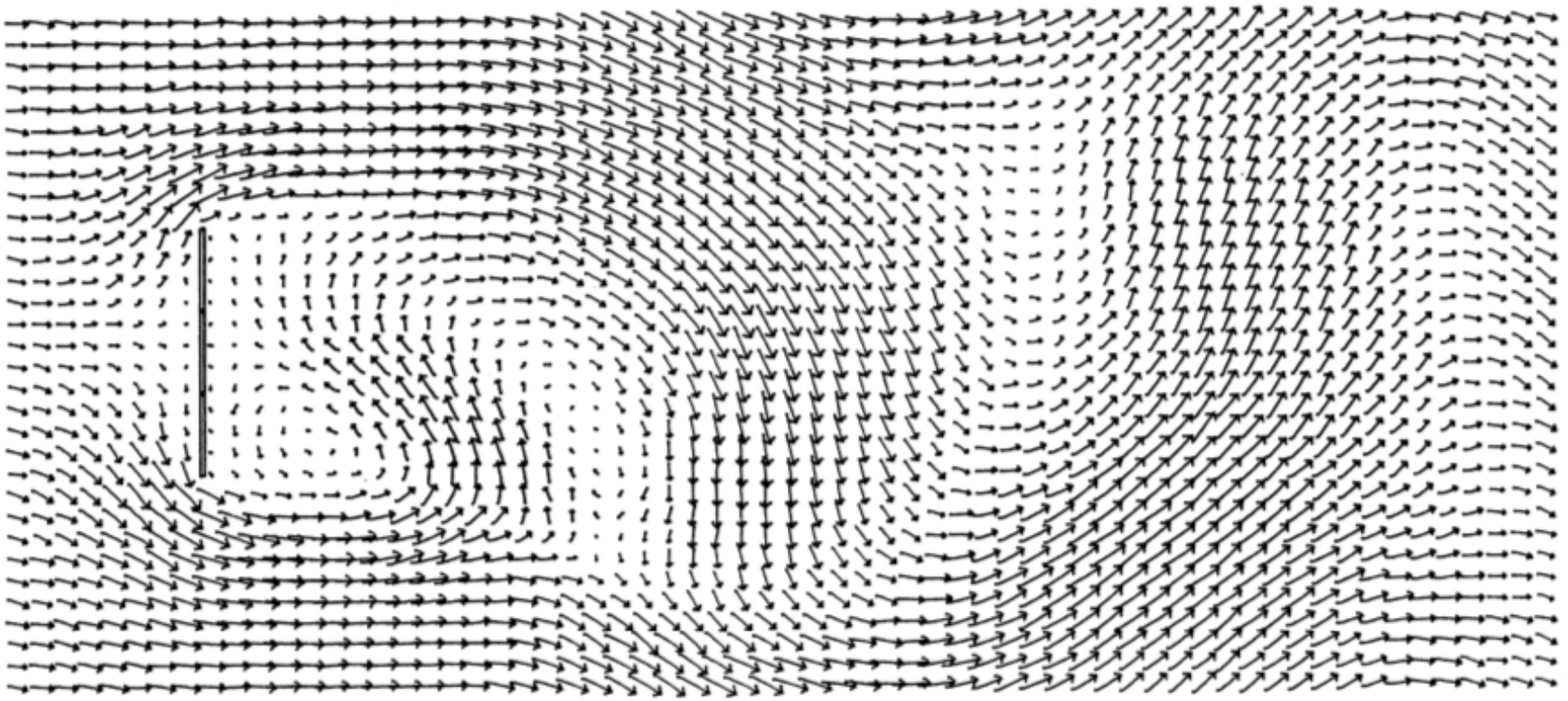


FIG. 3. Two-dimensional flow past a flat plate, simulated using the FHP lattice gas (d'Humières, Pomeau, and Lallemand, 1985). The Reynolds number is approximately 70.

Rev. Mod. Phys., Vol. 66, No. 4 (1994)

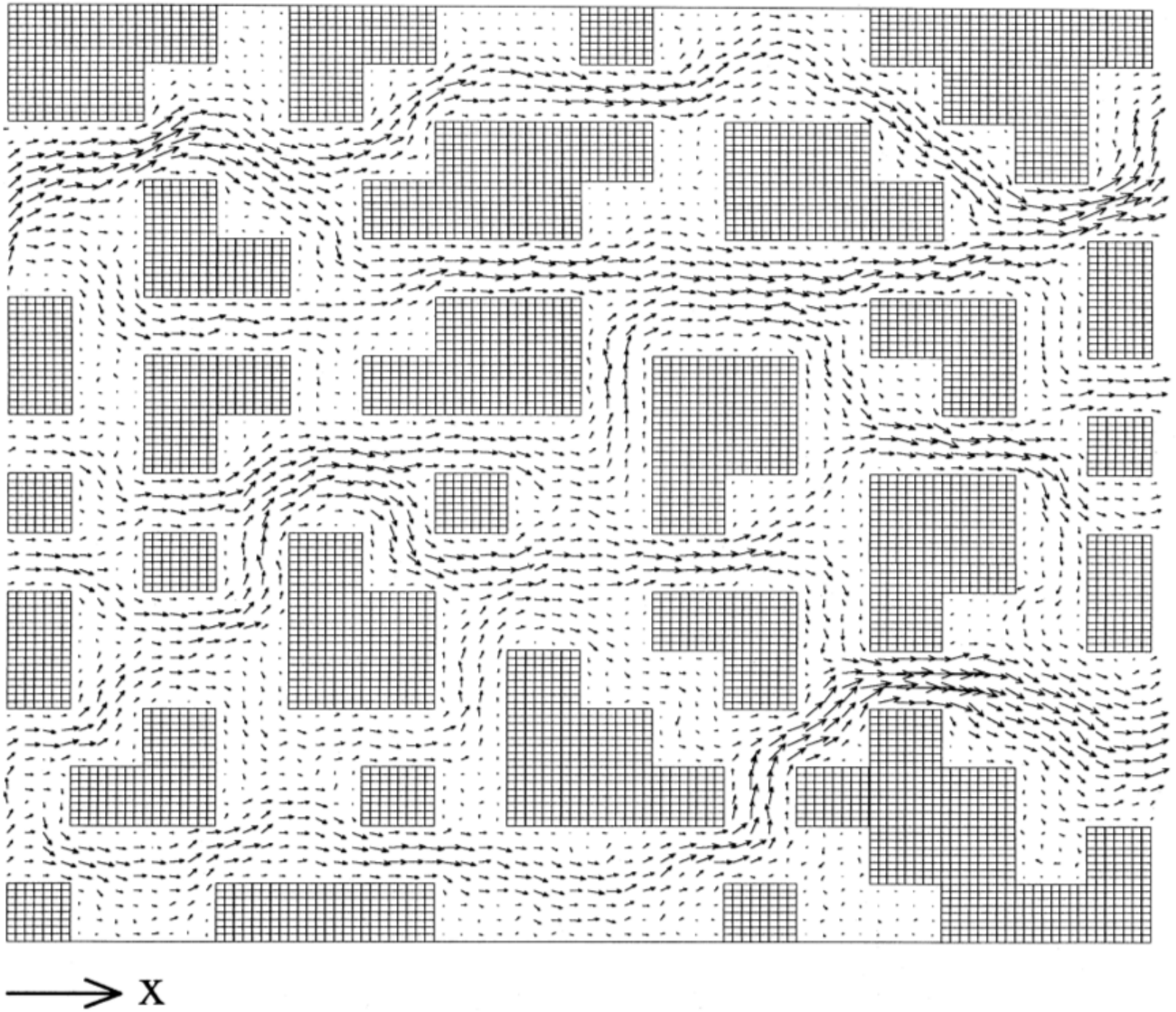


FIG. 4. Lattice-gas simulation of flow through a two-dimensional porous medium (Rothman, 1988). The fluid is forced from left to right.

Two phase flow

- Water + air (free surface)
- Oil + water
- Oil + oli
- etc. (more come later)

Immiscible* Lattice Gas (ILG)

- Start with standard FHP
- Two species (**red**, **blue**) particles

* nie mieszający się

Immiscible* Lattice Gas (ILG)

- Start with standard FHP
- Two species (**red**, **blue**) particles
- Particles attraction
  - **red** → directed towards **red** etc.

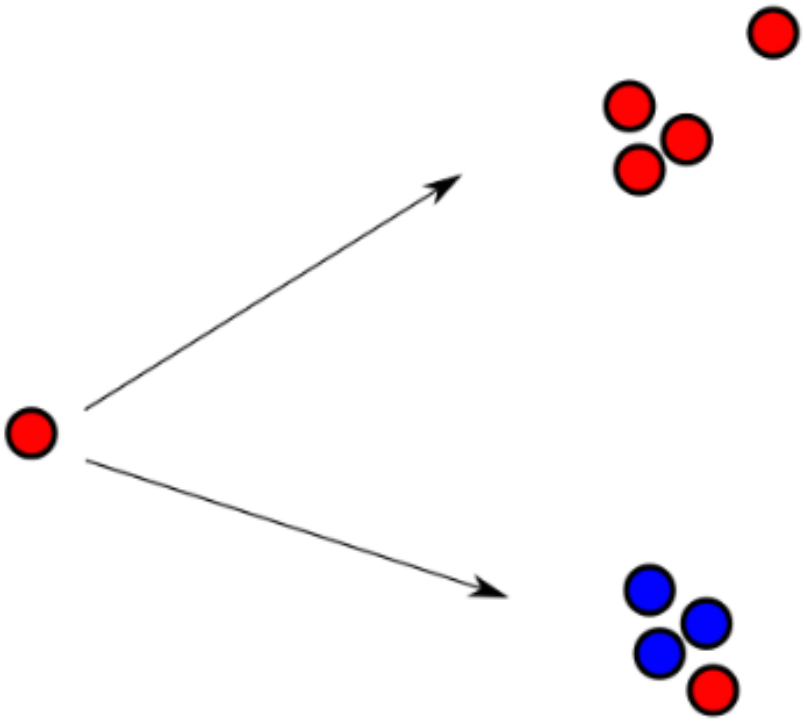
* nie mieszający się

Immiscible* Lattice Gas (ILG)

- Start with standard FHP
- Two species (**red**, **blue**) particles
- Particles attraction
  - **red** → directed towards **red** etc.
- Maximization of

  color flux towards

  color gradient



Rev. Mod. Phys., Vol. 66, No. 4 (1994)

* nie mieszający się

ILG results

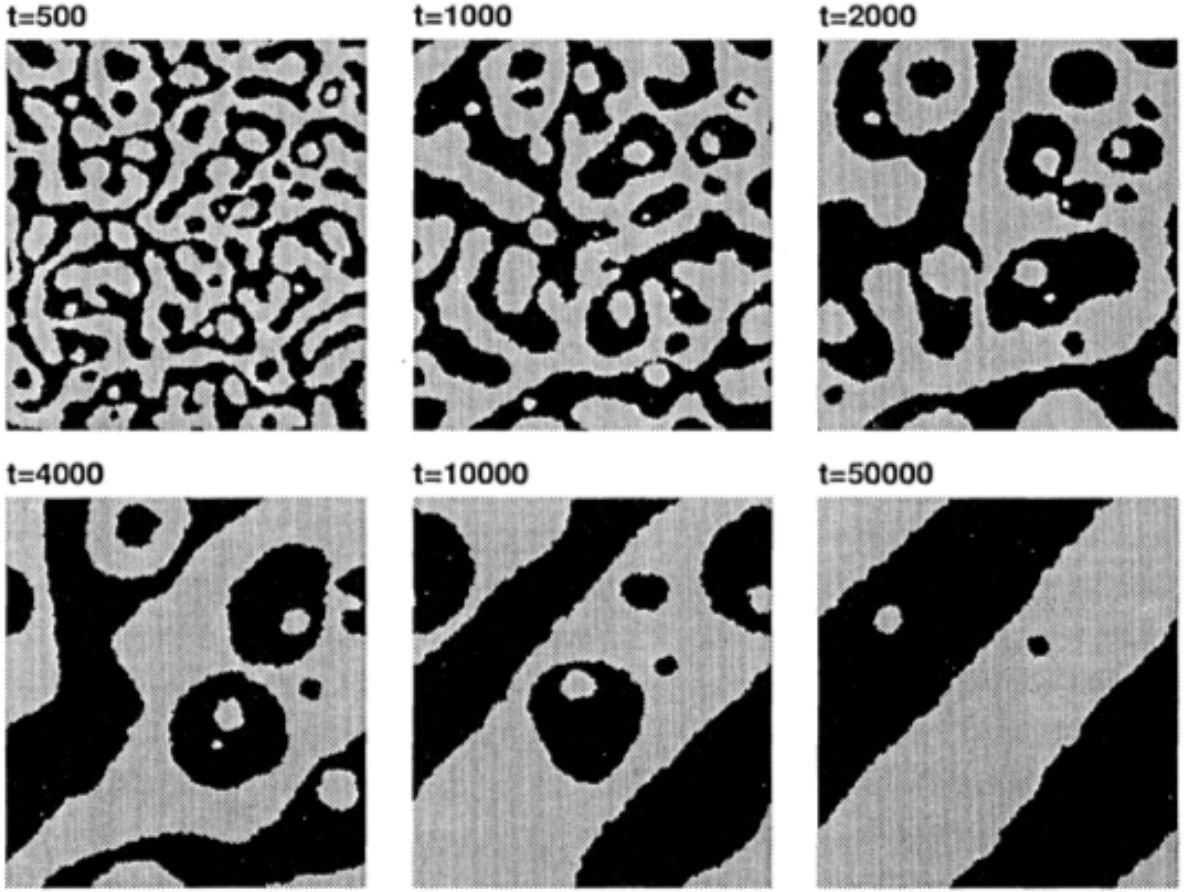


FIG. 6. Phase separation in the immiscible lattice gas. The initial condition was a homogeneous random mixture, with 50% red (black) particles, and 50% blue (gray). Time t is given in time steps. Boundaries are periodic in both directions. From Rothman (1992).

(animate?)

Anyone?

Project idea

- Re-implement FHP with two phases
- Render **phase separation movie**
- Add 3rd fluid
- Add gravity (?)
- Do splash of liquid drop

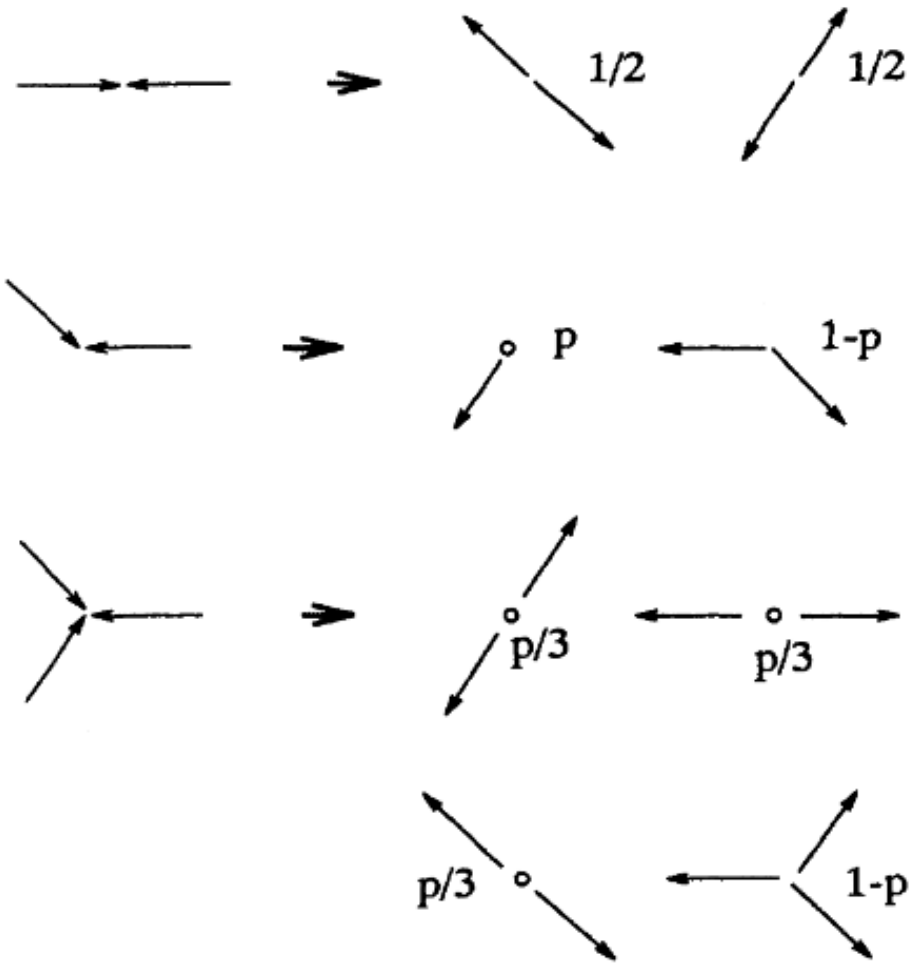
Granular flow using LGA

- Rest particle introduced
- Energy dissipation (rough particles)
- Temporal site occupation by >1 particle
- Rest particle may move along gravity direction
- Bounce back - probability

(figures?)

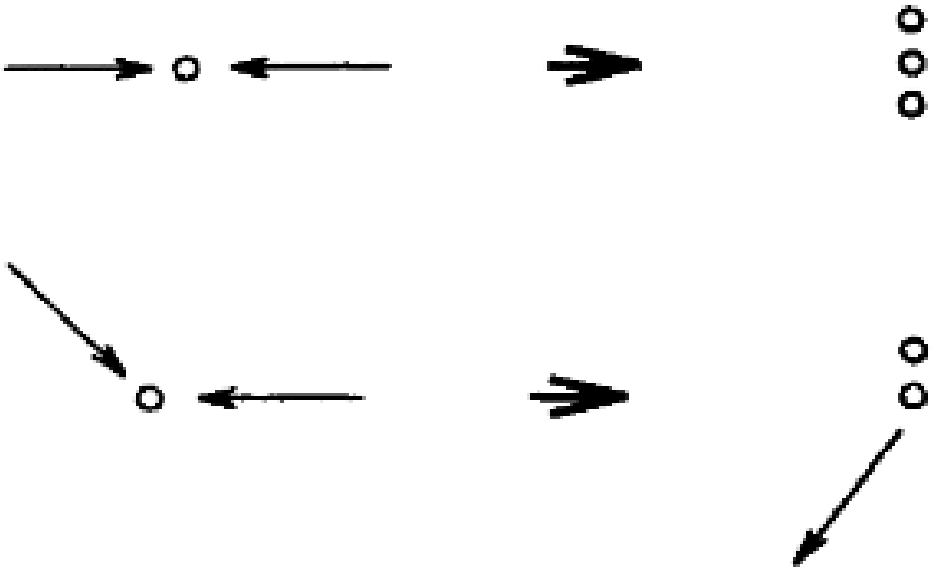
Phys. Rev. E 49, R1796(R) (1994)

Granular flow using LGA - basics



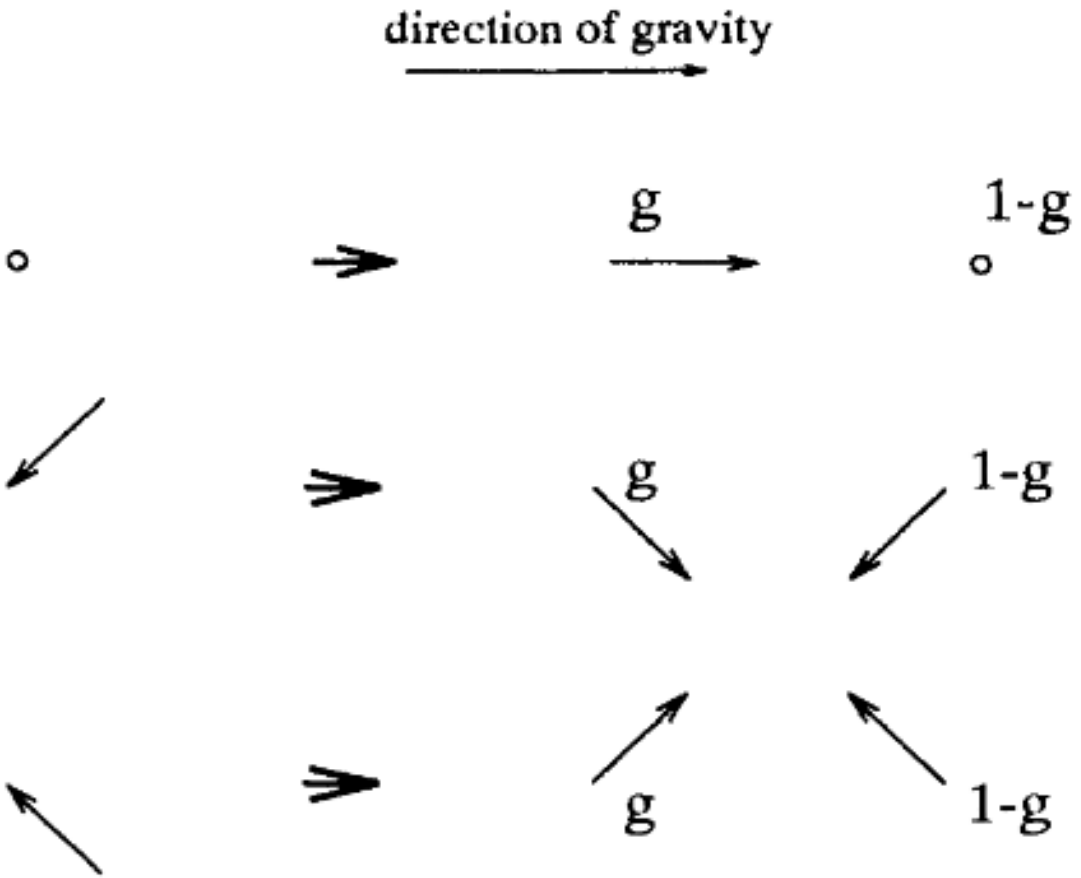
Phys. Rev. E 49, R1796(R) (1994)
Phys. Rev. E 51, 1745 (1995)

Collisions with rest particle



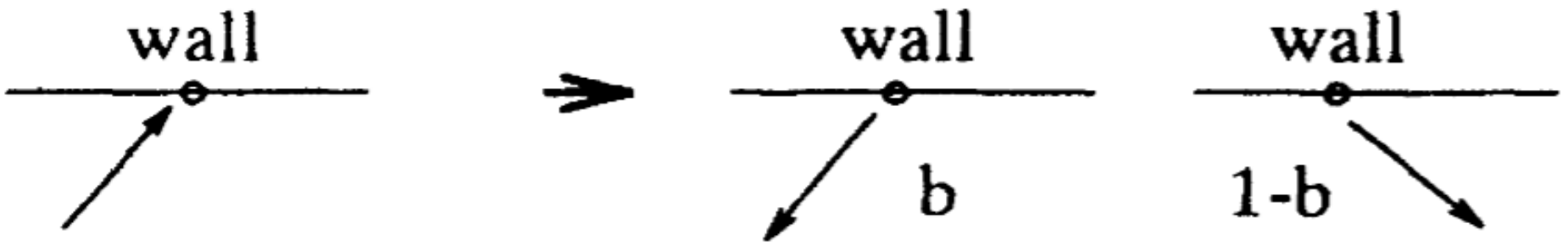
Phys. Rev. E 49, R1796(R) (1994)
Phys. Rev. E 51, 1745 (1995)

Effect of Gravity



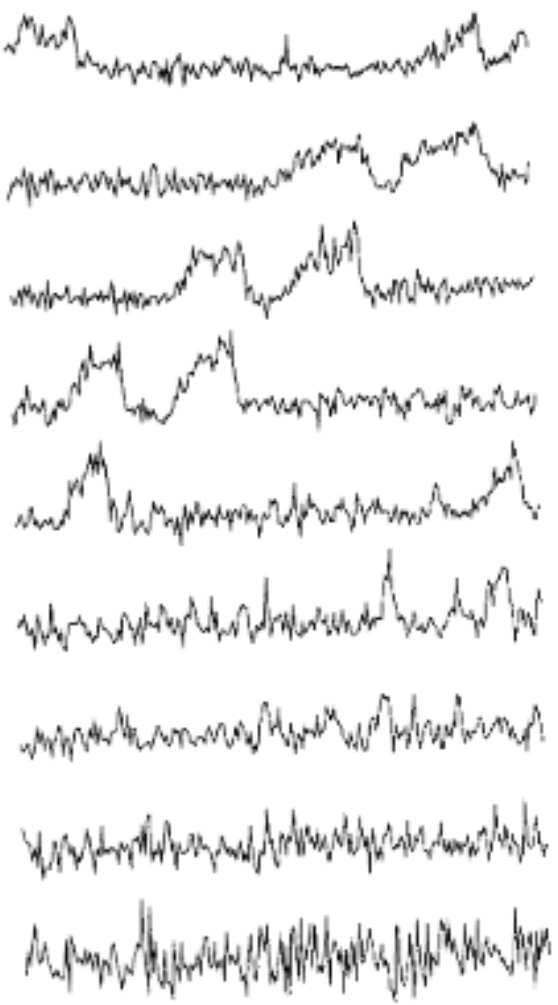
Phys. Rev. E 49, R1796(R) (1994)
Phys. Rev. E 51, 1745 (1995)

Walls



Phys. Rev. E 49, R1796(R) (1994)
Phys. Rev. E 51, 1745 (1995)

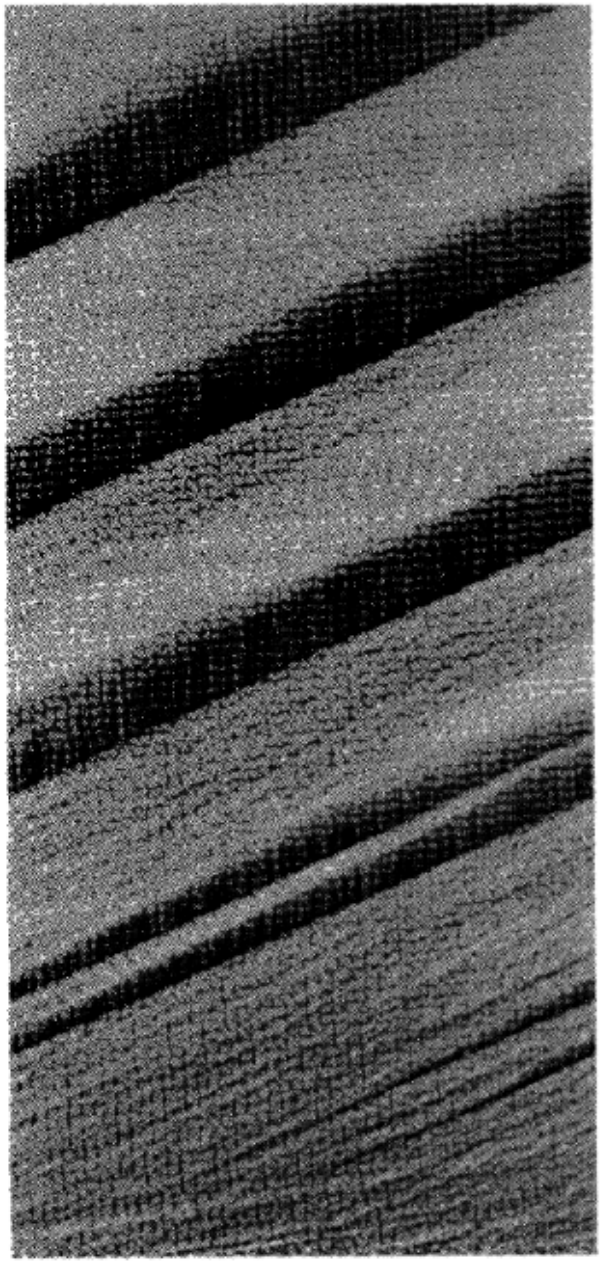
Density waves in granular media



Gravity

time

T=0



(b)

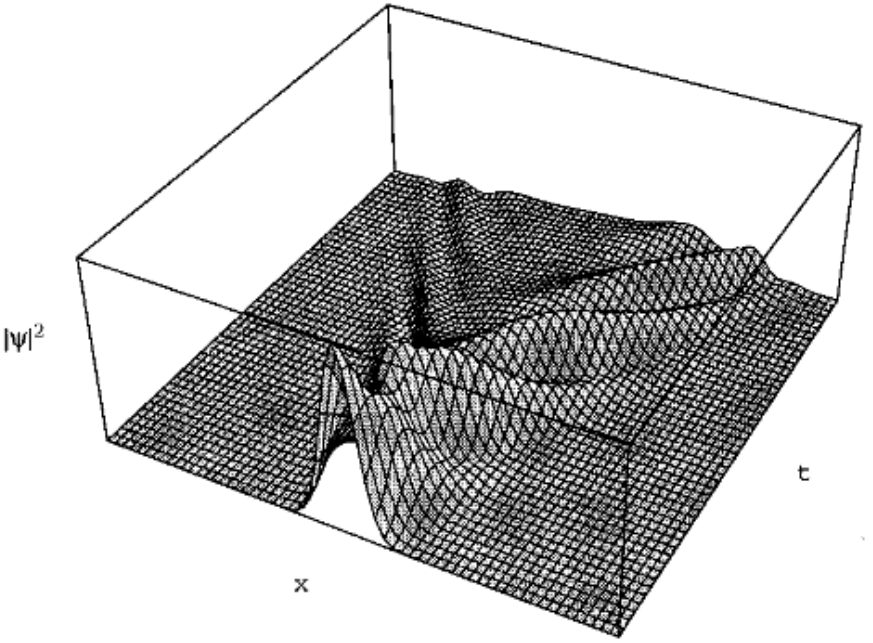
Phys. Rev. E 49, R1796(R) (1994)
Phys. Rev. E 51, 1745 (1995)

Anyone?

Project idea

- Implement granular media LGA model
- Produce **movies** (time evolution)
- Make an experiment in tube (!)
- Assume another geometry (may be hard)

Quantum Lattice Gas (QLGA)

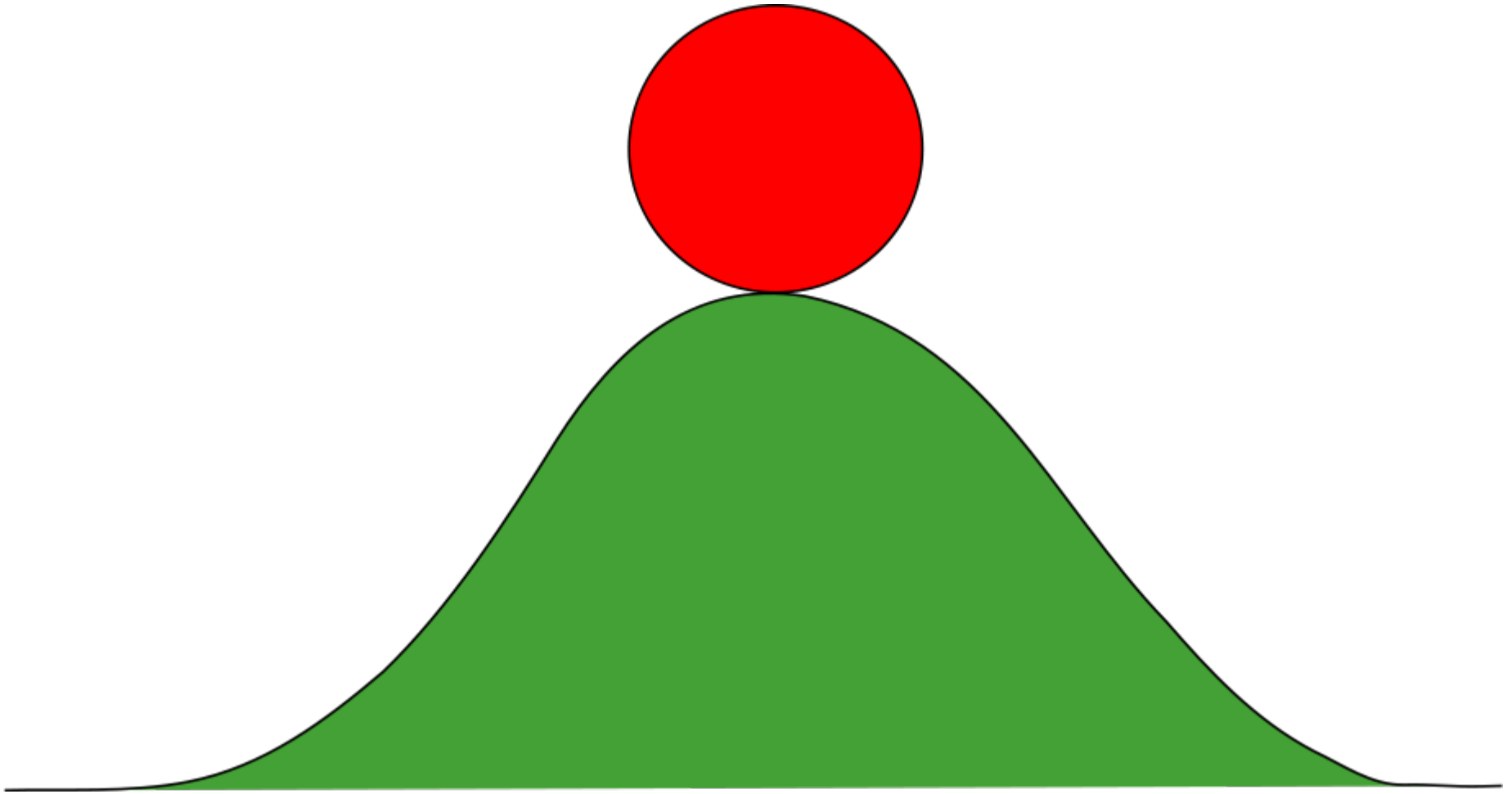


(Ciekawostka)

FIG. 7. Evolution of the $k_0 = \pi/32$ wave packet (4.1) with width $s = 32$; the rule parameters are still $\theta = \pi/3$, $\rho = \pi/4$. This wave packet disperses even more rapidly than the one shown in Fig. 6: left moving waves carry off some of the probability and an interference pattern is created.

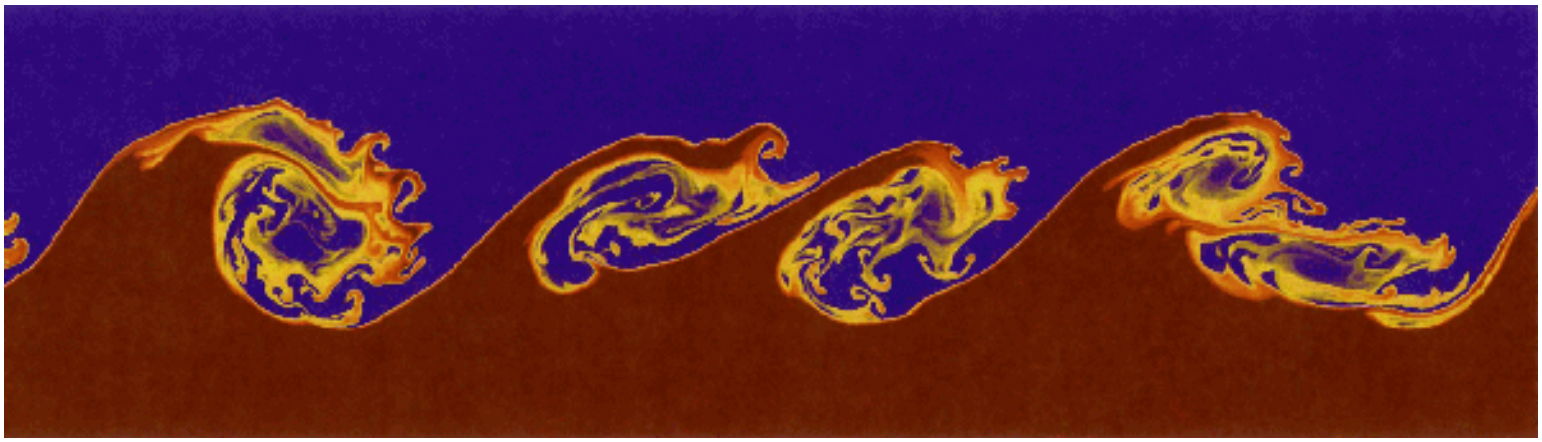
David A. Meyer, **Quantum mechanics of lattice gas automata: One-particle plane waves and potentials,** Phys. Rev. E 55, 5261 (1997)

Kevin Heimholtz instability

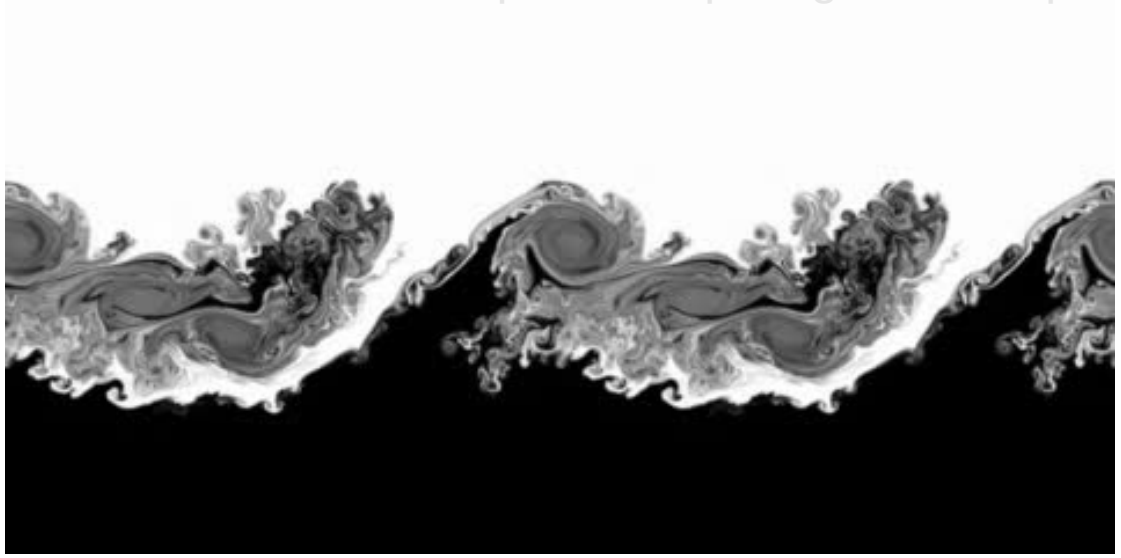


http://en.wikipedia.org/wiki/Instability

Kevin Heimholtz instability



http://www.aps.org/units/dfd/pressroom/gallery/2008/richards.cfm



http://en.wikipedia.org/wiki/Kelvin%E2%80%93Helmholtz_instability

Kevin Heimholtz instability

Figure 10.3 Kelvin–Helmholtz instability simulated on RAP-1, with a diffusive two-species version of the FHP-3 model. Snapshot after 1200 automaton steps.

J.-P. Rivet,J. P. Boon, *Lattice Gas Hydrodynamics,* Cambridge Univ. Press 2005

Kevin Heimholtz instability

Figure 10.5 Snapshot (after 4000 automaton steps) of the Kelvin–Helmholtz instability simulated on RAP-1, with a two-species reaction-diffusion version of the FHP-3 model. The reactive process is used to reduce the dispersive effect of diffusion, in order to obtain sharper contours. Note that this lattice gas model differs from the model used in the simulation illustrated in Figure 10.3.

J.-P. Rivet,J. P. Boon, *Lattice Gas Hydrodynamics,* Cambridge Univ. Press 2005

Fingering in porous medium

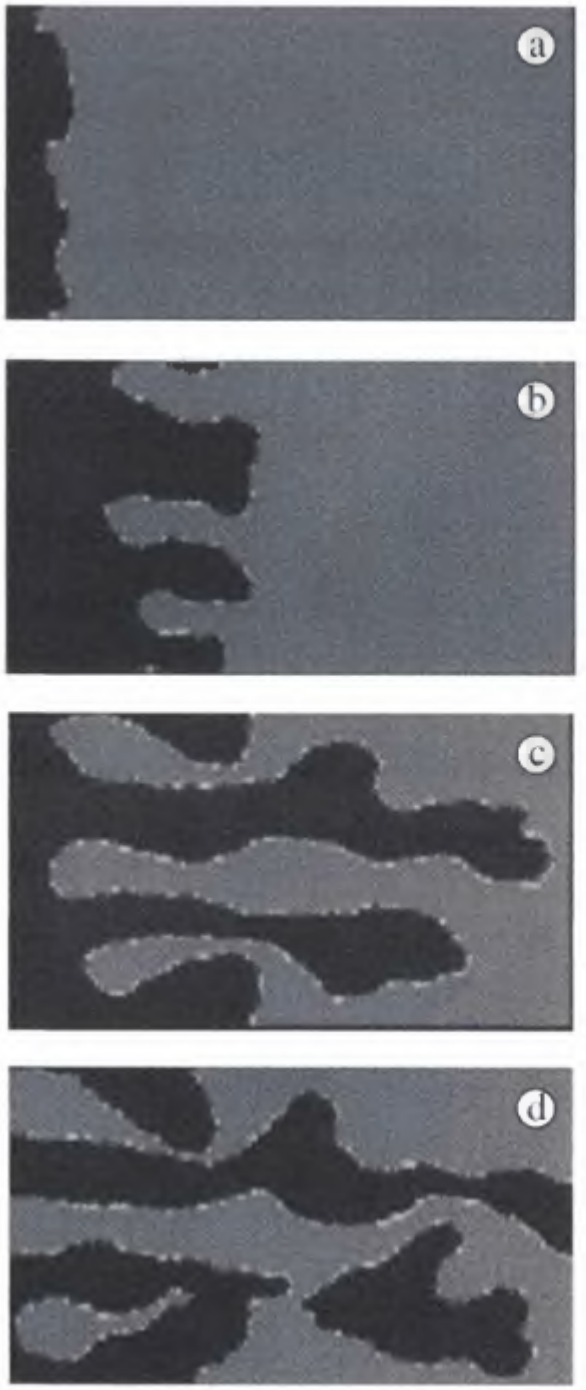


Figure 10.12
Two-dimensional lattice gas simulation of a two-phase fluid with surface tension in a porous medium. The interface is initially flat; an instability develops and produces the typical fingering patterns of the less viscous fluid (in black) penetrating the most viscous fluid (in gray). The four snapshots are taken after 30 000 (a), 50 000 (b), 100 000 (c), and 112 500 (d) time steps.

J.-P. Rivet,J. P. Boon, *Lattice Gas Hydrodynamics,* Cambridge Univ. Press 2005

FCHC-8 (on Cray supercomputer)

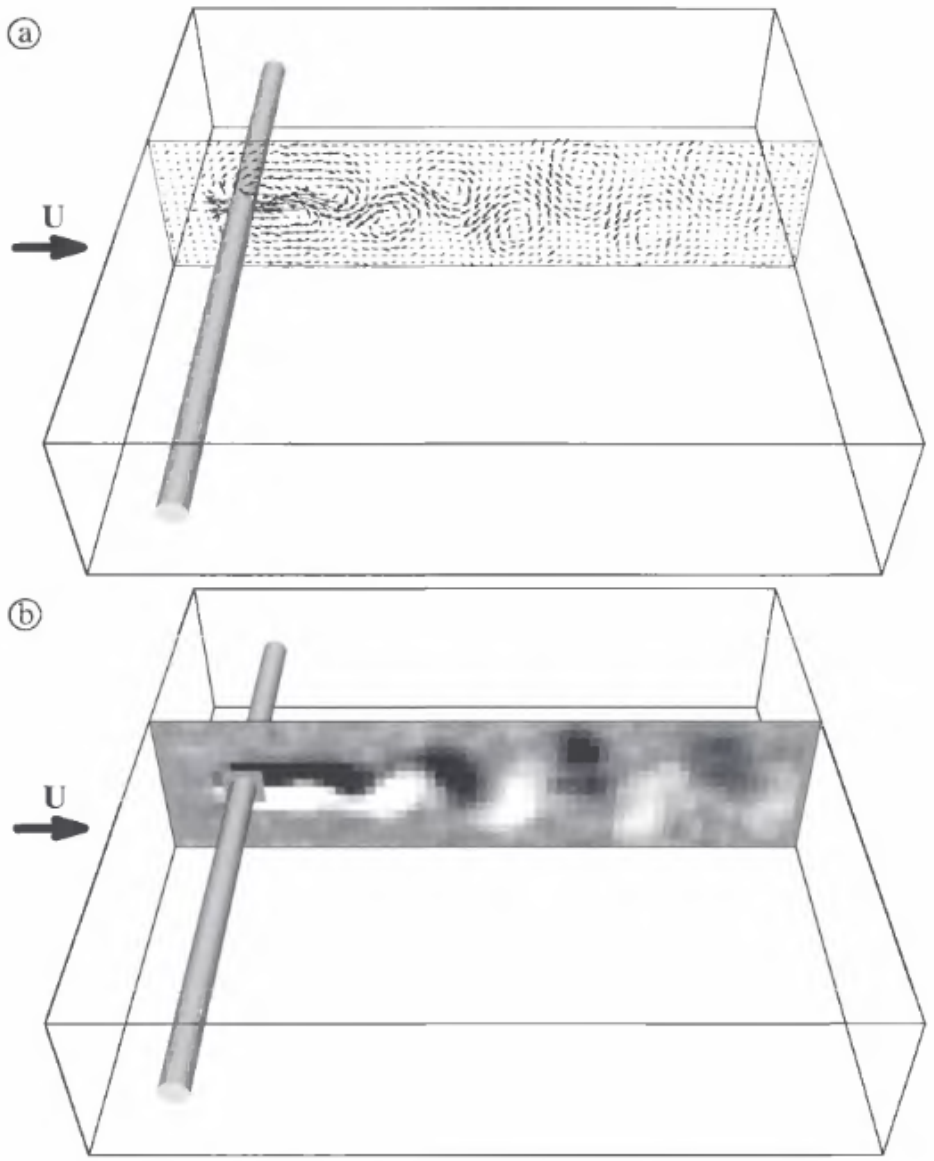


Figure 10.10 Three-dimensional incompressible flow around a cylinder in wind-tunnel geometry, simulated with the FCHC-8 lattice gas model on a CRAY-2 computer. The velocity field (a), and the span-wise component of the vorticity (b) are shown in a cut-plane transverse to the cylinder, 16 000 time steps (74.4 circulation times) after the impulsive start of the flow.

J.-P. Rivet,J. P. Boon, *Lattice Gas Hydrodynamics,* Cambridge Univ. Press 2005

Summary 1

- FHP: simplicity (+)
- FHP: computational speed (+)
- FHP: **noise** (-)

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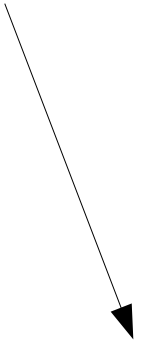

Large grids

Summary 1

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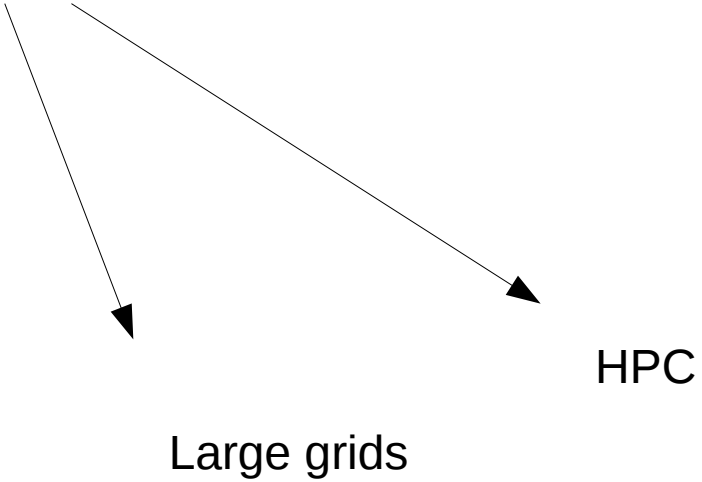

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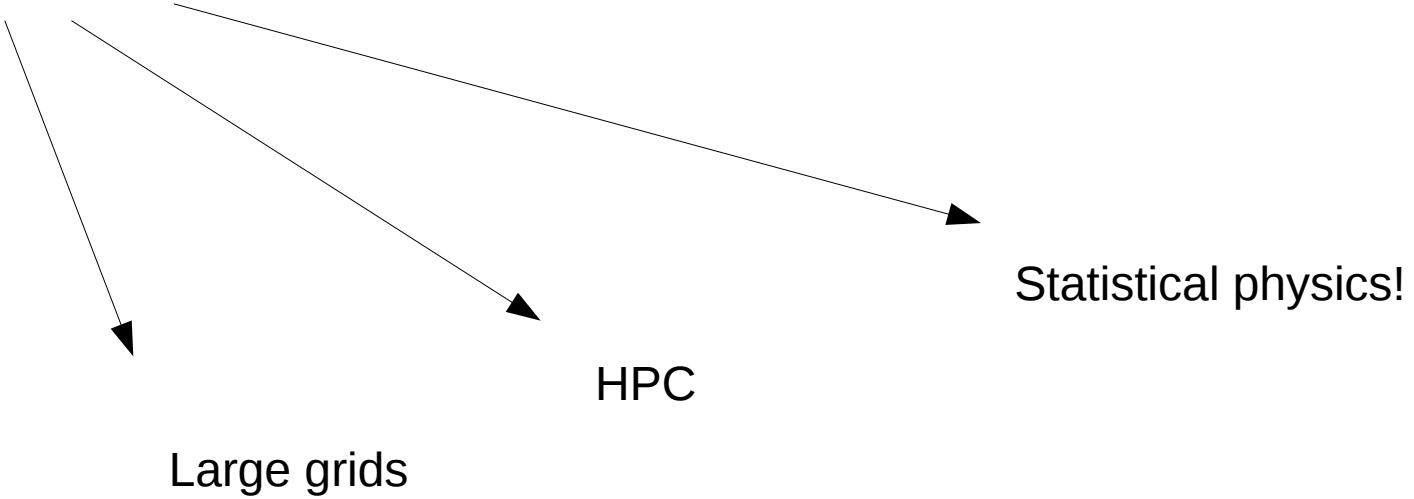


The Lattice Boltzmann Model

VOLUME 61, NUMBER 20 PHYSICAL REVIEW LETTERS 14 NOVEMBER 1988

Use of the Boltzmann Equation to Simulate Lattice-Gas Automata

Guy R. McNamara and Gianluigi Zanetti[(a)]

The Research Institutes, The University of Chicago, 5640 South Ellis Avenue, Chicago, Illinois 60637

(Received 29 July 1988)

We discuss an alternative technique to the lattice-gas automata for the study of hydrodynamic properties, namely, we propose to model the lattice gas with a Boltzmann equation. This approach completely eliminates the statistical noise that plagues the usual lattice-gas simulations and therefore permits simulations that demand much less computer time. It is estimated to be more efficient than the lattice-gas automata for intermediate to low Reynolds number $R \lesssim 100$.

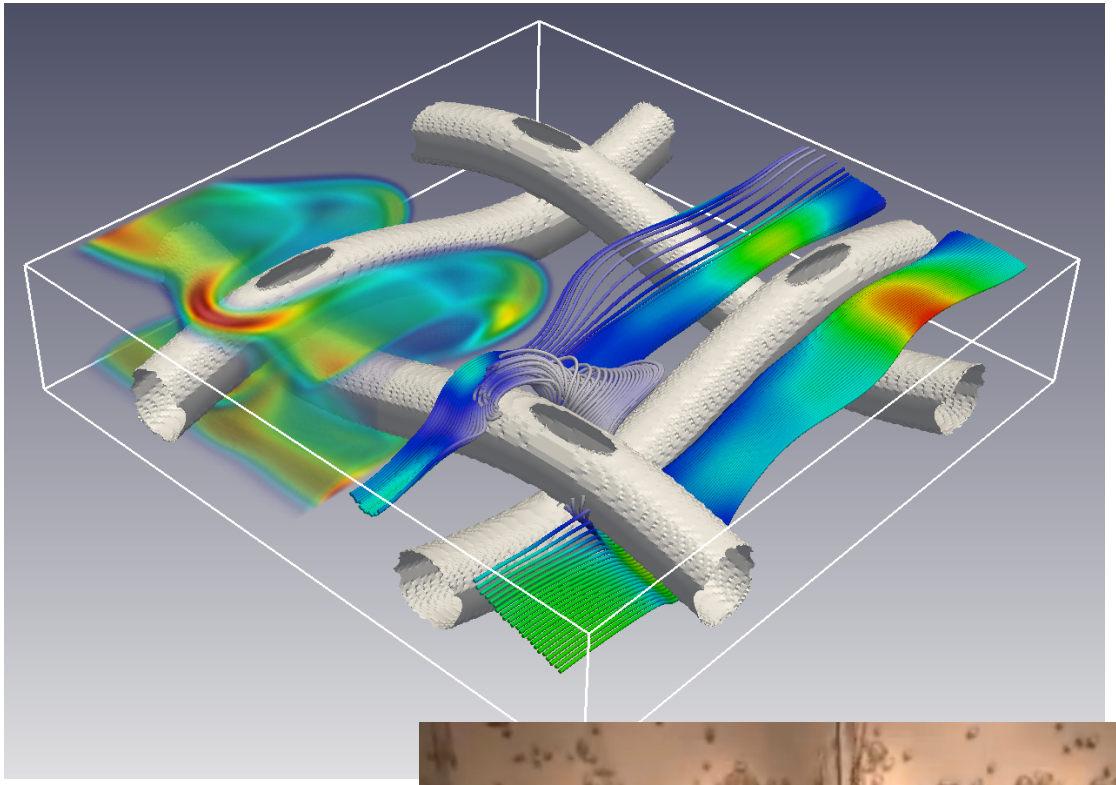
PACS numbers: 47.10.+g

Assumption in LBM

$$c_i = \{0, 1\} \rightarrow c_i \in [0; 1]$$

Boolean

Continous



LBM

http://www.jenszudrop.de/research/

http://pages.cpsc.ucalgary.ca/~ualim/research.html

http://youtu.be/p67-Qiad5zc

Summary 2

- FHP: simplicity (+)
- FHP: computational speed (+)
- FHP: noise (-)

next: **Lattice Boltzmann Methods** (LBM)

Lecture list

- 1 **Motivation** (part 1): Fluid dynamics, Turbulence, **24.02**
- 2 Lattice Gas automata (our work, technical): FHP5 (Diffusion), HPP, FHP models, part 1, **3.03**
- 3 Lattice Gas automata (technical, review): FHP, review, part 2, **10.03**

(17.03 – class cancelled)

- 4 **Motivation** (part 2): Review fluid dynamics & effects (**24.03**)

5 „CFD in Wrocław", Workshop, 31 – double lecture (9:30 – 14:00)

- 6 … Navier-Stokes equations (origin, math, analytics, millenium problem)
- 7 … **Motivation**, Navier-Stokes solvers: review
- 8 … Classification of PDE + solutions (Waves, Laplace, Poisson, Finite Differences)
- 9 … **Motivation**,The MAC Method (Finite Difference scheme + review)
- 10 … The MAC Method (Free surfaces, markers, boundary conditions)
- 11 … The MAC Method (Implementation)
- 12 … Finite Volume Methods (Overview, discretization, Algorithms, SIMPLE(R))
- 14 … **Motivation,** Smoothed Particle Hydrodynamics (SPH), Review, Theory Implementation
- 15 … **Motivation, LBM** , Review, Theory Implementation
- 16 … **Motivation,** Software: Paraview, VisIT, OpenFOAM, Palabos, Sailfish etc. (Review)