FHP3 fluid dynamics model

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1 The Objective

The main goal of the project was the proper implementation of the FHP3 model. Apart from simple tests like mass and momentum conservation, the main attempt was to simulate the Poiseuille flow.

2 The FHP1 Model

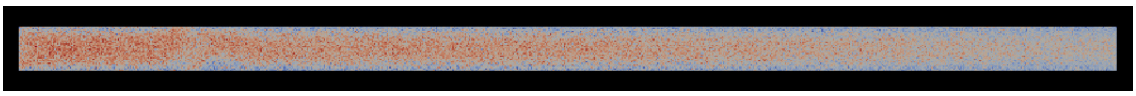
I have implemented two models. The first one was the FHP1 model, that is: a fluid on a hexagonal lattice, with particles in 6 possible directions and 4 collisions. The FHP1 model has been implemented with states evolution using a formula with boolean arithmetics. The implementation was very successful. An example animation is attached (fhp1.avi). The animation was recorded using Paraview.

3 The FHP3 Model

In the second part of the project the goal was to implement FHP3 model. The difference from the previous one is an addition of one more particle (without direction) and many collisions. I have started with the same approach as the last time. Unfortunately the literature around the subject is very vague. After some time spent on proving mistakes in one mans PhD thesis I gave up and focused on the other approach (look-up tables for collisions).

After checking the mass and momentum conservation (attachment: FHP3_test.cpp, compilation using -lgsl -lgslcblas). I wanted to check the same situation (a drop of higher density) as in the FHP1 model. The result can be found in the file "fhp3.avi".

The last thing to do was to simulate the Poiseuille flow.



Poiseuille flow
l=1600, h=64, averaging=2x2, $\rho_1 = 0.6$, $\rho_2 = 0$, steps=3000

We know that the parameters of the Poiseuille flow can be calculated analitically. Using Navier-Stokes equation, we can obtain [as shown in Sebastian Szkoda MSc Thesis, UWr]:

$$u_x(y) = -\frac{\Delta p}{2\eta l} y(y-h), \tag{1}$$

where u is a velocity, Δp a difference of pressure on each side, η a viscosity and l/h length(x)/height(y) of the pipe. Now, an important thing to calculate is the difference of pressure. Using Bernoulli equation

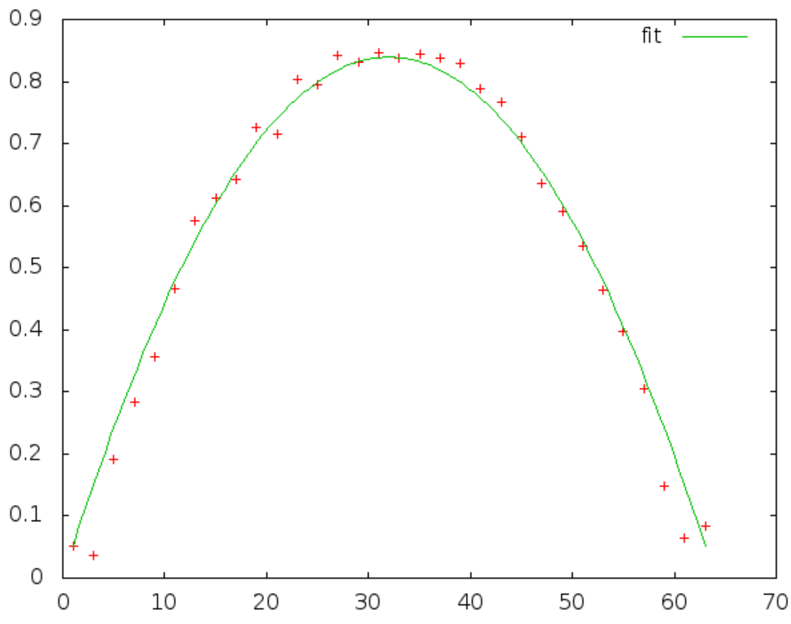
$$P_1 + \frac{1}{2}\rho_1 v_1^2 = P_2 + \frac{1}{2}\rho_2 v_2^2 \tag{2}$$

$$\Delta p = \frac{1}{2}(\rho_1 v_1^2 - \rho_2 v_2^2) \tag{3}$$

Now, if we can estimate the velocity on the sides, as they have fixed density and only the paricles in the direction to the inside of the pipe give contribution. Therefore: $v = 2\rho$.

$$\Delta p = 2(\rho_1^3 - \rho_2^3) \tag{4}$$

We can now compare our velocity profile to the theoretical predictions.



Velocity profile
l=1600, h=64, averaging=2x2, $\rho_1 = 0.6$, $\rho_2 = 0$, steps=3000

As we see, the shape of the profile fits very well. From the fit we can get the viscosity for our model. $\eta = 0.23$. The average density in the flow is 0.3. Reading out literature data for the FHP3 model, as in [Dieter A. Wolf-Gladrow, *Lattice-Gas Cellular Automata and Lattice Boltzmann Models*, Springer (2000), page 78], for our density we have $\eta_{lit} = 0.28$.

4 Results

For the sample set of parameters:

$$\eta_{lit} = 0.28 \tag{5}$$

$$\eta = 0.23 \tag{6}$$

We can see that both models simulate a simple fluid dynamics. Although the FHP3 model is surely more accurate, as it has more collisions implemented. It can be seen while comparing the animations.

The Poiseuille flow has also been successfuly simulated. Obtained parameters are not far from the literature.

Although we got satisfactory results, we could also see some drawbacks. The most important ones are the fluctuations and the long time of termalization. Those important problems are reduced in e.g. LB models.