

# Quasicrystals Investigated Using DFT: Bulk, Surface and Adsorbates

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- Quasicrystals
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  - Structural models of quasicrystalline surfaces
  - Quasiperiodic monolayers and multilayers
  - Catalytic properties of quasicrystalline surfaces

# Introduction

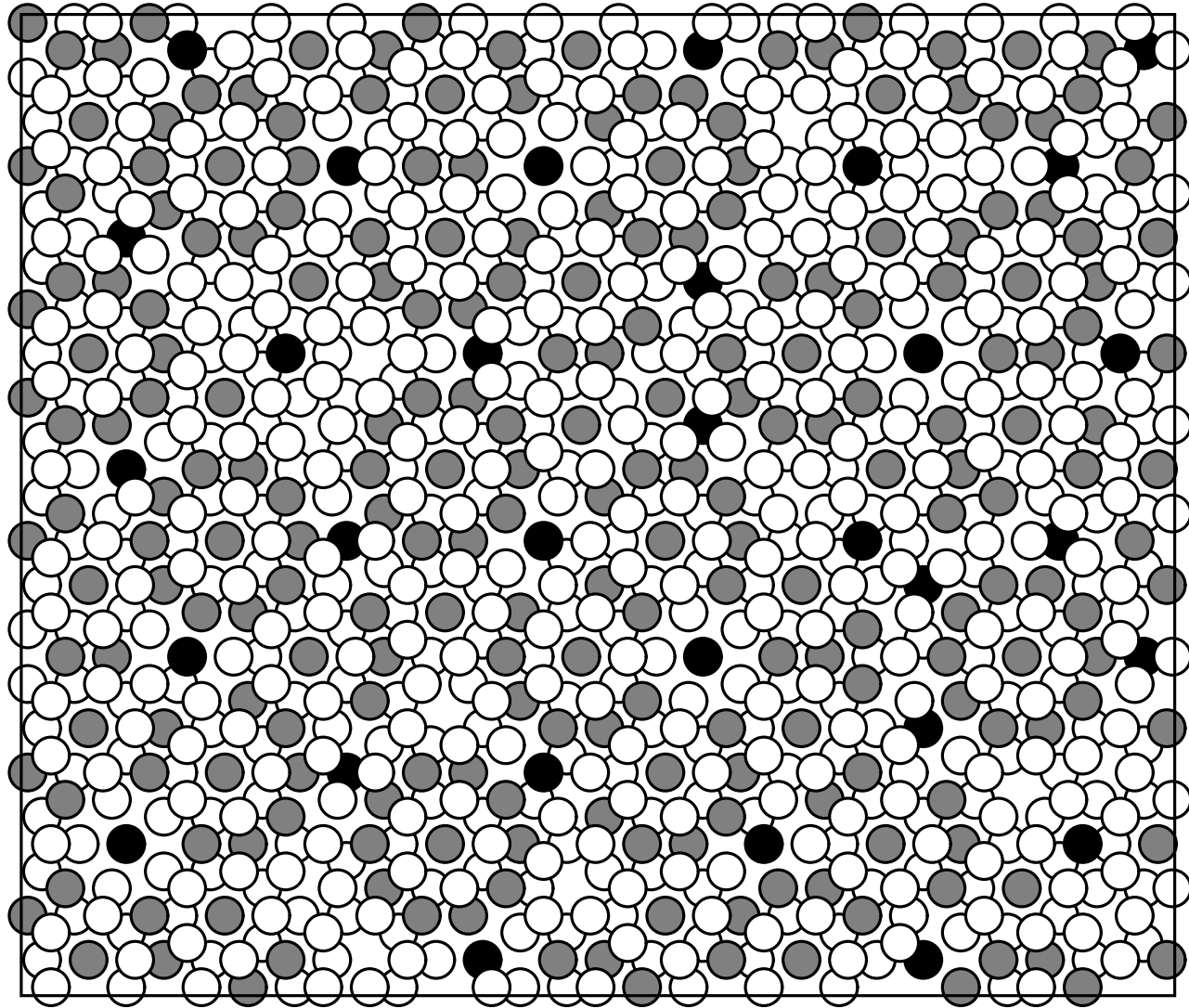
# Quasicrystals

Discovered by Shechtmanom in 1982 at study of AlMn alloys

## Basic characteristics

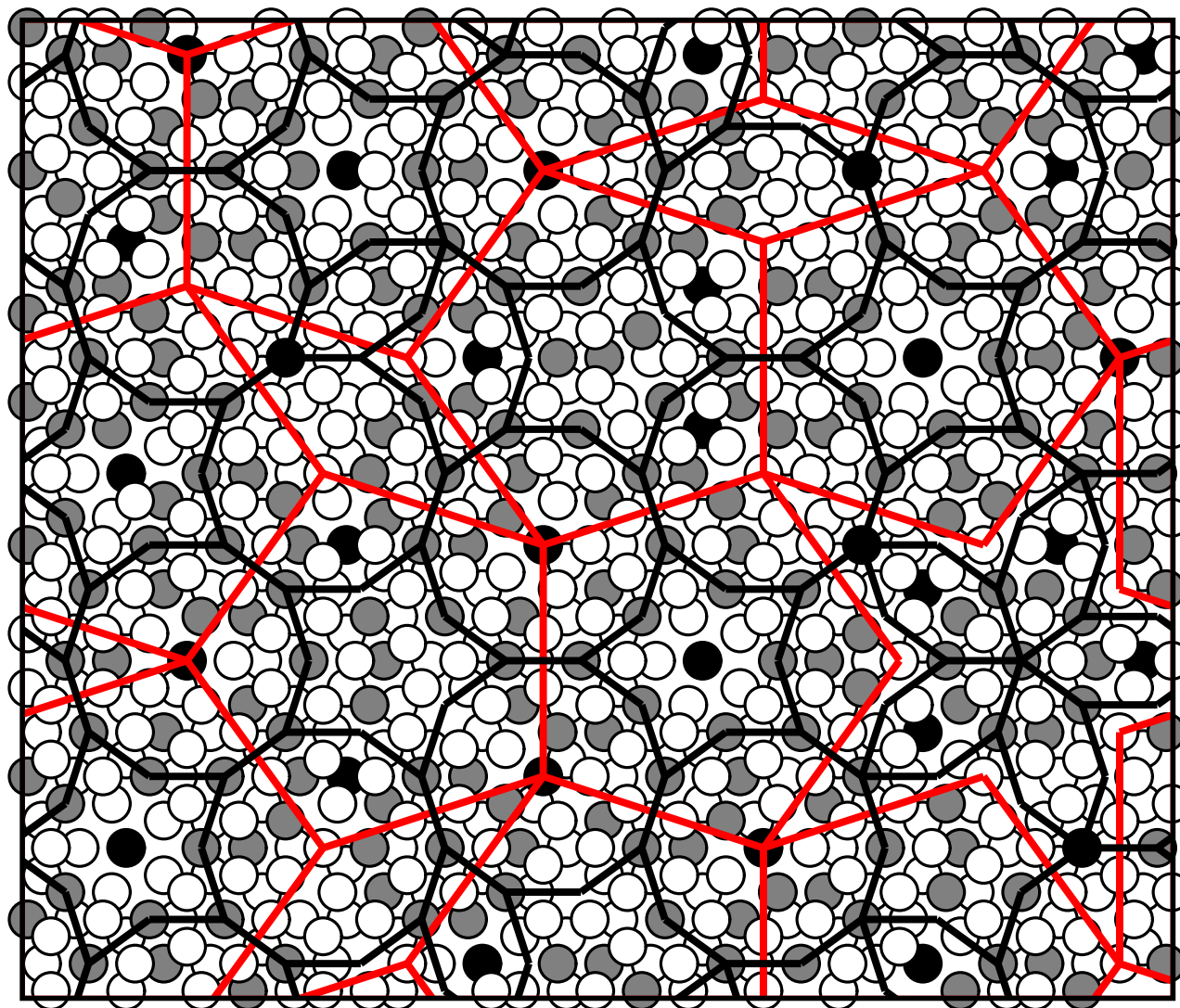
- Regular but not periodic - quasiperiodic arrangement of atoms
- „Forbidden“ symetries of diffraction patterns
- Structure described in high-dimensional space
- Metallic phases
- Anomalous physical properties

## Quasiperiodic arrangement of atoms



5-fold surface of icosahedral AlPdMn quasicrystal

## Quasiperiodic arrangement of atoms



5-fold surface of icosahedral AlPdMn quasicrystal

# Classes of quasicrystals

- Icosahedral
- Decagonal
- Dodecagonal
- Octagonal
- .....

## Quasicrystals as intermetallic phases

- Al-Mg (AlZnMg, AlCuLi, AlMgLi, ...) - Frank-Kasper
- Al-Mn (AlMn, AlPdMn, AlPdRe, ...) - Mackay
- Zn-Mg-RE (RE=Y, Dy, Gd, Ho, ...)
- Ti-Zr-Ni
- Cd-Yb
- .....

Quasicrystals - (Al-based) complex metallic phases



# Quasicrystals

# Basic notions

Geometry of icosedral and decagonal quasicrystals is related to properties of the Golden Mean

$$\tau = (1 + \sqrt{5})/2 = 1.61803398\dots$$

and the Fibonacci numbers  $F_n$ ,  $n = 0, 1, 2, \dots$

$$F_0 = 0, F_1 = 1, F_{n+1} = F_n + F_{n-1}$$

$$F_n = 0, 1, 1, 2, 3, 5, 8, 13, 21, \dots$$

$$\lim_{n \rightarrow \infty} F_{n+1}/F_n = \tau$$

$F_{n+1}/F_n$  - rational approximant to  $\tau$

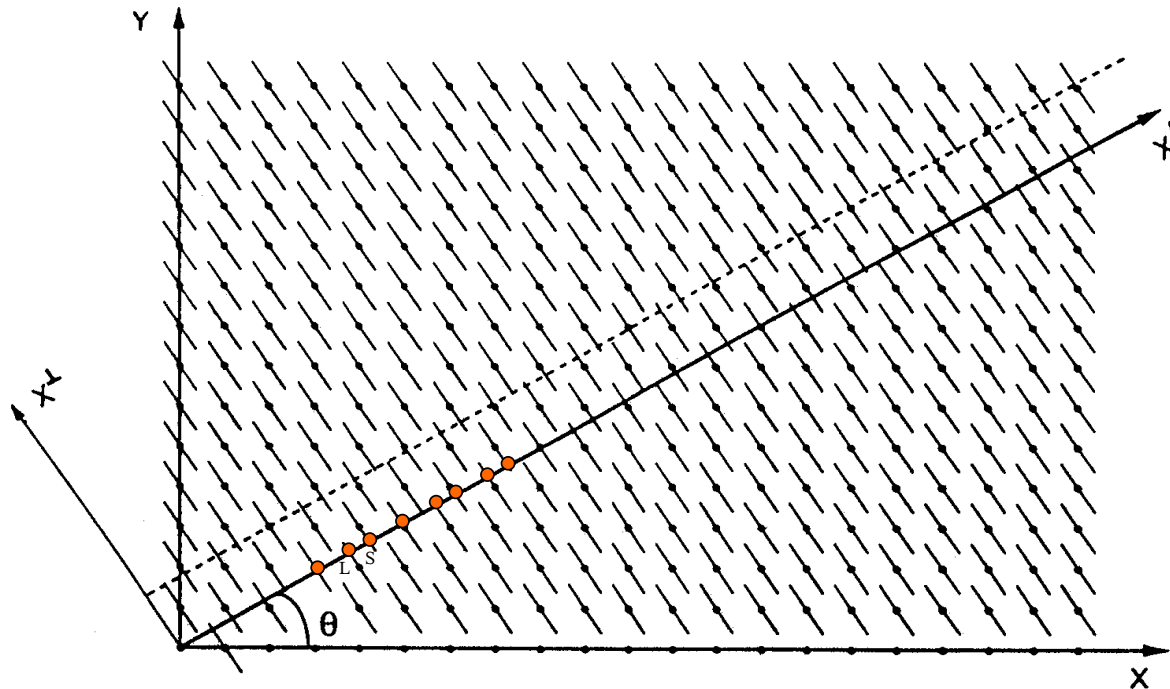
1/1, 2/1, 3/2, 5/3, 8/5, ... – approximants to a quasicrystal

Approximants to a quasicrystal  $\rightarrow$  crystals with large unit cells

# Structural models of quasicrystals

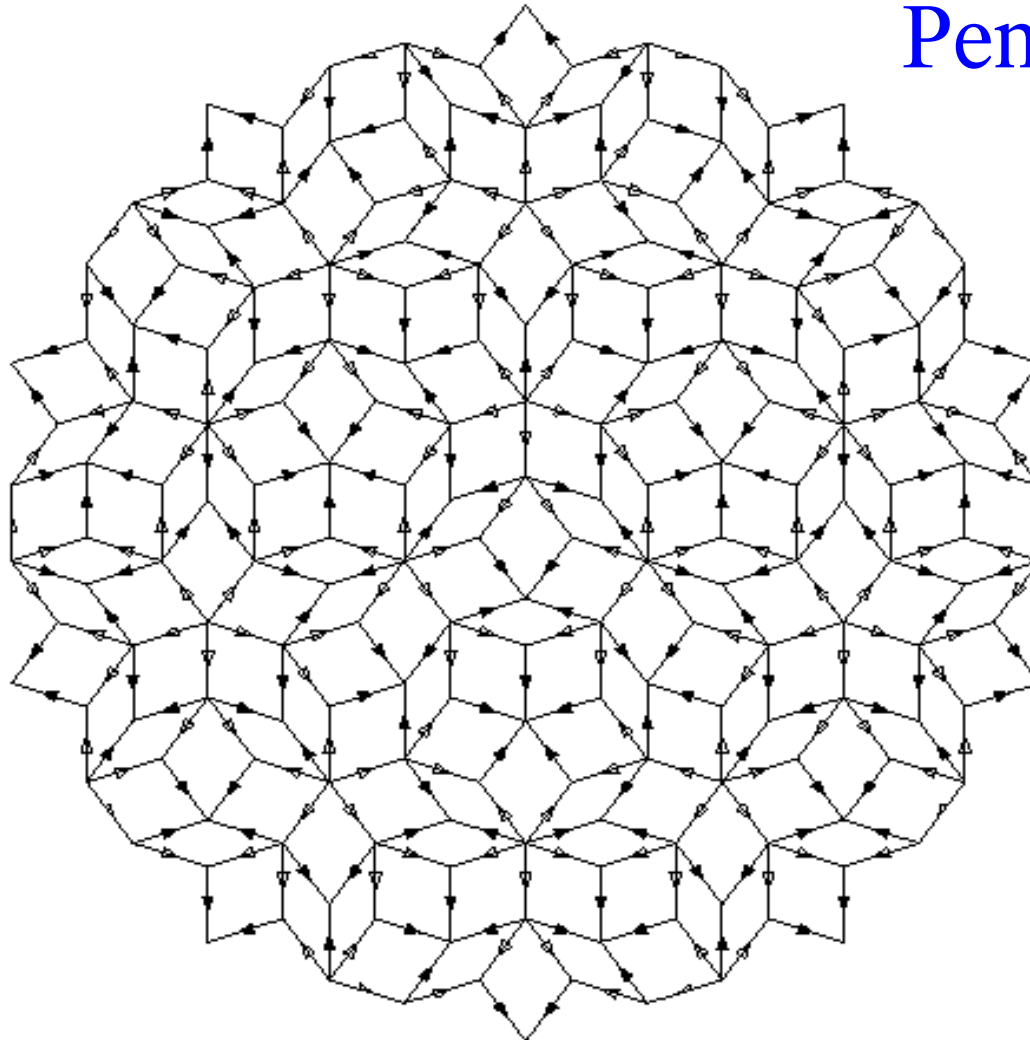
# Projection method

Construction of a 1D (quasi)periodic sequence



2D (square) lattice decorated by 1D acceptance domains (AD)  
Intersections of parallel space with AD define a (quasi)periodic sequence  
LSLLSLSL....

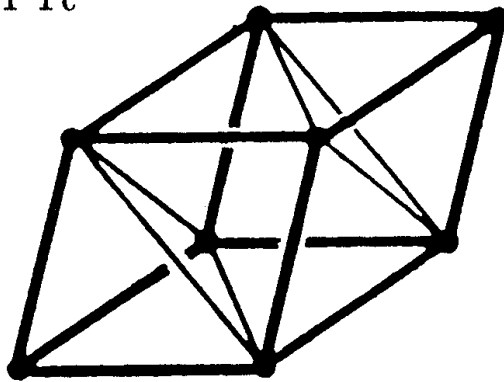
# Penrose tiling



- two types of tiles:
  - fat rhombus
  - skinny rhombus
- matching rules

# 3D Penrose (Ammann) tiling

PR

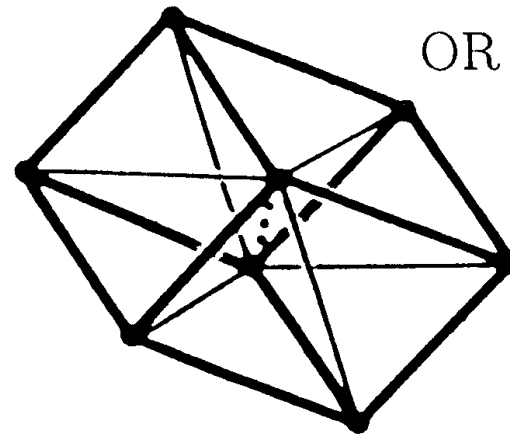


$$a = 1$$

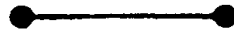


PR – prolate rhombohedron

OR



$$b \cong 1.05$$

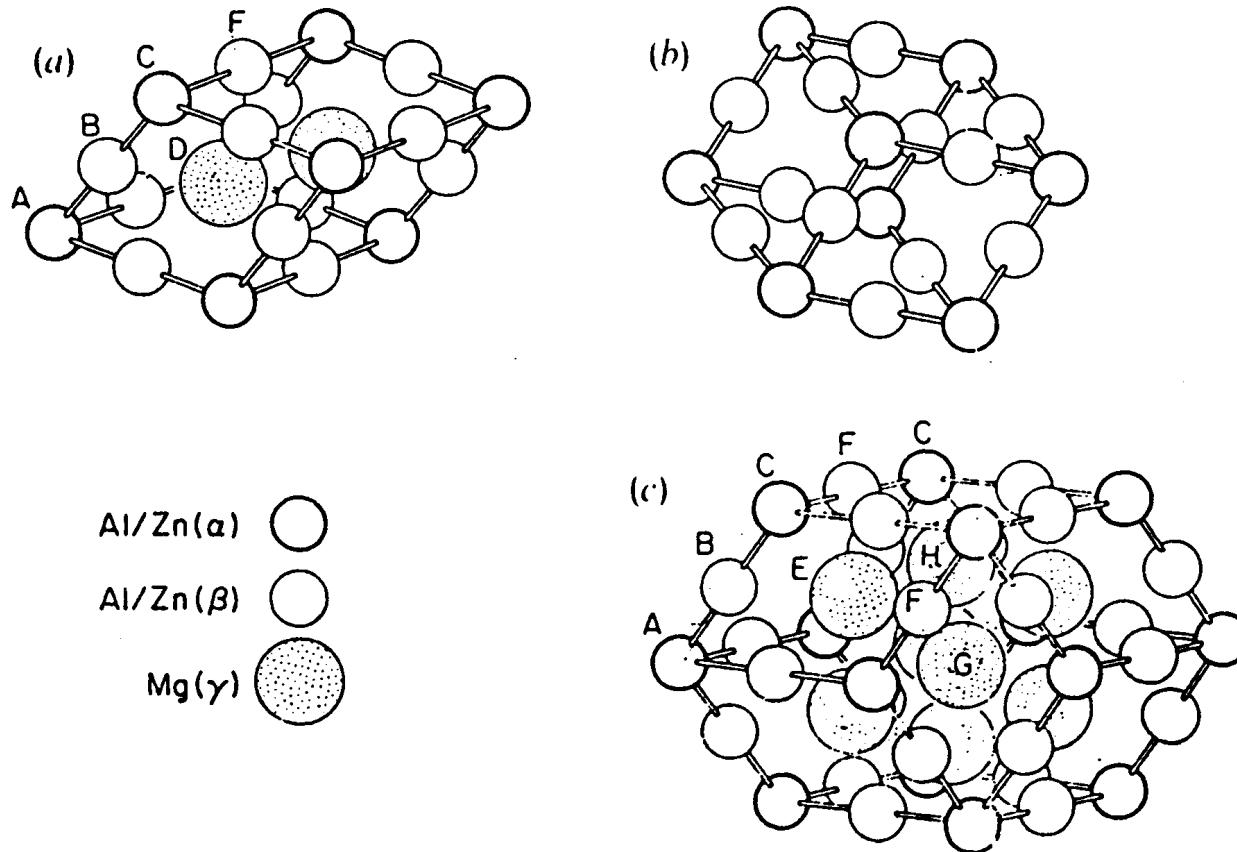


$$c \cong 0.56$$



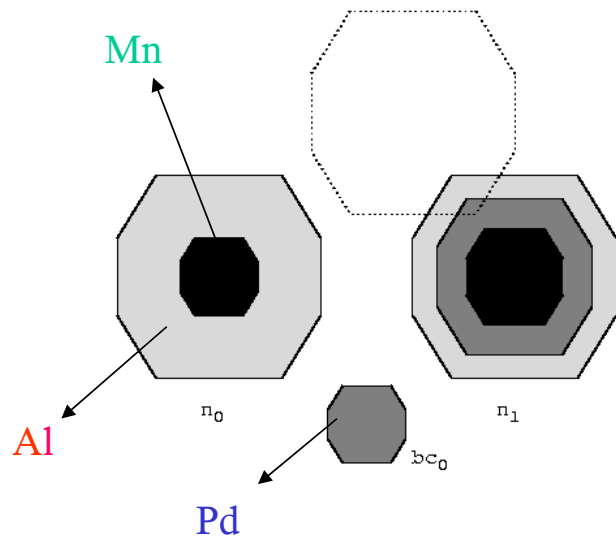
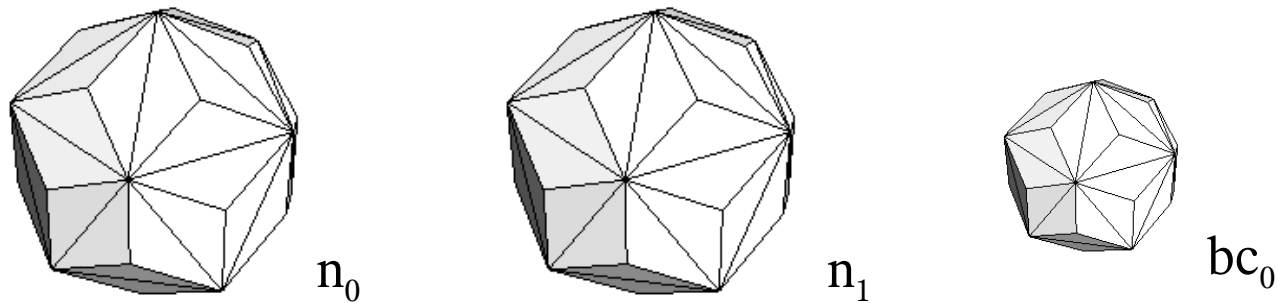
OR - oblate rhombohedron

## Structural model AlZnMg – decoration of PR a OR by atoms

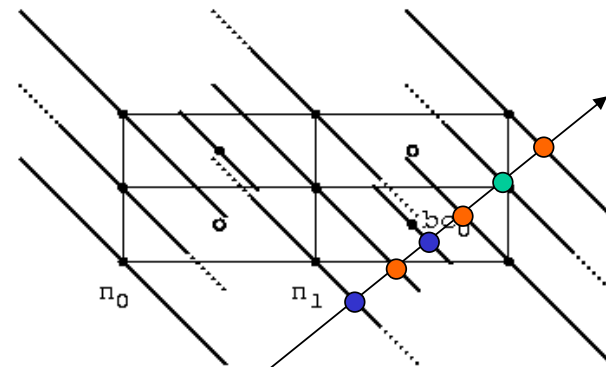


# 6D model of icosahedral AlPdMn

Acceptance domains (AD) – 3 polyhedra - triacontaheders

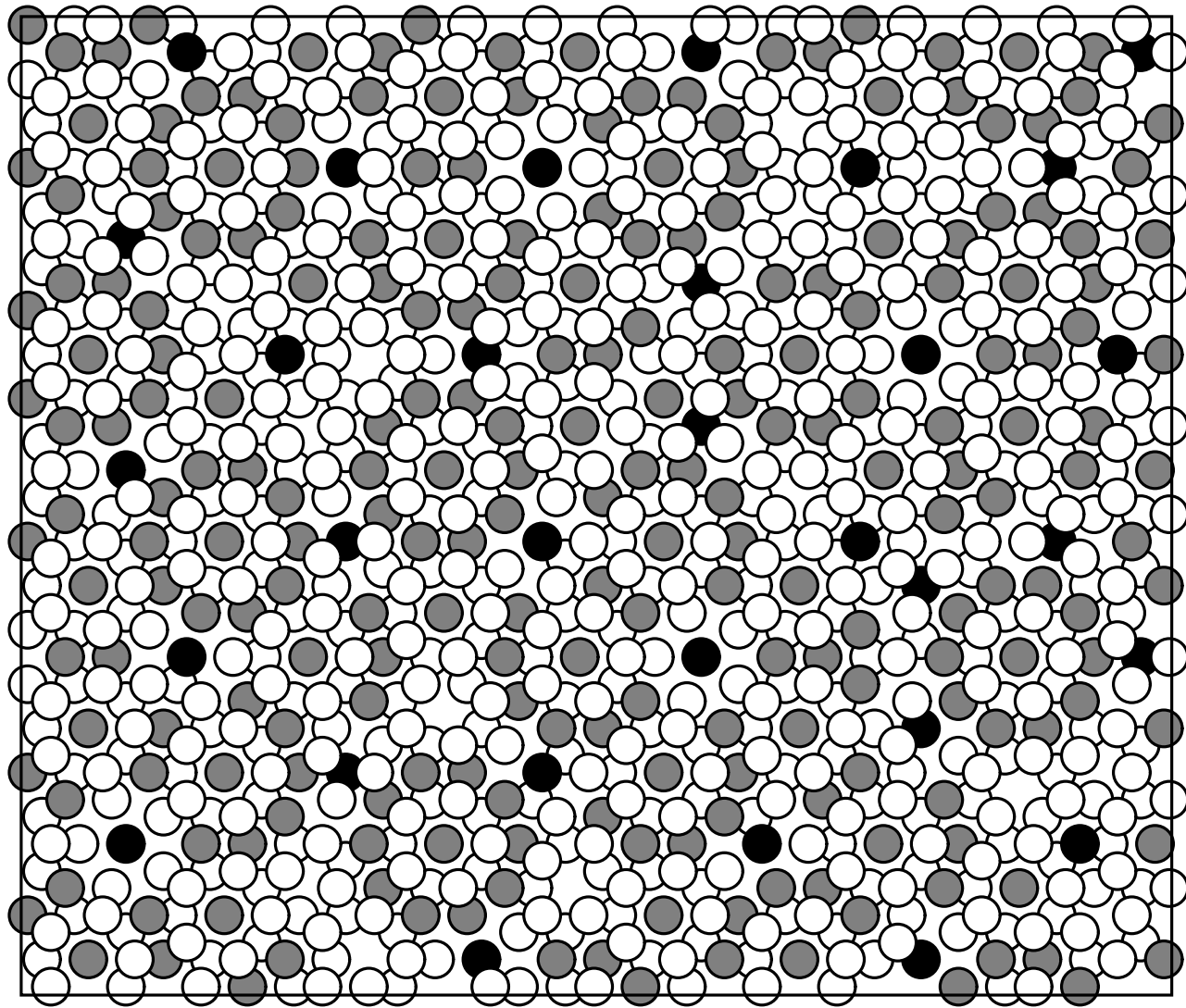


Internal structure of AD define chemical ordering of atoms



6D cubic lattice decorated by AD

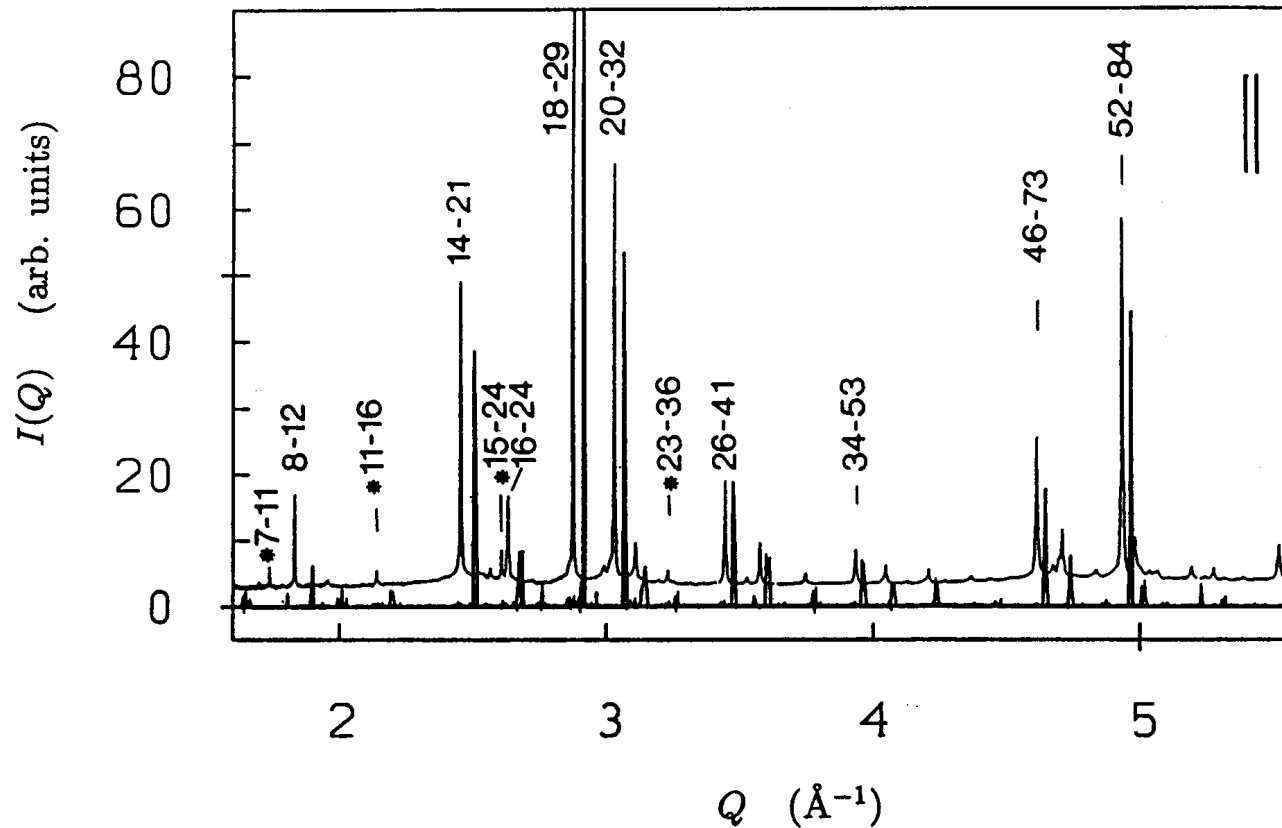




6D model of icosahedral AlPdMn quasicrystal

# i-AlPdMn

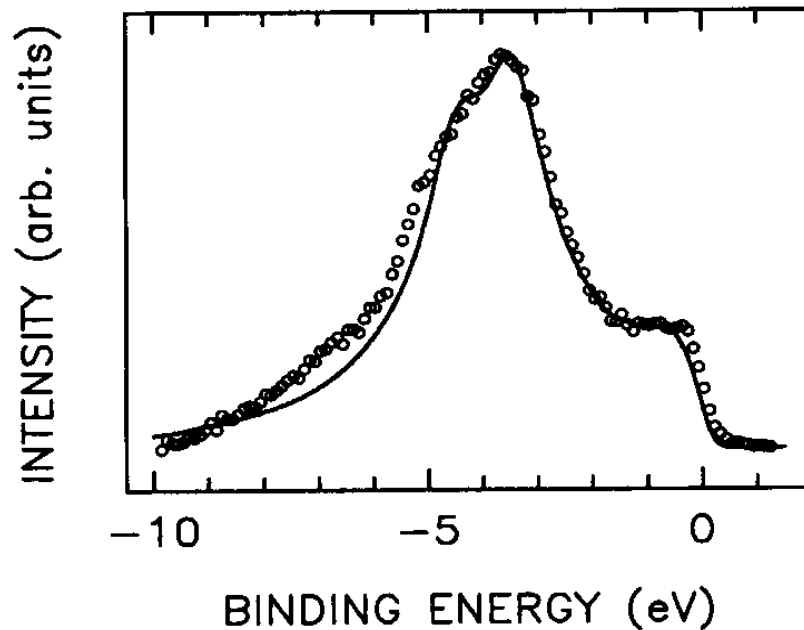
Comparison of diffraction pattern from 6D model with experimentom



M.K., J. H., et al.: PRB **51**, 17355 (1995)

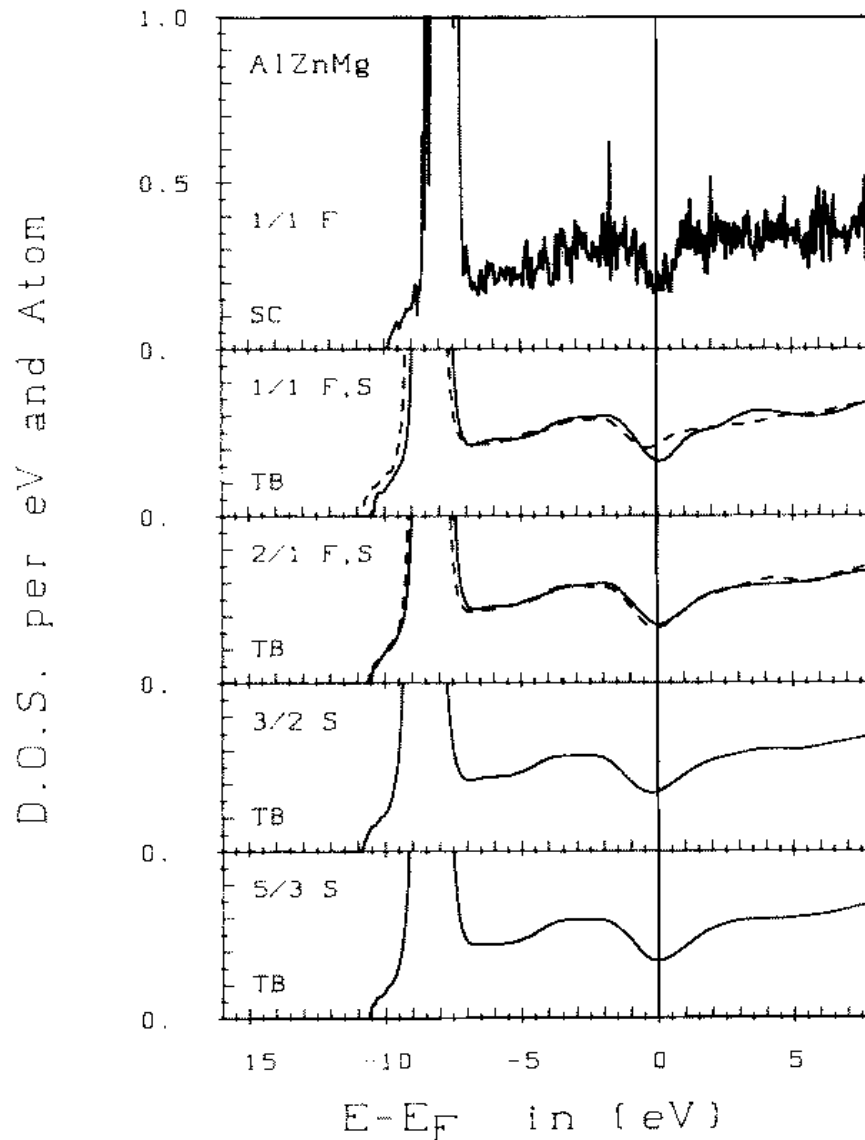
## *i*-AlPdMn

Comparison of a PES from 6D model with experiment



**Fig. 8.11.** Comparison of the valence band of *i*-Al<sub>70</sub>Pd<sub>20</sub>Mn<sub>10</sub> measured at  $h\nu = 100$  eV (open circles, Zhang et al. 1994) with the broadened theoretical DOS for the 8/5 approximant to the *i* phase (solid line, Krajčí et al. 1995). After Zhang and Stadnik (1995).

# Electronic structure of quasicrystals



## DOS of quasicrystalline approximants

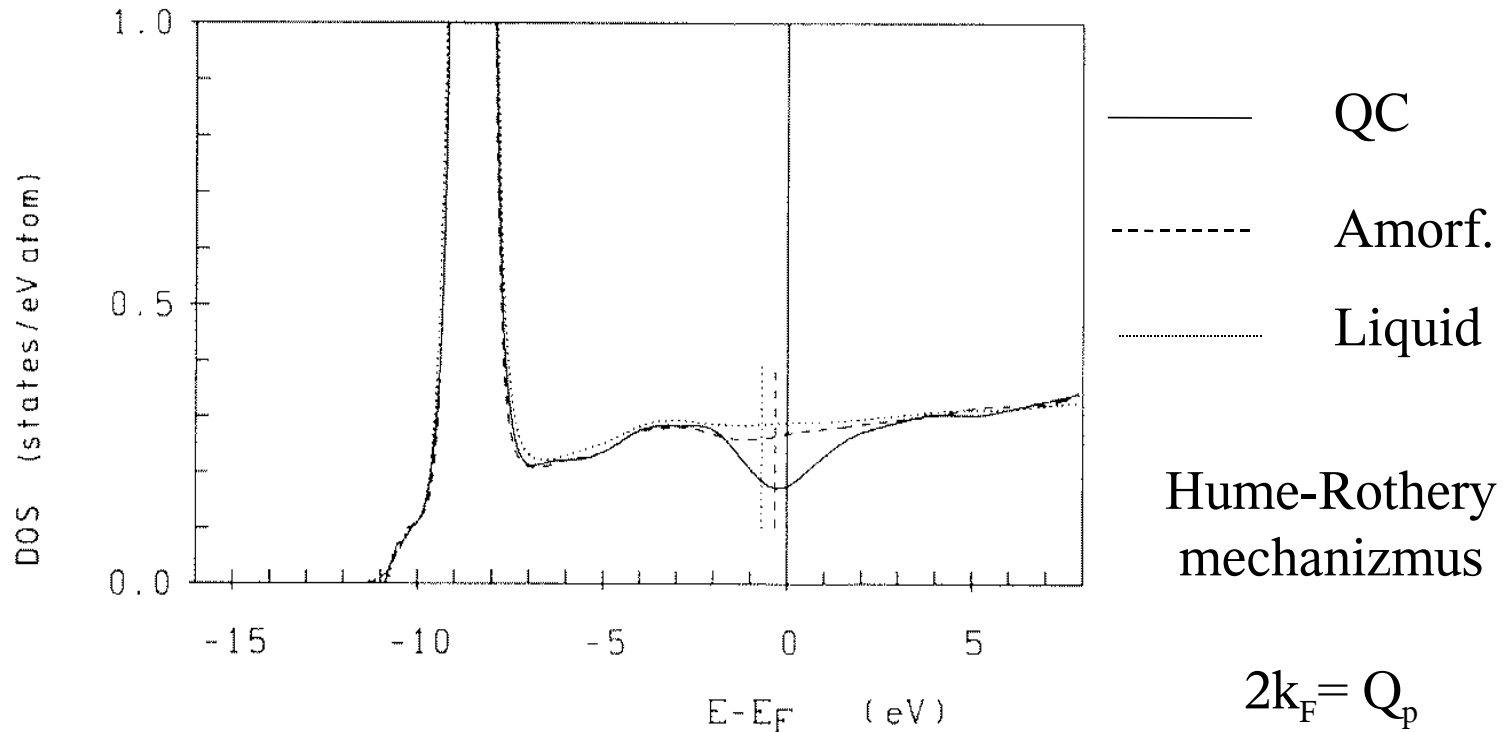
AlZnMg

- 1/1 - 162 atoms
  - 2/1 - 680 atoms
  - 3/2 - 2920 atoms
  - 5/3 - 12560 atoms
- 
- „spiky“ character
  - pseudogap at  $E_F$
  - fast convergence

J. H. and M. K.: PRB **47**, 11795 (1993)

# DOS of quasicrystalline, amorphous and liquid system

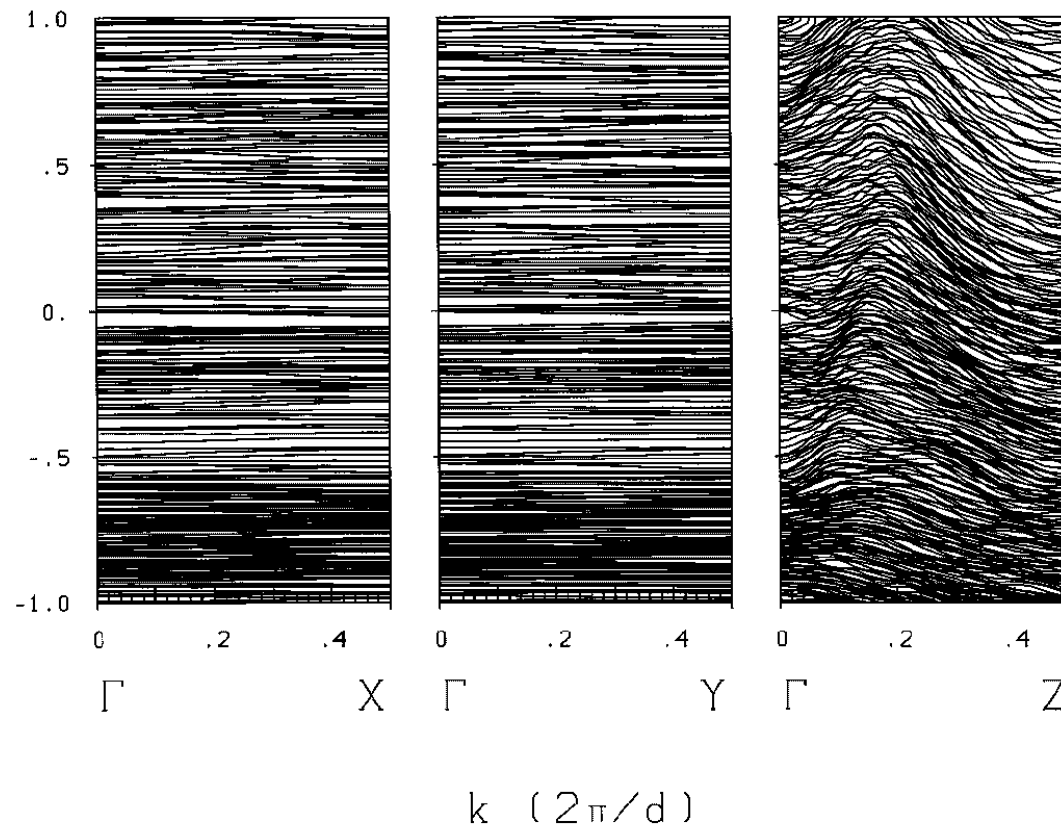
- 3/2 – approximant i-AlZnMg
- 2920 atoms
- MD -> liquid state
- relaxed to amorphous state



J. H. and M. K.: PRB **47**, 11795 (1993)

## Band structure of a d-AlCuCo approximant

x, y – quasiperiodic plane  
z - decagonal axis



$37.7 \text{ \AA} \times 32.07 \text{ \AA} \times 4.18 \text{ \AA}$ , 352 atoms

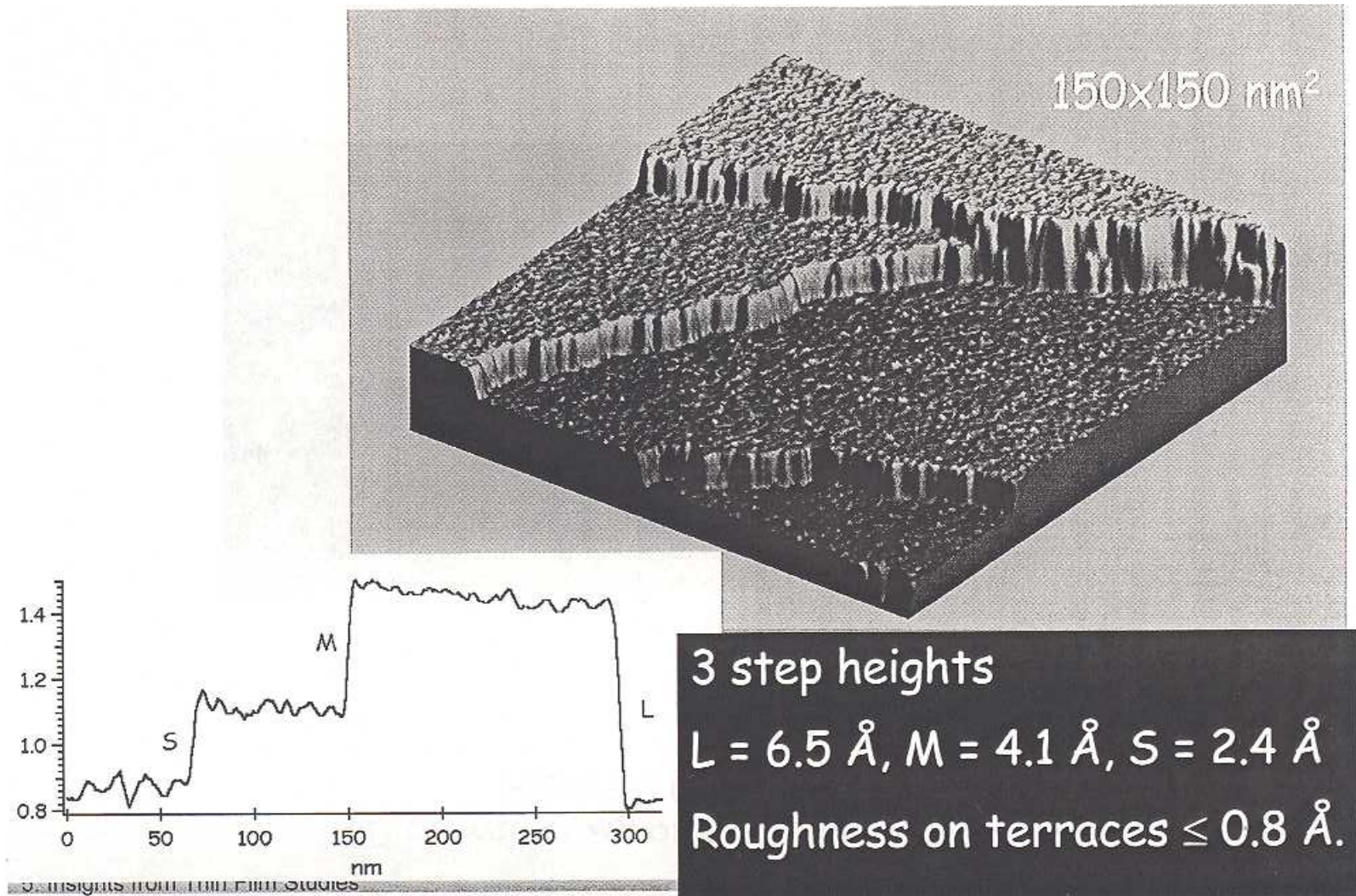
# Quasicrystalline surfaces



# Surfaces of quasicrystals

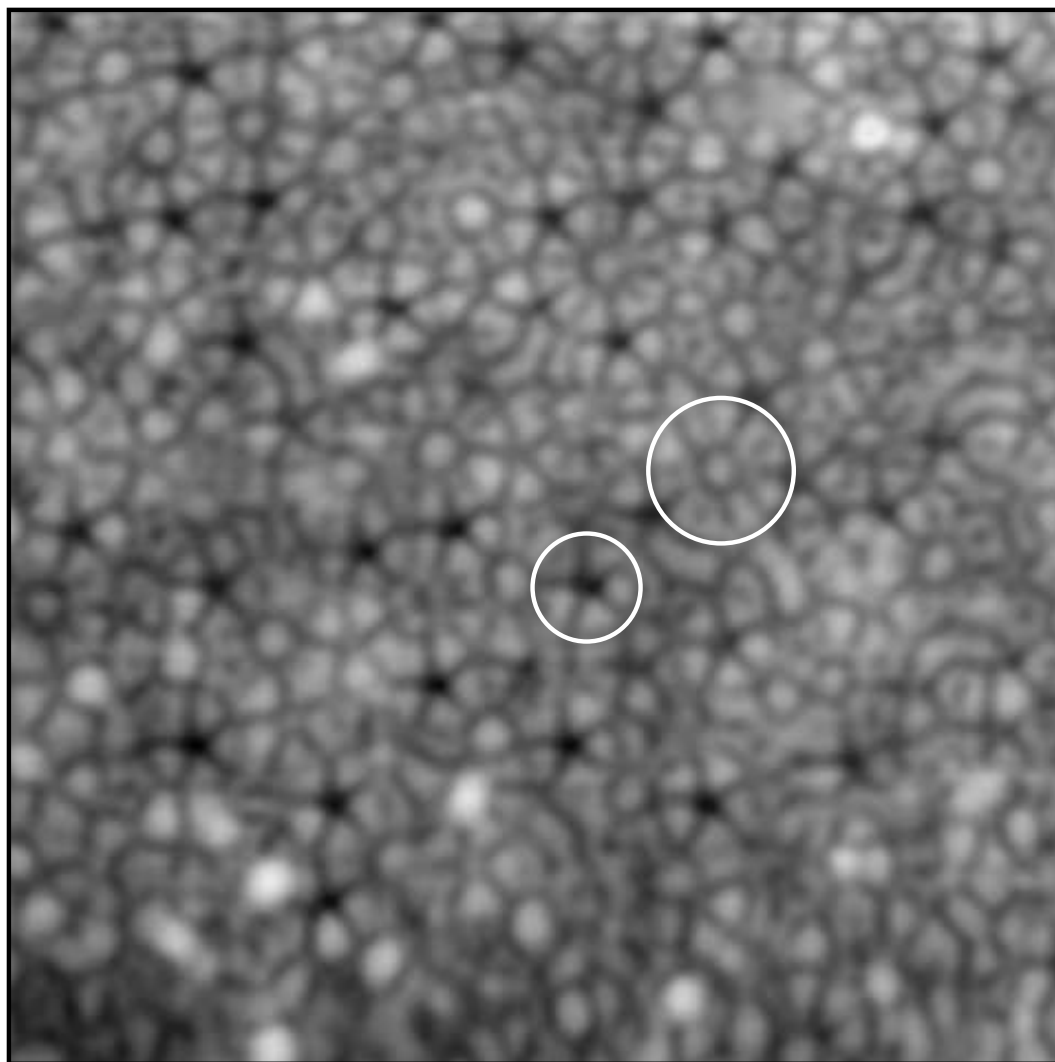
- Quasicrystals - complex metallic alloys → mechanical properties of bulk - brittle
- Despite quasiperiodic order – surfaces are atomically flat
- Surfaces - possible technical applications
- Extraordinary tribological properties (wear, wetting, ...)
- Extraordinary chemical reactivity - adsorption and catalytic properties
- Surface of stable quasicrystals i-AlPdMn and d-AlNiCo - intensively studied.

# Low resolution STM image of the surface of i-AlPdMn.



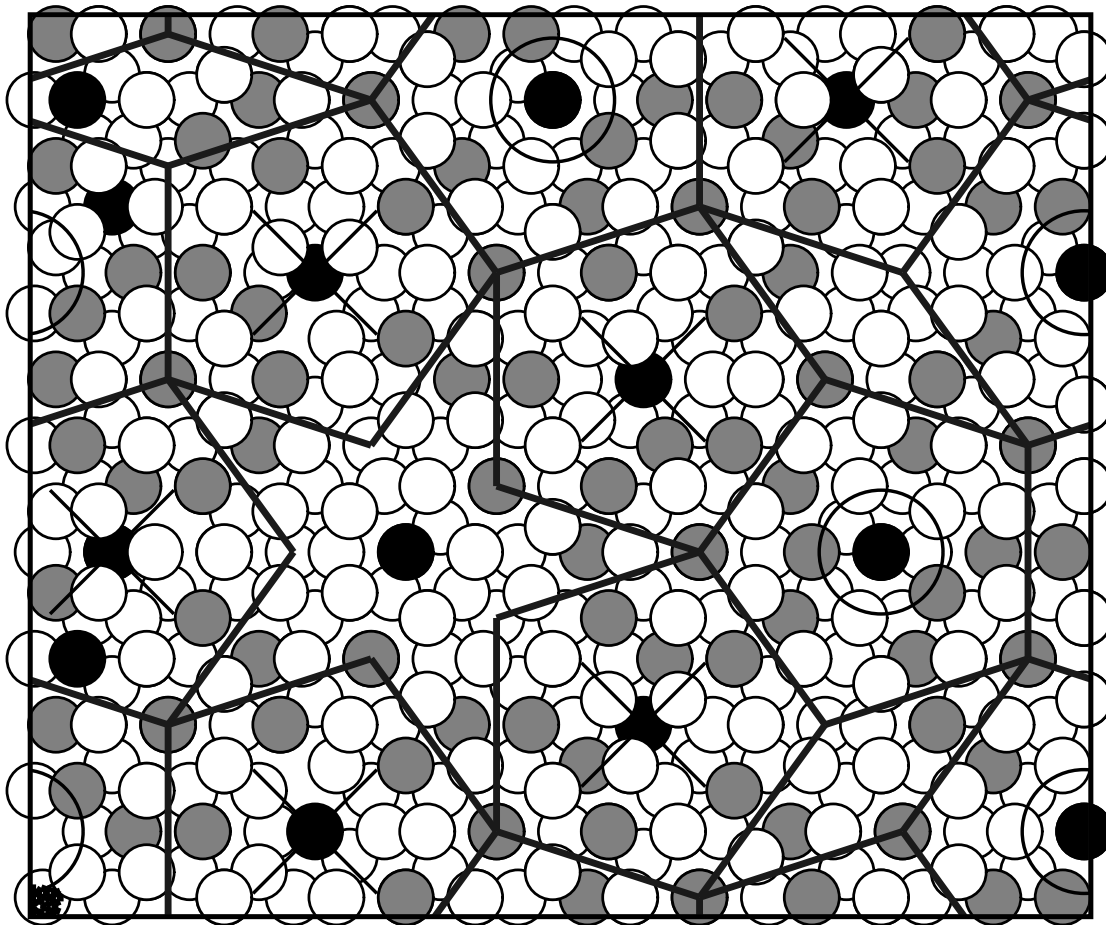
$150 \times 150 \text{ nm}$ , Thiel and McGrath 2004

## High resolution STM image of the surface of i-AlPdMn.



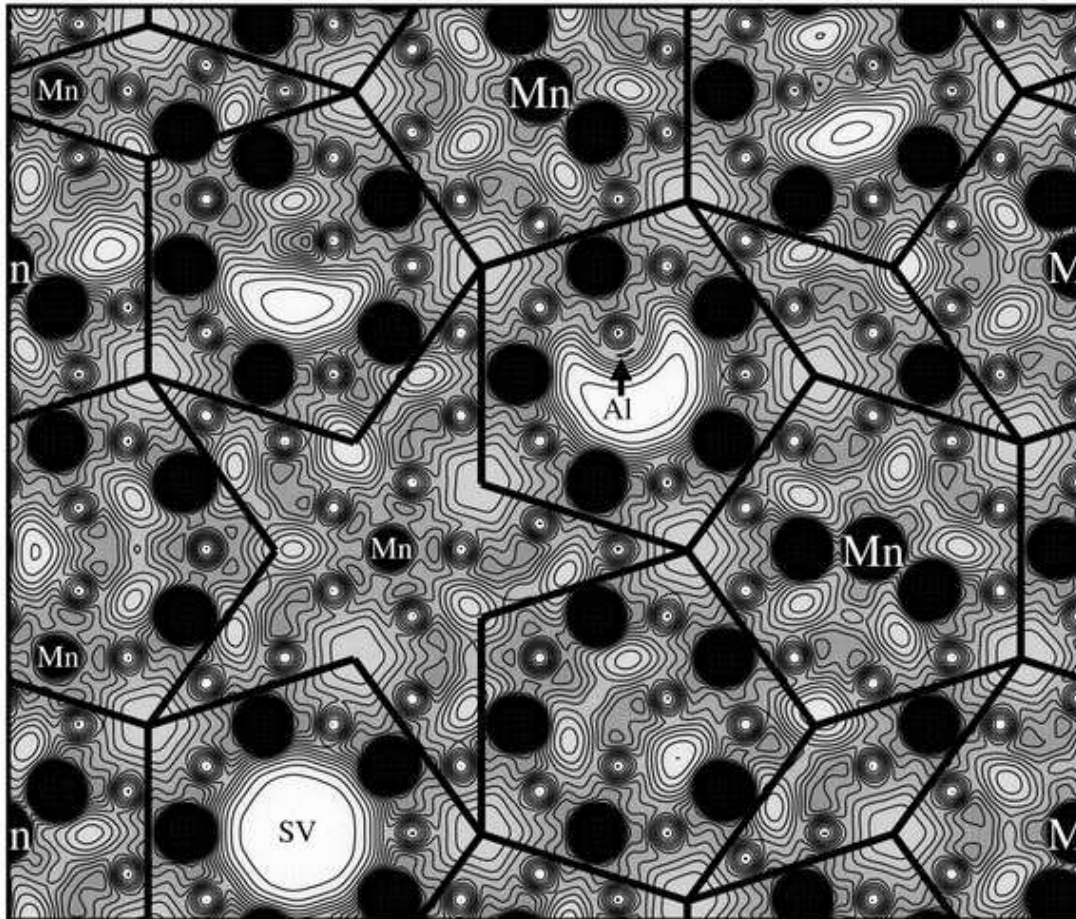
- $10 \times 10$  nm
- White flower (WF)
- Dark star (DS)
- Ledieu and McGrath 2005
- Atomic structure not known (from experiment)

# Structural model of the 5-fold surface of i-AlPdMn



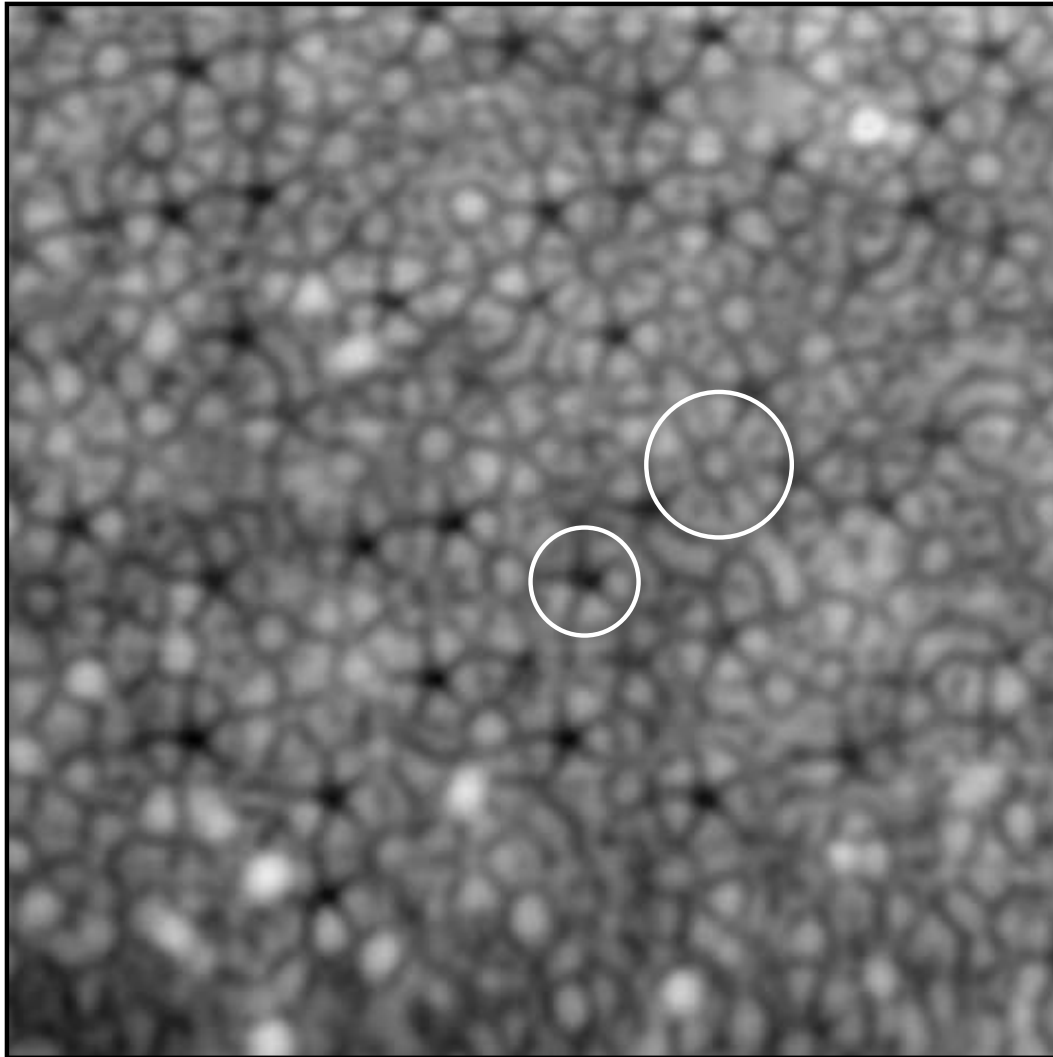
- KGB model
- 3/2-approximant  
38.63 Å × 32.86 Å
- Al – open circles
- Pd – gray circles
- Mn – dark circles
- Quasiperiodic order – **P1-tiling** (Papadopoulos et. al 2002)
- Pentagons – "**top**" and "**bottom**" orientations

# Charge density distribution in a surface layer



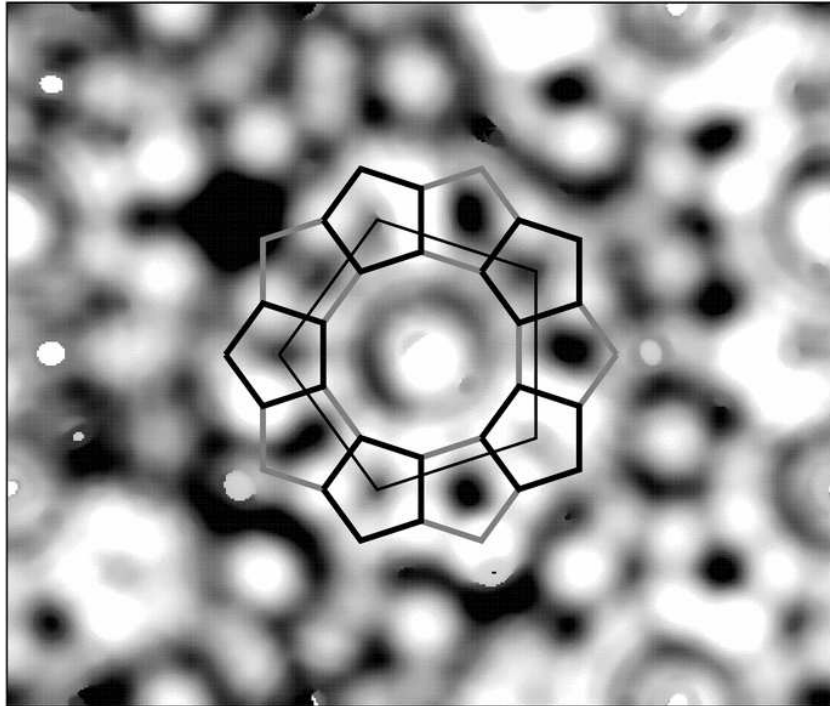
- "top" and "bottom" pentagons - central Mn atom
- pseudo-Mackay ( $M$ ) and Bergman-type ( $B$ ) clusters
- Surface vacancies ( $SV$ )

# High resolution STM image of the i-AIPdMn surface

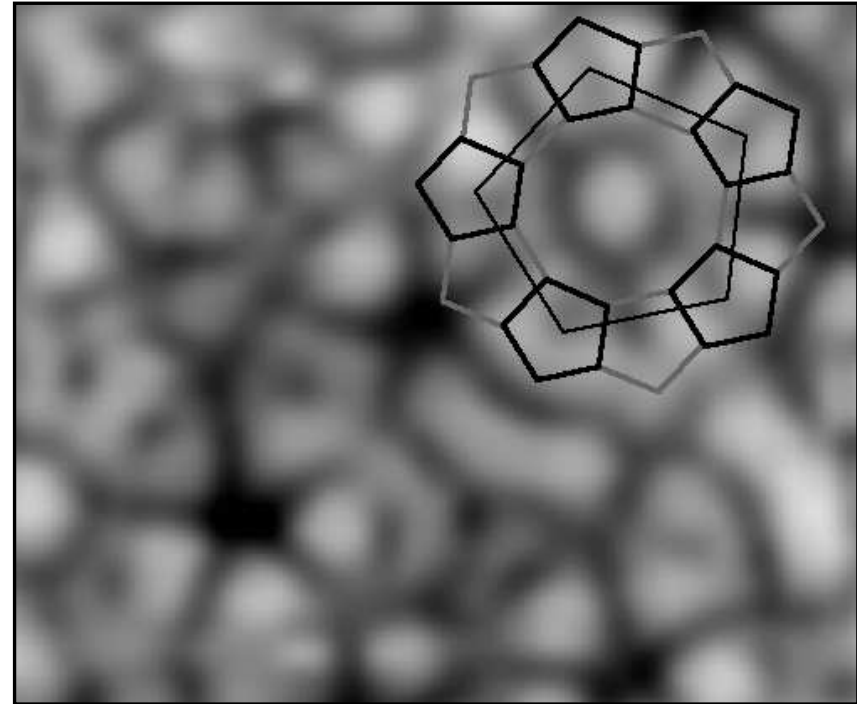


- exp. STM image from Ledieu and McGrath 2005
- 10×10 nm
- White flower (WF)
- Dark star (DS)

## Detail of STM image – white flower (WF)



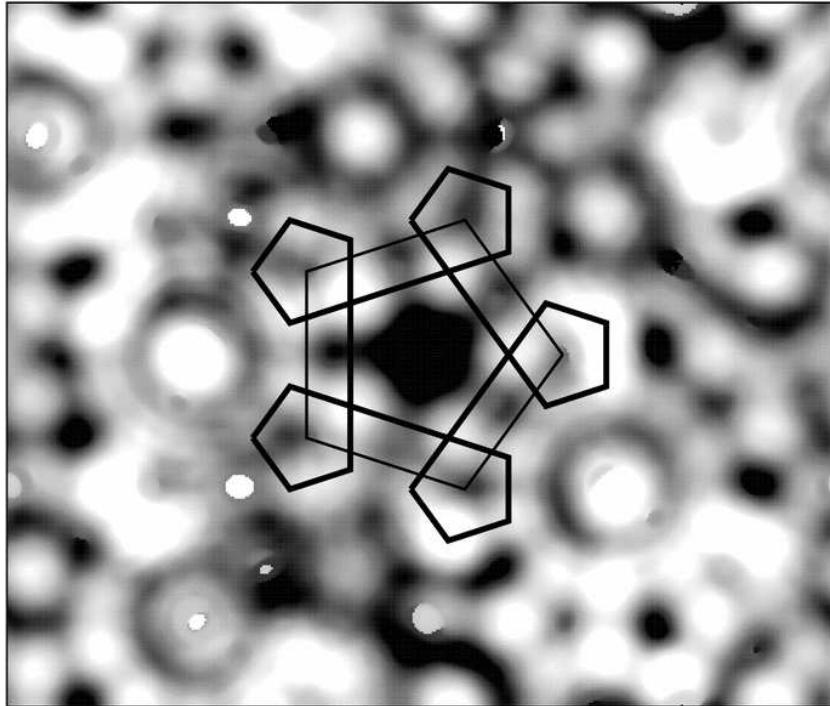
structural model



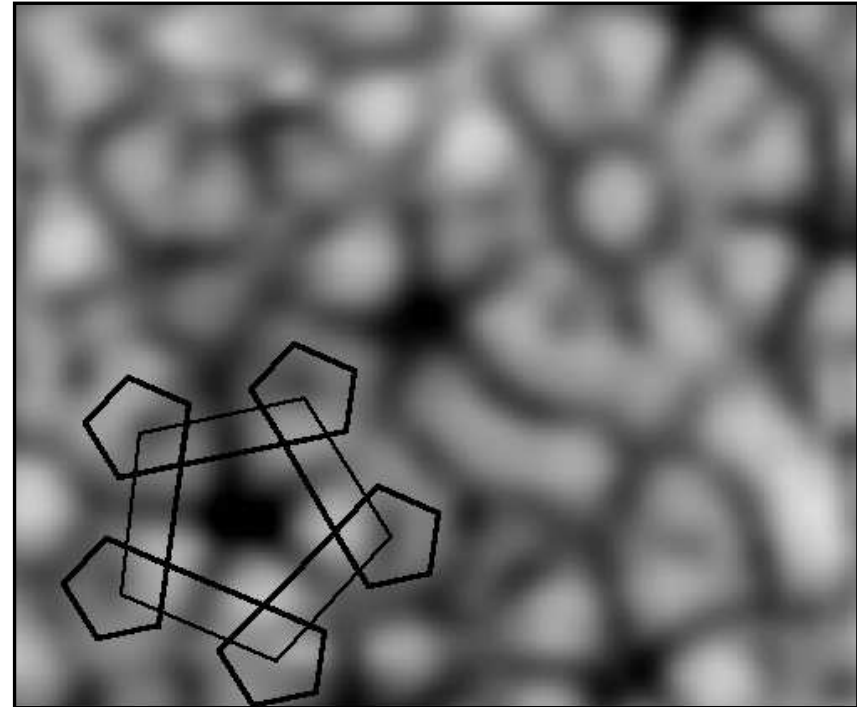
experiment

- **Identification of positions and chemical types of individual atoms:**  
Small pentagons – Al pentagons with an edge  $2.96 \text{ \AA}$  – leaves of WF  
In the center – a Mn atom
- WF – "top" pentagonal tiles of the P1 tiling
- Experimental STM: Ledieu and McGrath 2005

## Detail of STM image – dark star (DS)



structural model

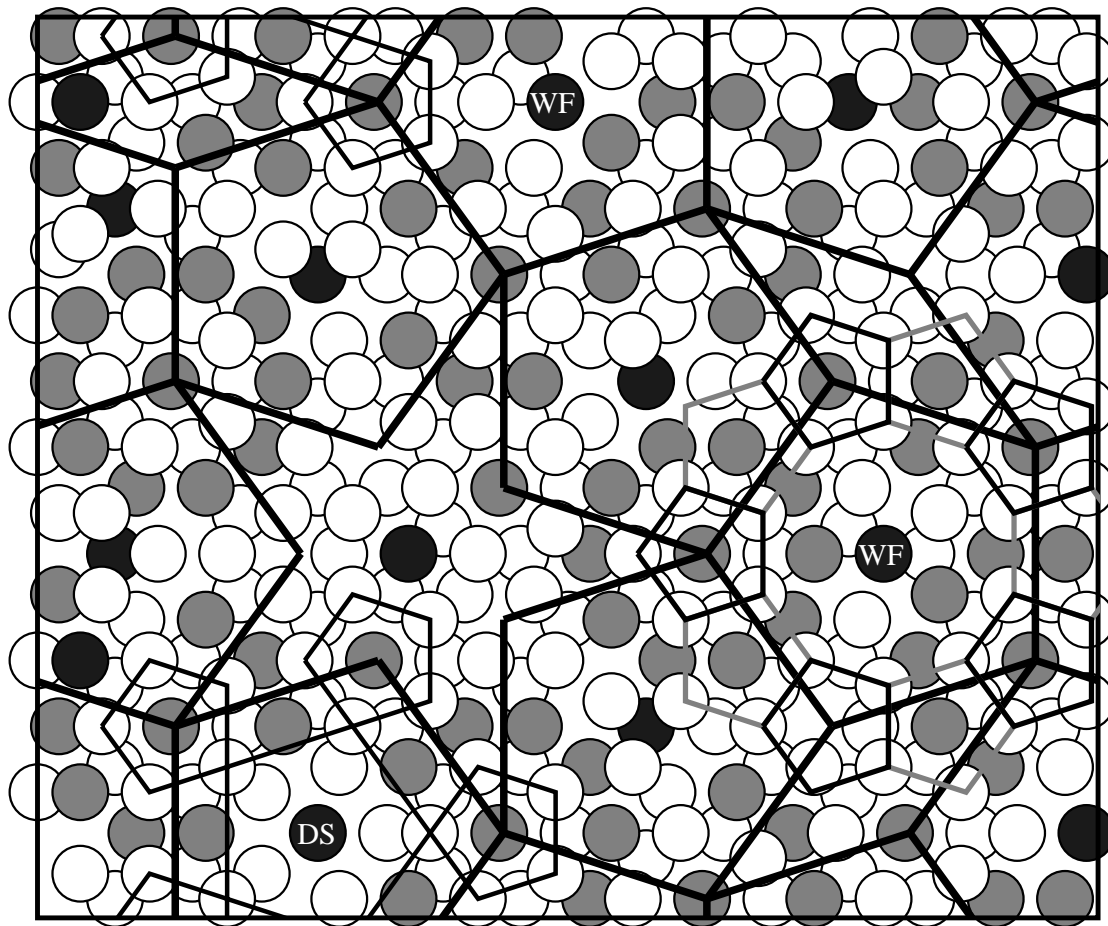


experiment

- DS – inside some of "bottom" pentagonal P1 tiles
- Atomic structure: **surface vacancy** surrounded by the ten-fold ring of atoms – a pentagon of Al atoms with the edge 4.79 Å and a pentagon of Pd atoms.  
The contrast between the bright Al atoms and the dark Pd atoms forms the characteristic pentagonal shape of the DS



# Structural model of the 5-fold surface of i-AlPdMn



- WF - "top" pentagonal tile
- DS - "bottom" pentagonal tile
- M. K., J. H., J. Ledieu, and R. McGrath: PRB **73** 024202 (2006)

Satisfactorily realistic structural model of clean i-AlPdMn surface

# Quasiperiodic monolayers and multilayers

- Ab-initio studies: monolayers on the surface of i-AIPdMn and d-AlCoNi

- PRB **71** 054202 (2005) : clean i-AIPdMn surface
- PRB **71** 184207 (2005) : Bi, Sb, and Sn monolayers on i-AIPdMn surface
- PRB **73** 024202 (2006) : clean i-AIPdMn surface - STM images
- PRB **73** 134203 (2006) : clean d-AlCoNi surface
- PRB **73** 184202 (2006) : Bi on d-AlCoNi surface
- PRB **75** 224205 (2007) : alkali metal monolayers and multilayers on i-AIPdMn
- PRB **77** 134202 (2008) : monolayers on i-AIPdMn - elements from group I to III
- PRB **79** 134206 (2009) : alkali metal monolayers on i-AIPdMn - Shukla et al.
- PRB **79** 165430 (2009) : Pb clusters on i-AIPdMn - Ledieu et al.
- PRB **81** 085417 (2010) : Pb monolayers on i-AIPdMn - structure and electronic properties

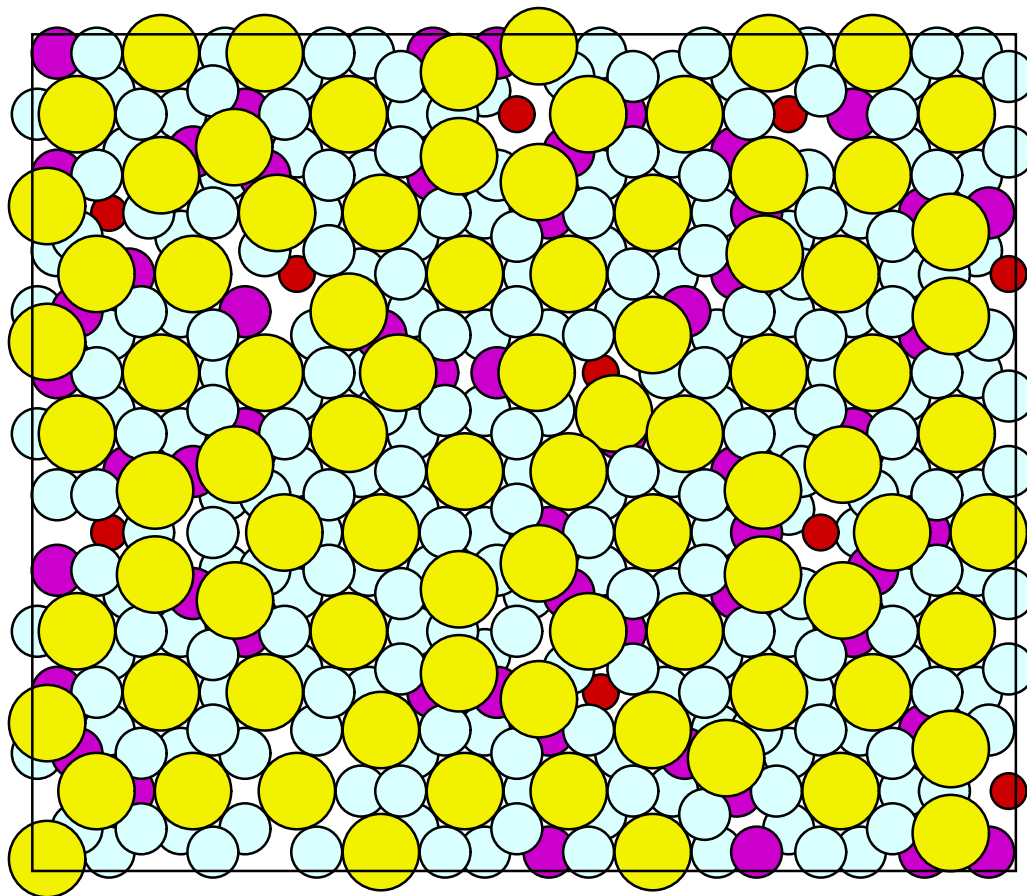
- Experiment: most attempts to grow quasiperiodic overlayers failed!

# Which elements form regular quasiperiodic monolayers?

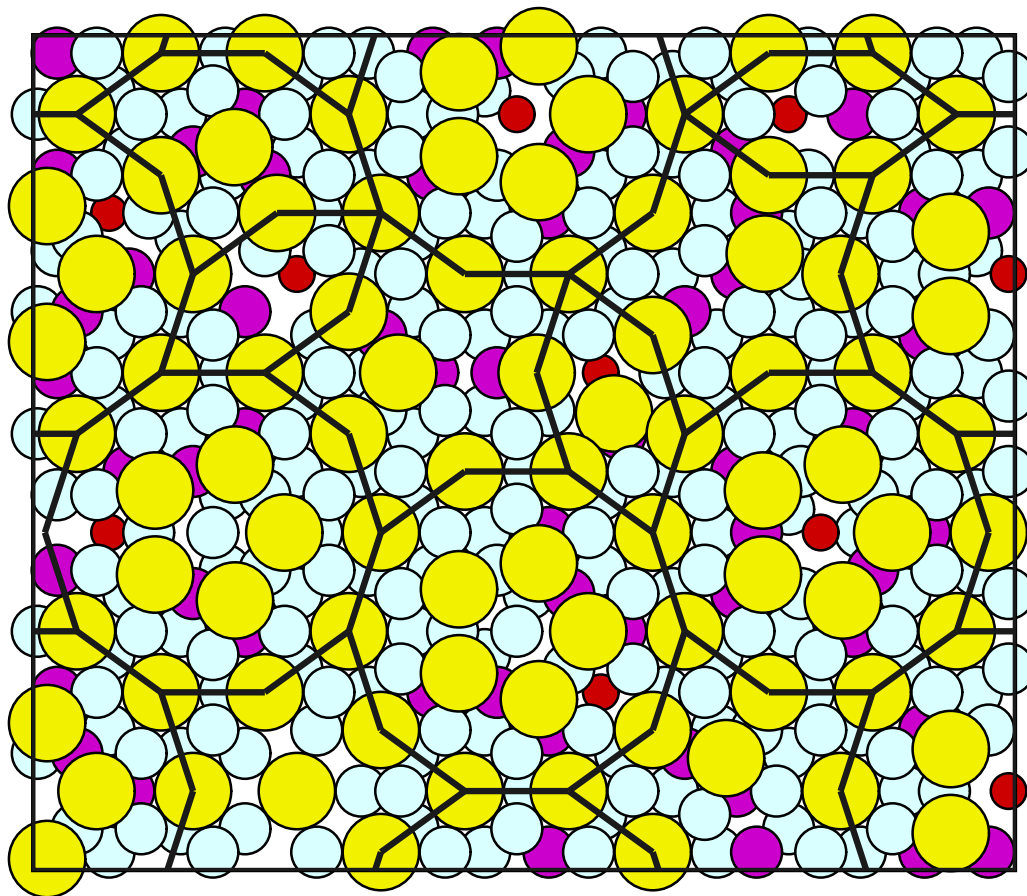
Investigated elements:

- Alkali metals: Li, Na, K, Rb, Cs
- Alkaline earth metals: Be, Mg, Ca, Sr, Ba
- Group 3 elements: Sc, Y, La
- Rare-earth elements: Gd
- Pb group: Pb, Bi

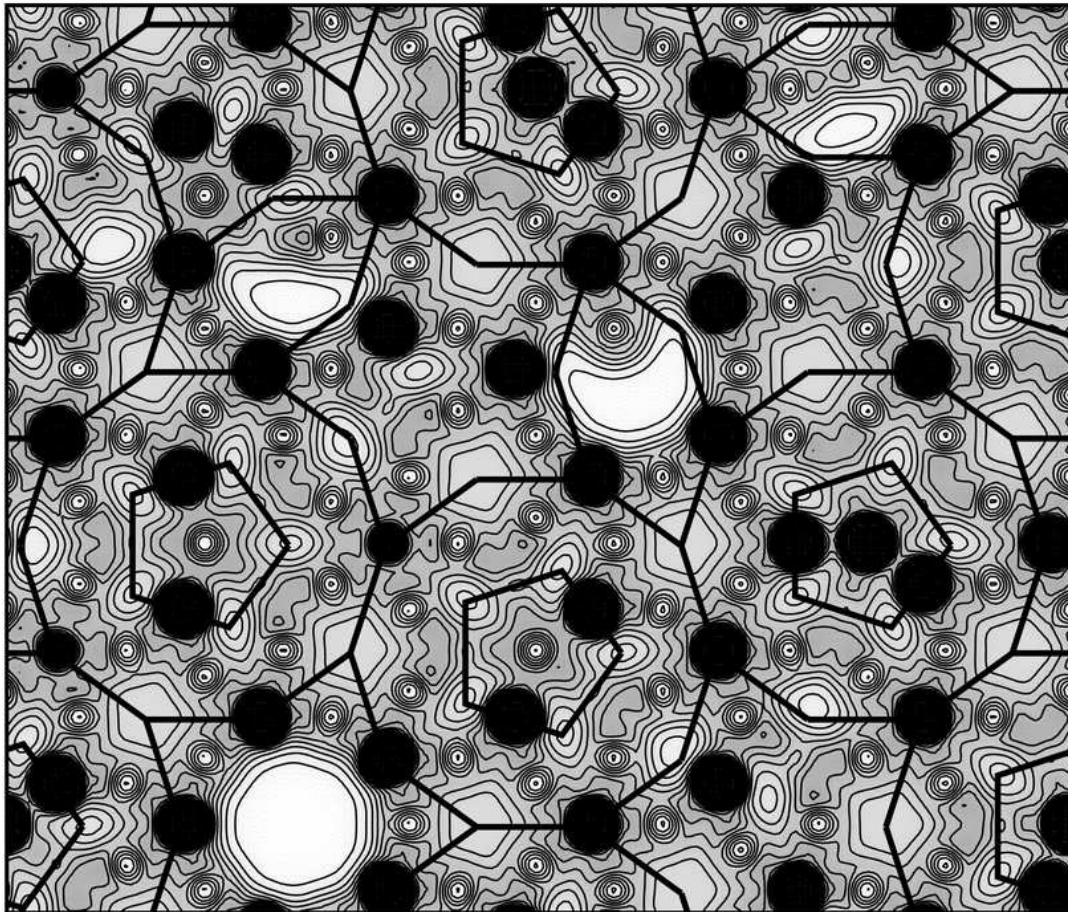
## Na monolayer on 5-fold i-AlPdMn surface



## Na monolayer on 5-fold i-AlPdMn surface



# Distribution of adsorption sites

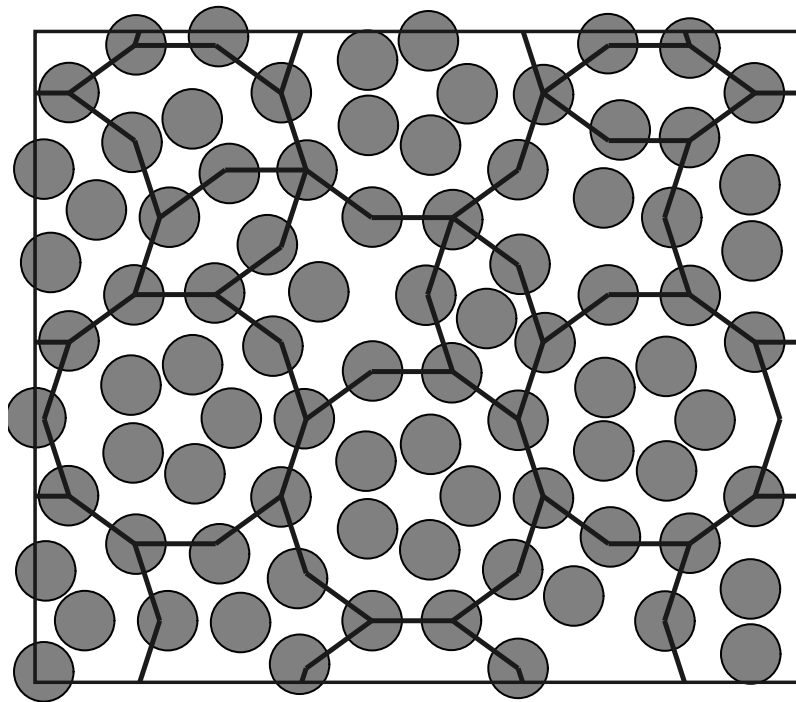


(a)

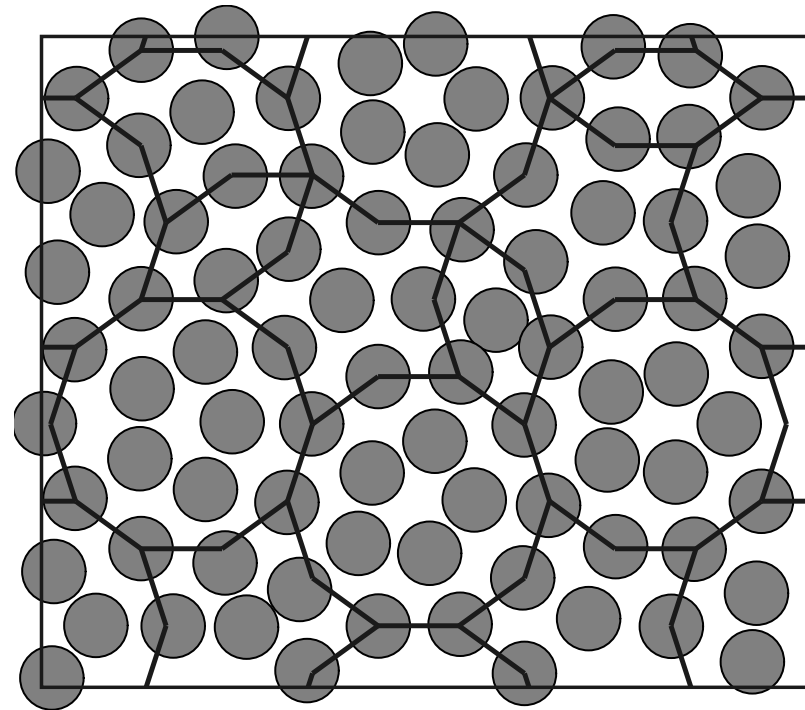
- Sites inside surface vacancies - do not belong to a monolayer
- Hollow sites between Al atoms → **DHBS tiling**
- Charge density minima → **small pentagon** inside the D tile

Quasiperiodic ordering in an overlayer enforced by the substrate

## Na vs. K monolayer on 5-fold i-AlPdMn surface



Na



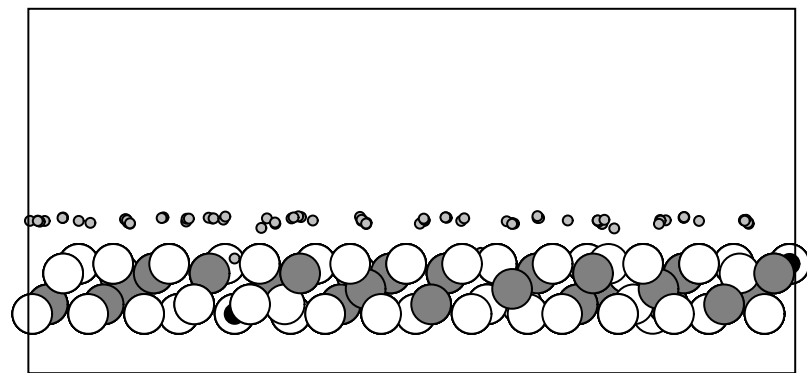
K

- size of atom  $d_{\text{Na-Na}} \approx 3.7 \text{ \AA}$  vs.  $d_{\text{K-K}} \approx 4.6 \text{ \AA}$
- atomic density of i-AlPdMn 5-fold surface  $0.132 \text{ atoms/\AA}^2$
- Na, K coverage  $0.066 \text{ atoms/\AA}^2$   $\Theta \approx 0.50$

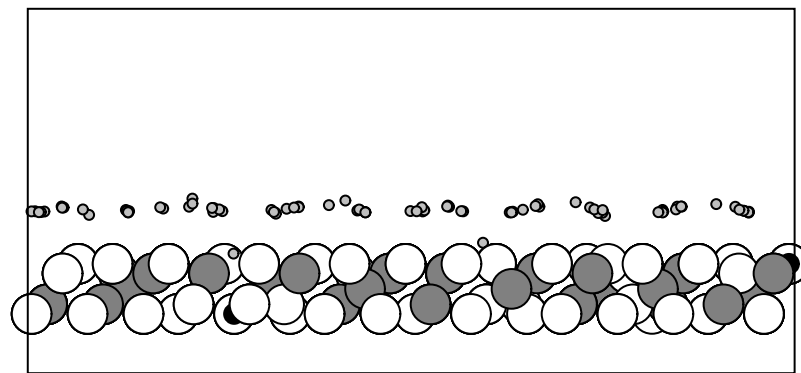


# Na vs. K monolayer on 5-fold i-AlPdMn surface

side view



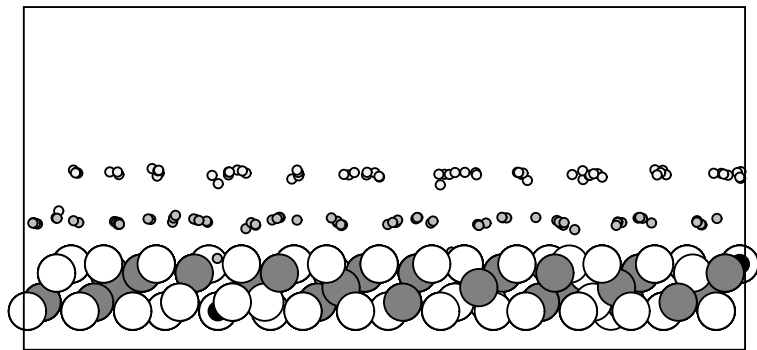
Na



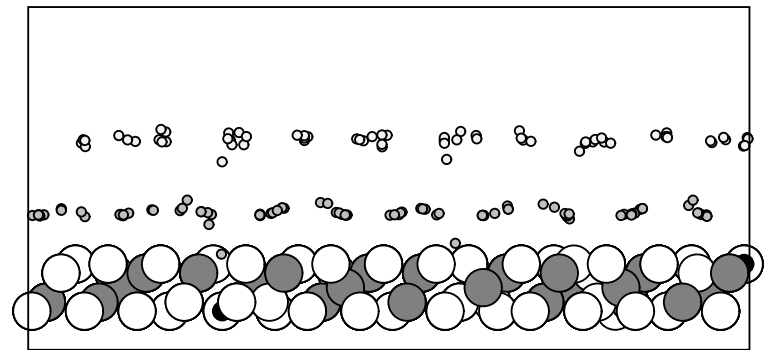
K

# Na and K bilayer on 5-fold i- $\text{AlPdMn}$ surface

side view

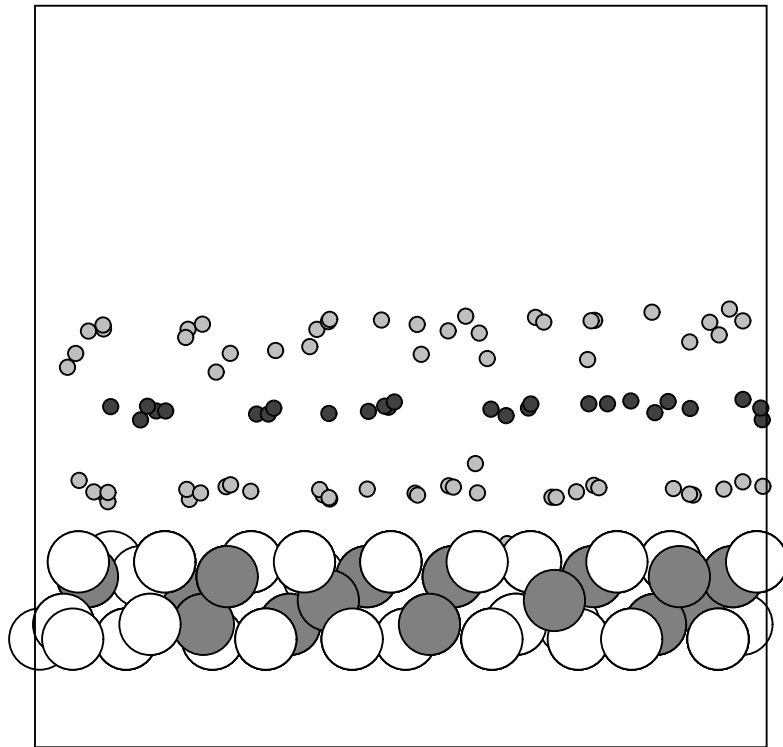


Na

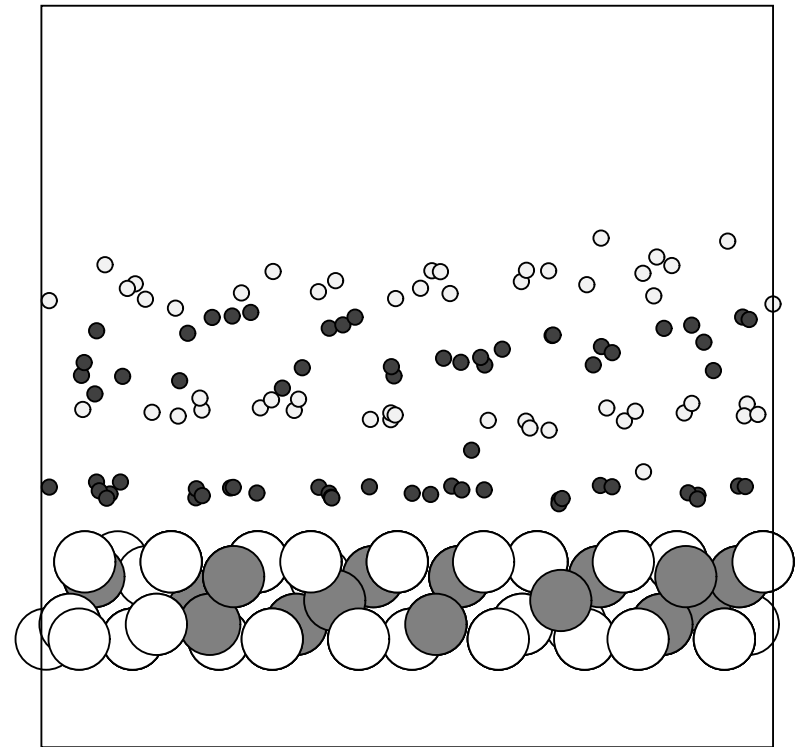


K

## Na multilayers on 5-fold i-AIPdMn surface



Na - 3 layers

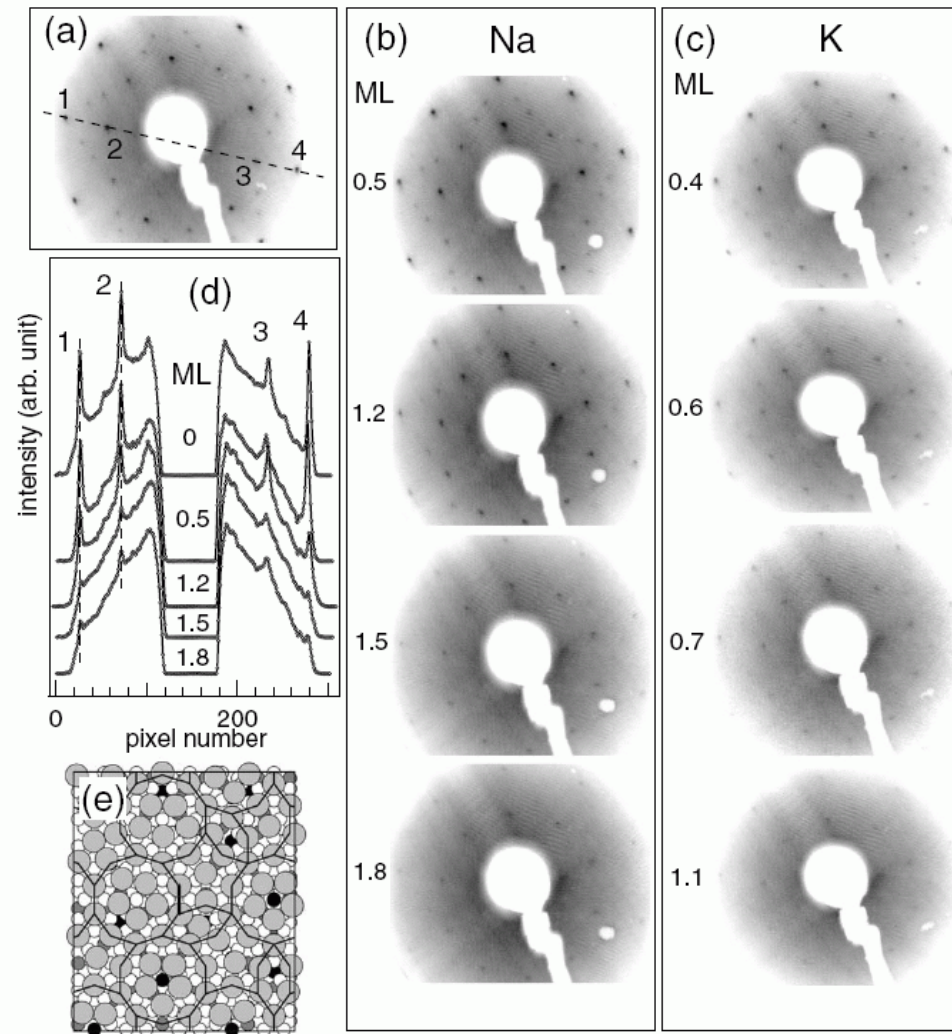


Na - 4 layers

quasiperiodic ordering does not propagate beyond a bilayer

# Na and K multilayers on 5-fold i-AlPdMn surface

SHUKLA *et al.*



A. K. Shukla *et al.*: PRB **79**, 134206 (2009)

# Short summary

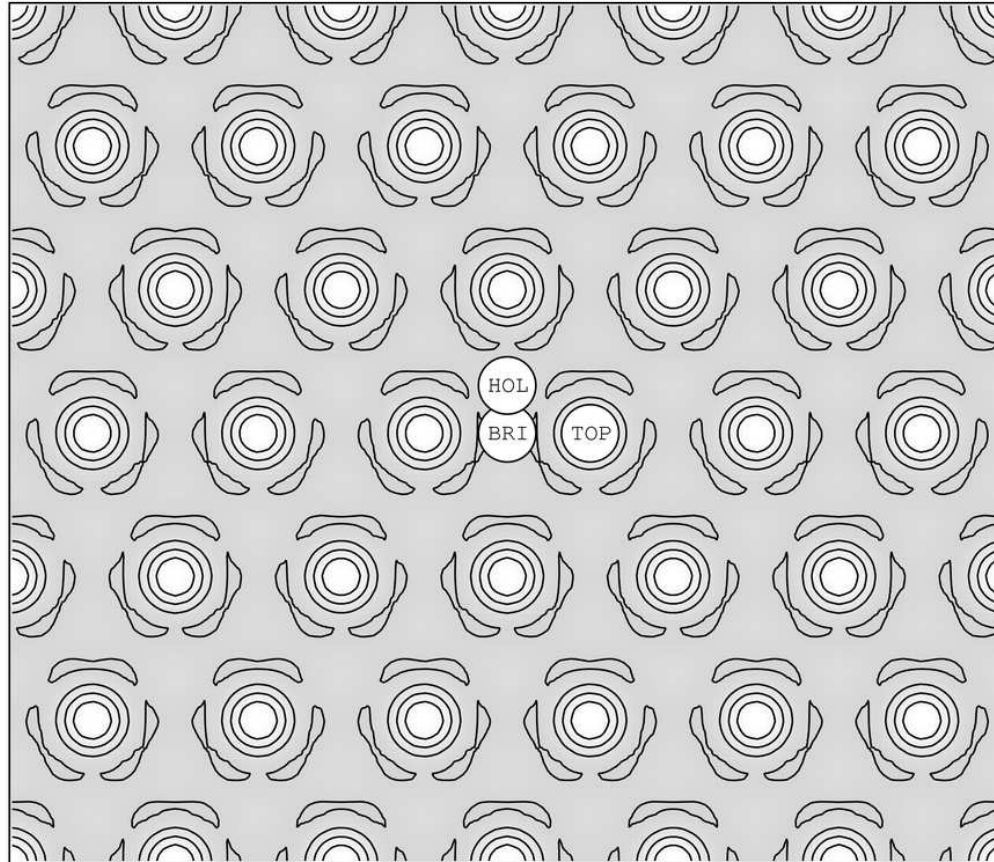
- Quasiperiodic ordering in an overlayer enforced by the substrate
- Unsupported quasiperiodic monolayers - not stable!
- Which atoms form quasiperiodic monolayers?
  - size of adatoms should be comparable with distances between adsorption sites
  - for i-AlPdMn surface optimal size of adatoms 3-4 Å.

# Catalytic properties of surface quasicrystalline approximant $\text{Al}_{13}\text{Co}_4$

# Catalytic properties of quasicrystals

- Surfaces of ordinary metals - few inequivalent adsorption sites
- Surfaces of quasicrystals - many inequivalent adsorption sites
- $\text{Al}_{13}\text{Co}_4$  - approximant to decagonal AlCoNi quasicrystal
- Experiment:  $\text{Al}_{13}\text{Co}_4$  surface - good catalyst for acetylene hydrogenation

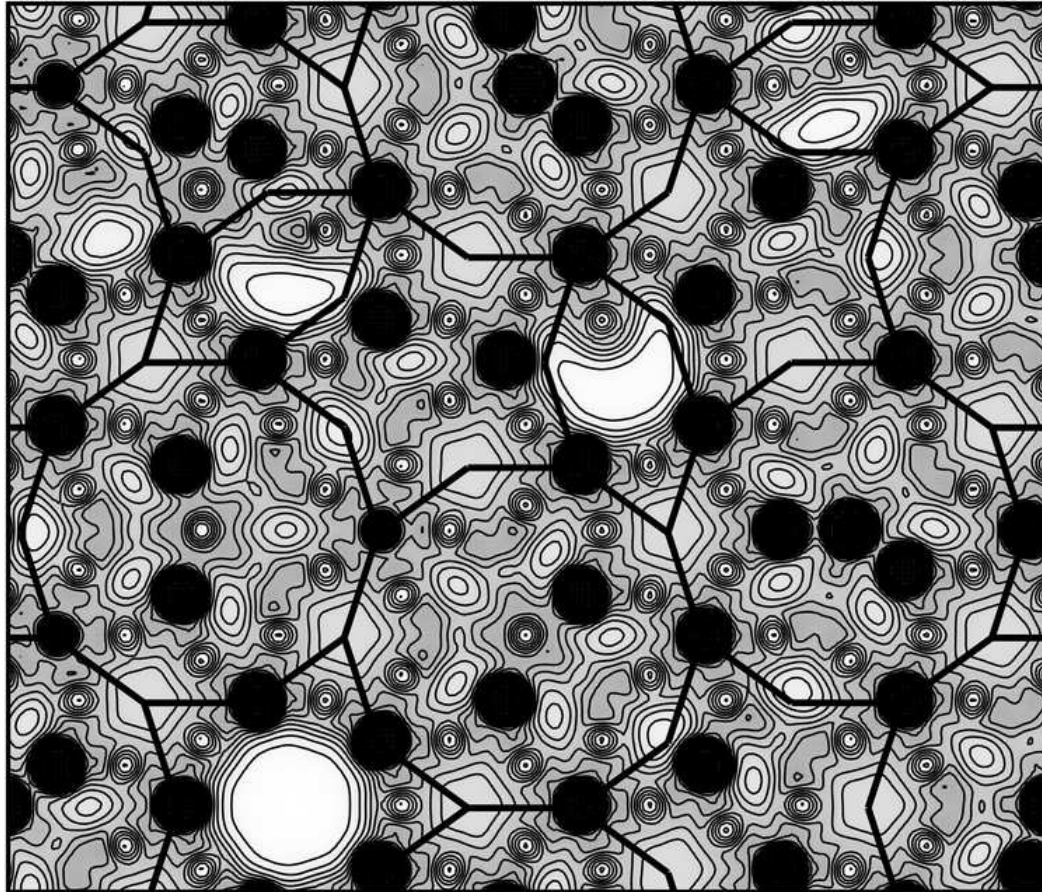
## Al(111) surface



Surface of simple metal

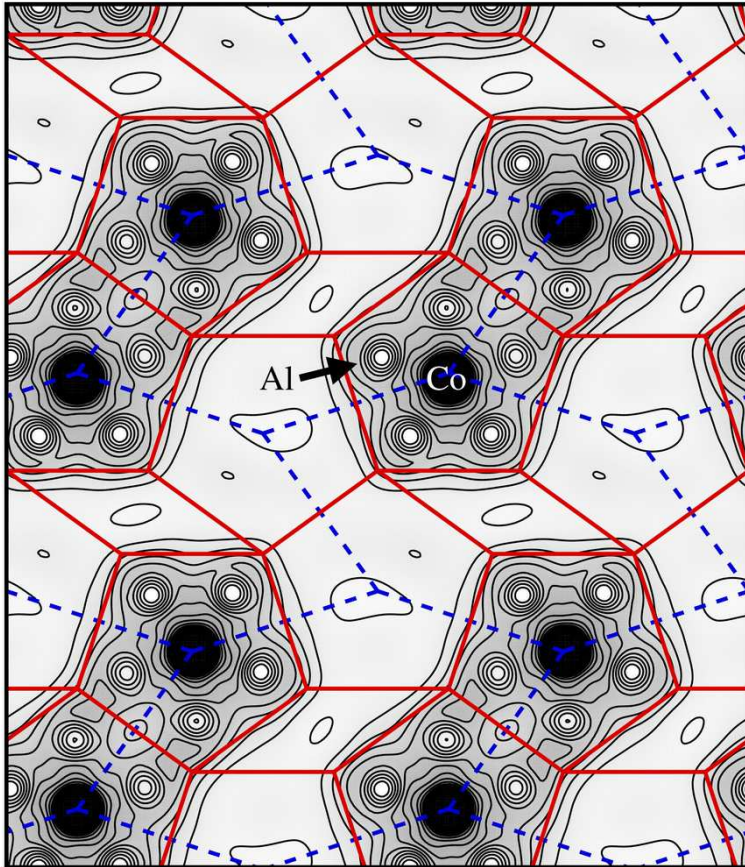


## 5-fold i-Al-Pd-Mn surface

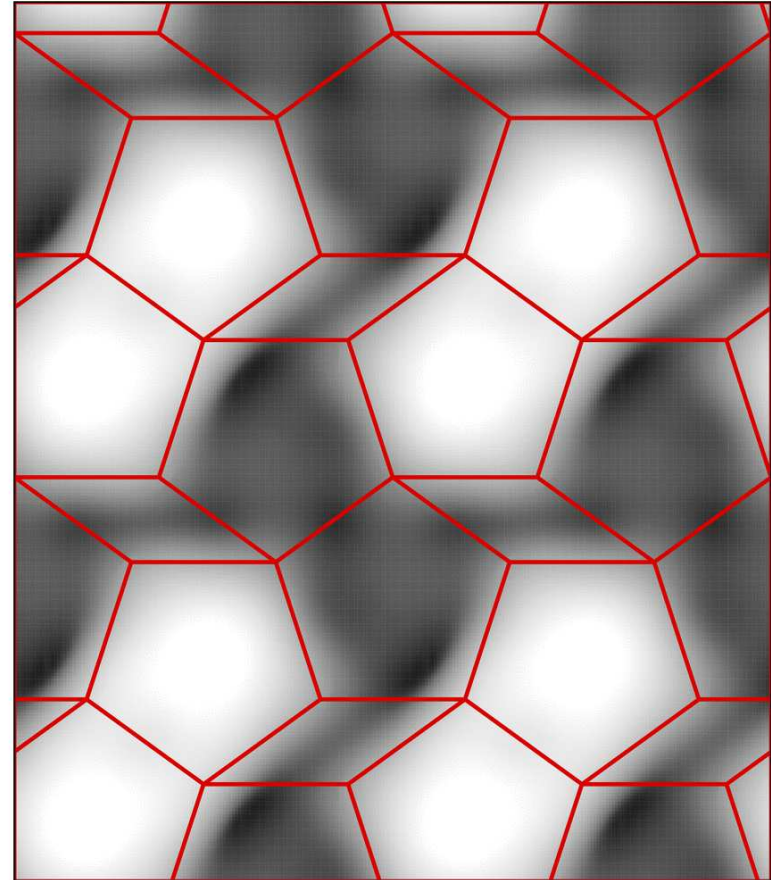


Complex metallic surface

# Surface of o-Al<sub>13</sub>Co<sub>4</sub>

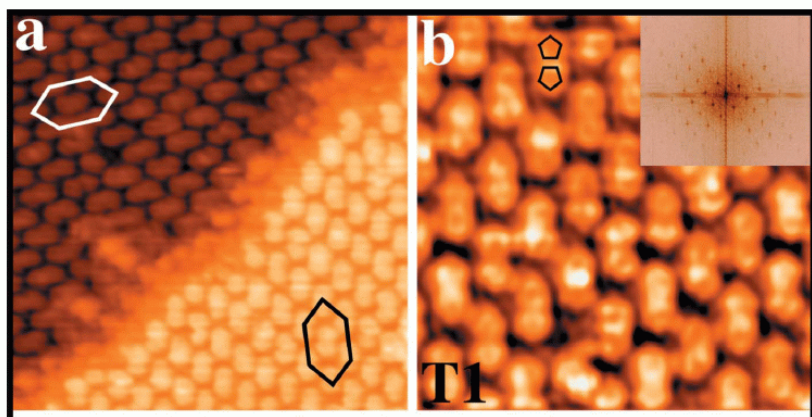


charge density



calculated STM

# Surface of o-Al<sub>13</sub>Co<sub>4</sub>

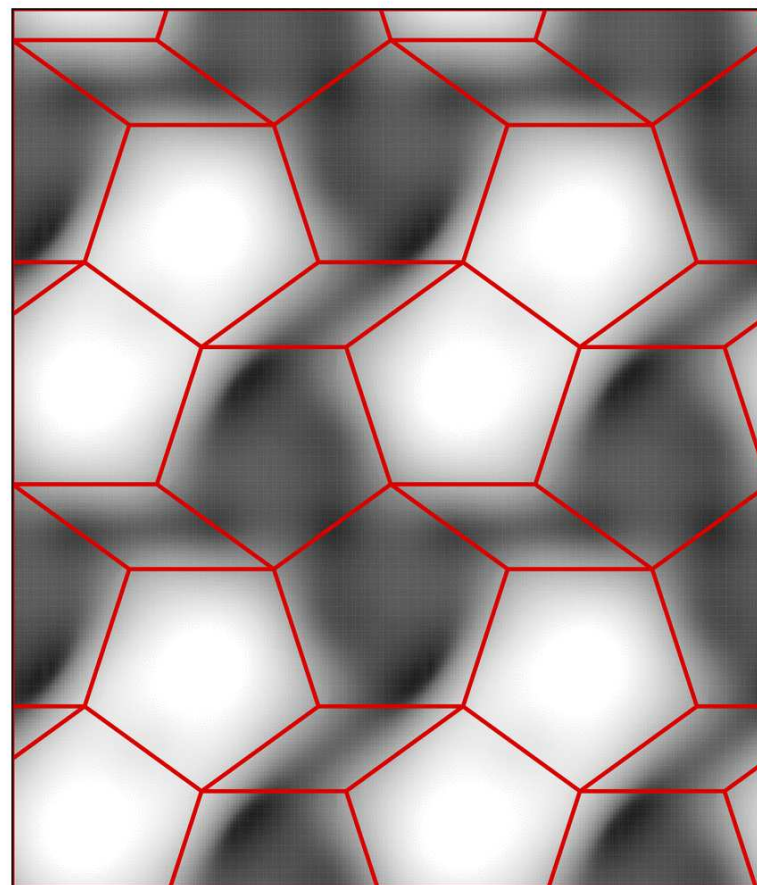


experimental STM

R. Addou et al. : PRB **80**, 014203 (2009)

Interesting catalytic properties

M. Armbrüster et al. (2009)

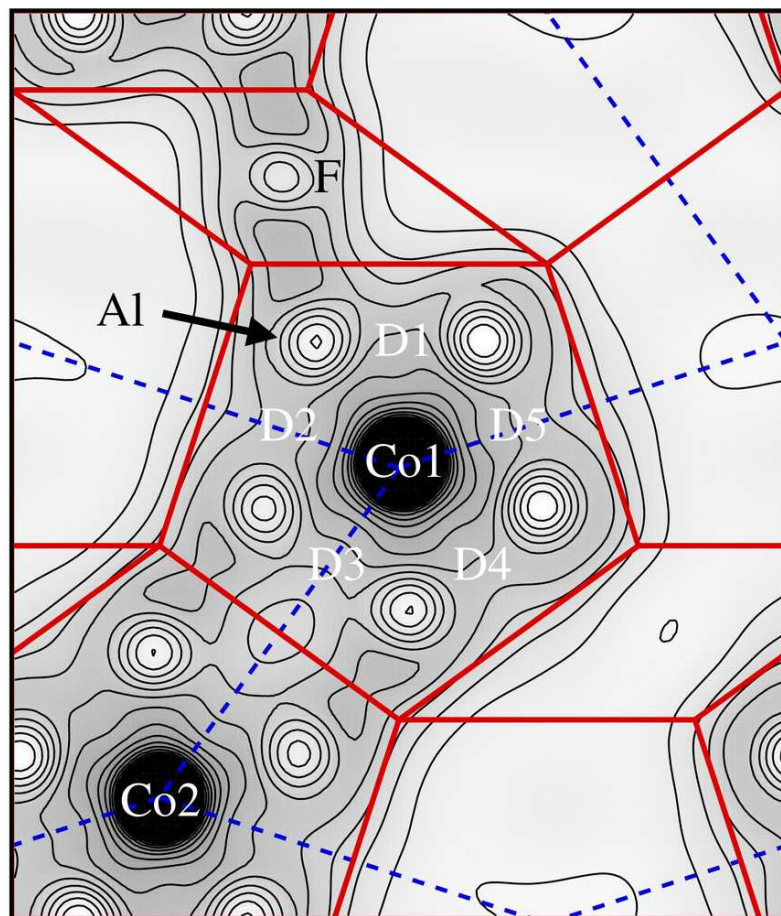


calculated STM

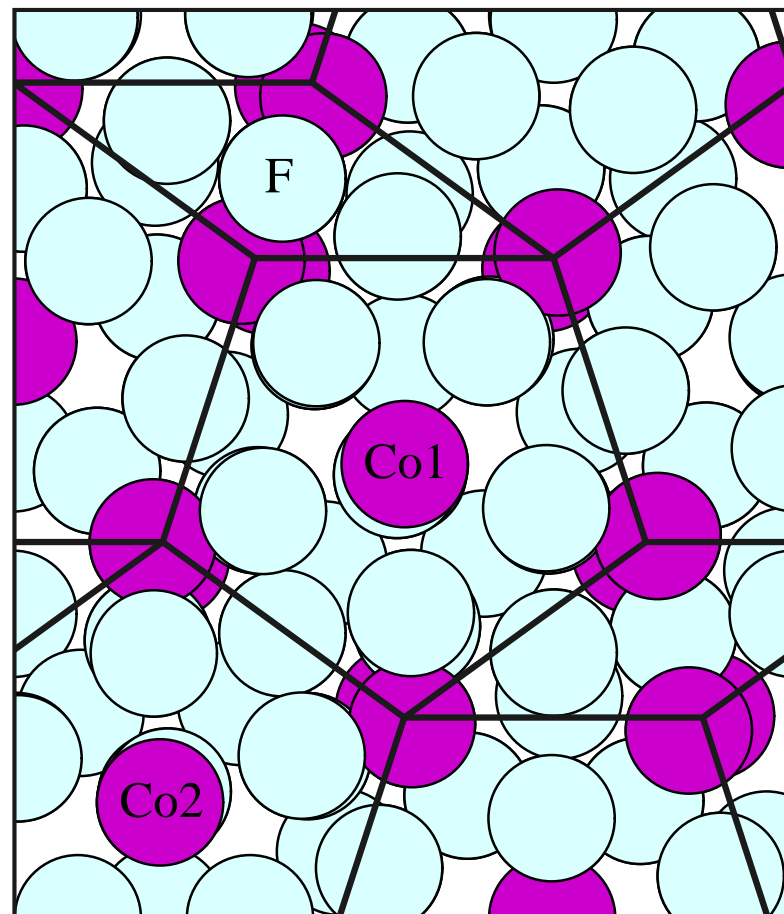
# Acetylene hydrogenation

- Acetylene -  $C_2H_2$
- Ethylene (ethen) -  $C_2H_4$
- Hydrogenation  $C_2H_2 + H_2 \rightarrow C_2H_4$
- Important sub-process at industrial production of polyethylene
- Ethan -  $C_2H_6$
- Hydrogenation  $C_2H_4 + H_2 \rightarrow C_2H_6$  - undesired
- Pd catalyst - expensive ( $\approx 10\,000\$/kg$ )
- Effort to replace Pd by non-precious metals
- Role of the catalyst - reduce activation energy barrier
- Good catalyst - Activity vs. Selectivity

## Surface of o-Al<sub>13</sub>Co<sub>4</sub>



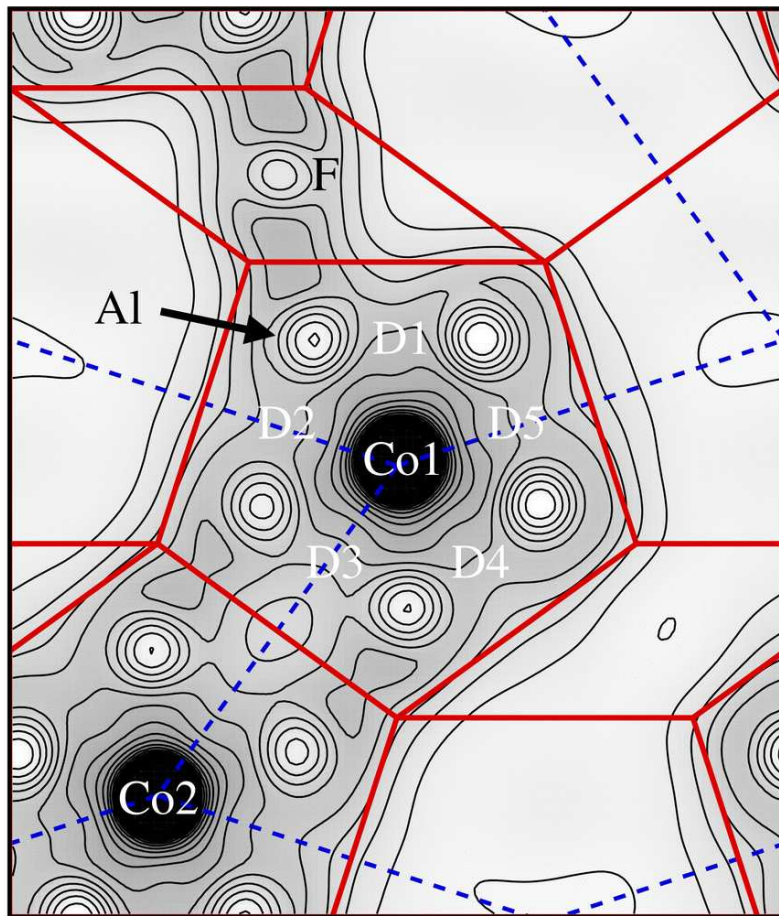
charge density



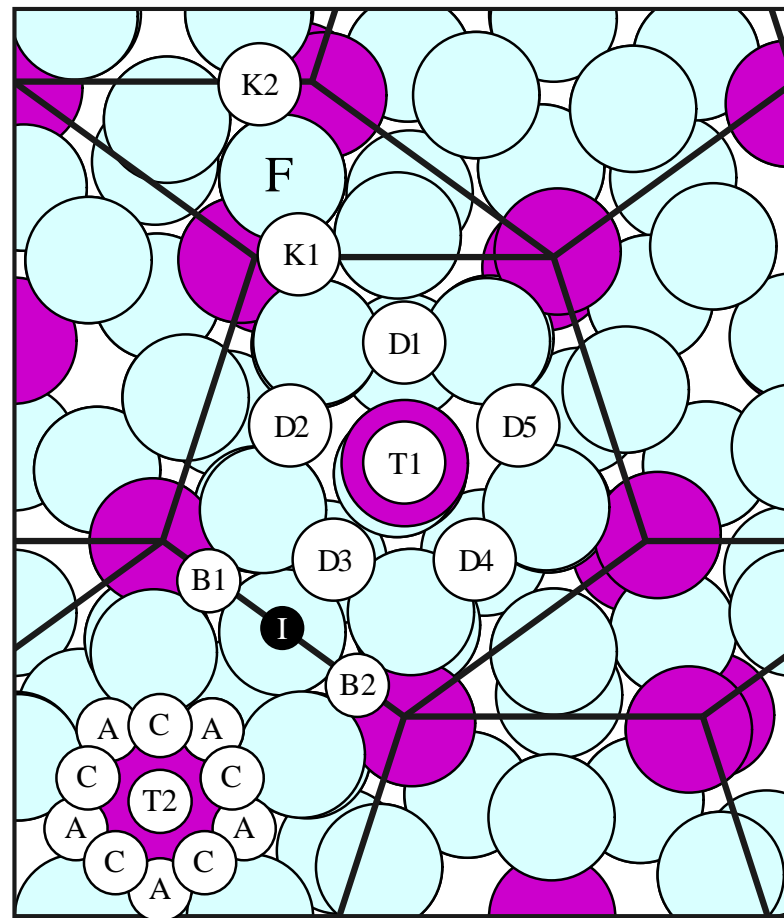
structural model

Preferable adsorption sites for H<sub>2</sub> and C<sub>2</sub>H<sub>2</sub> molecules?

## Surface of o-Al<sub>13</sub>Co<sub>4</sub>



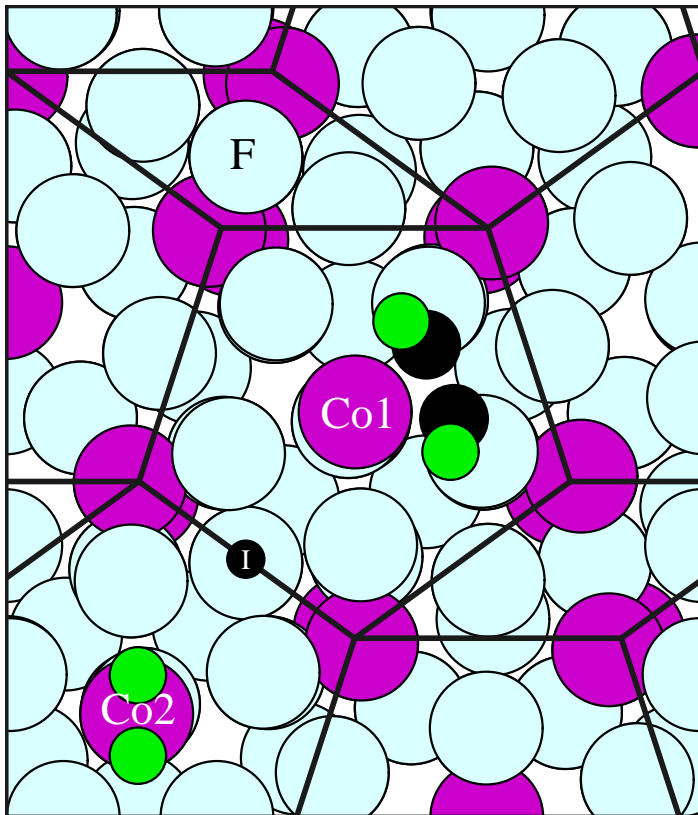
charge density



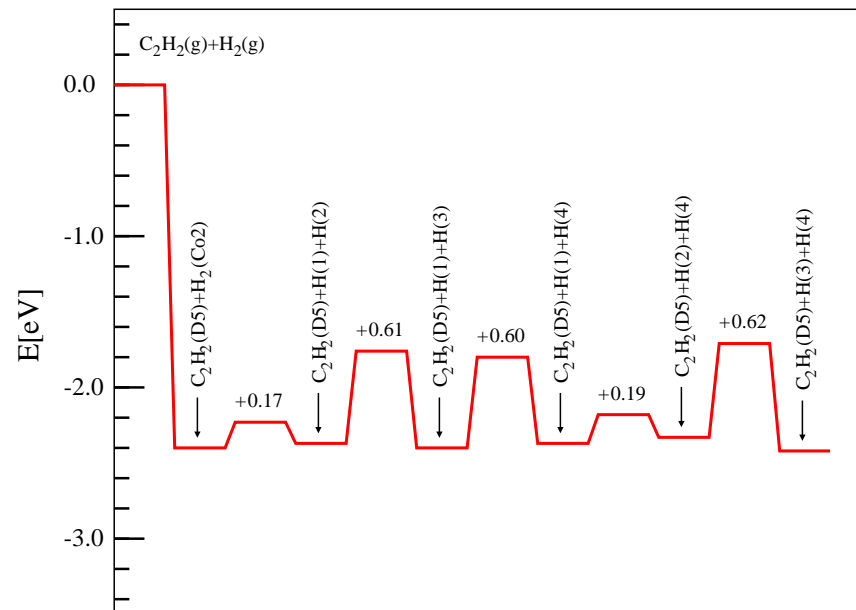
structural model

Preferable adsorption sites for H<sub>2</sub> and C<sub>2</sub>H<sub>2</sub> molecules?

# Acetylene hydrogenation on the surface of o-Al<sub>13</sub>Co<sub>4</sub>



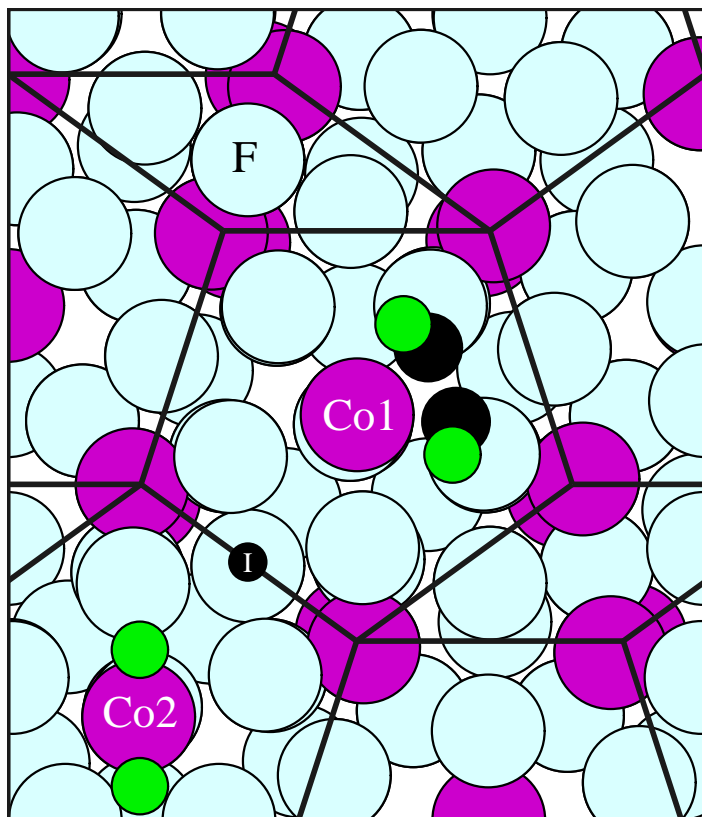
structural model



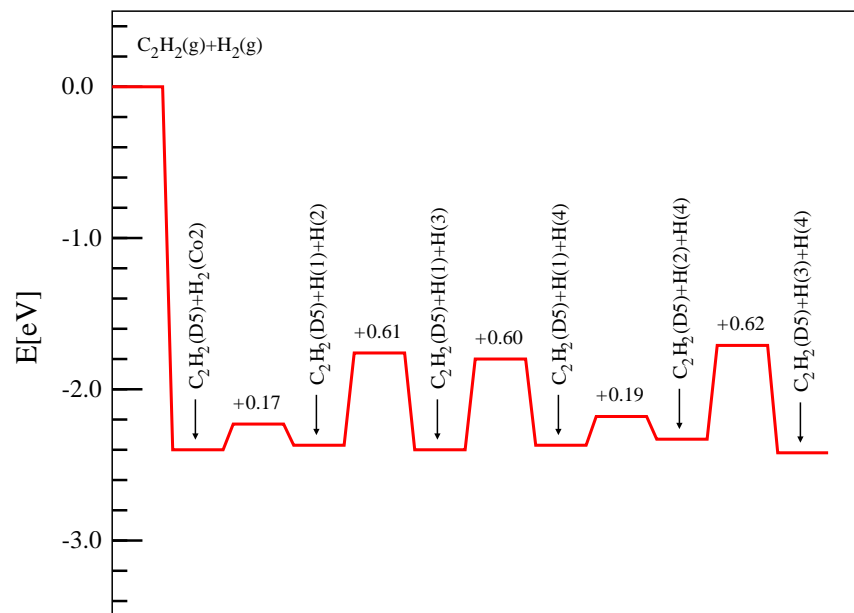
energy profile

C<sub>2</sub>H<sub>2</sub> bonding - di- $\sigma$  at D5 vs.  $\pi$ -bond at Co1  
H<sub>2</sub> at Co2 - easily dissociates

# Acetylene hydrogenation on the surface of o-Al<sub>13</sub>Co<sub>4</sub>



structural model



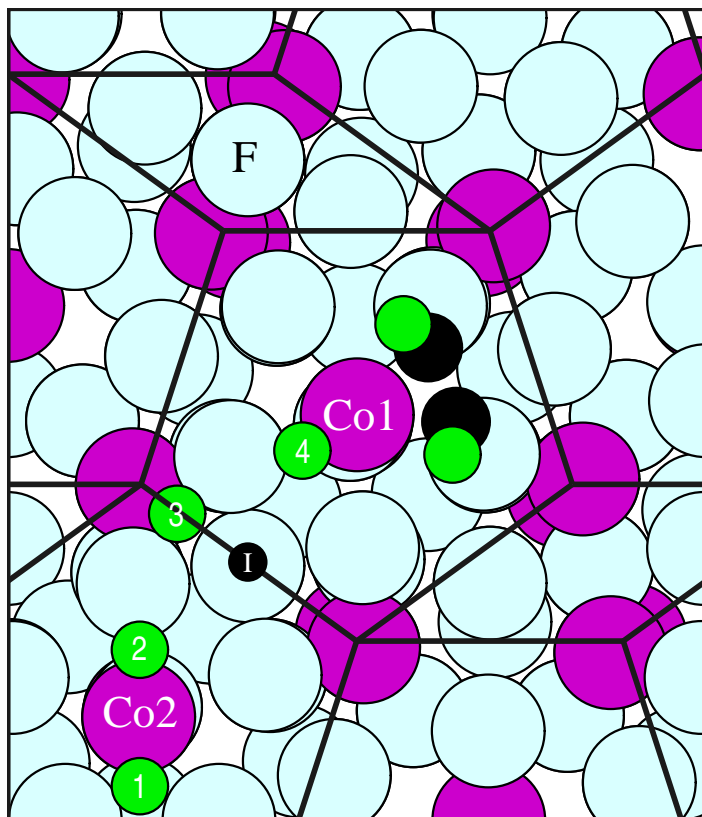
Reaction path

energy profile

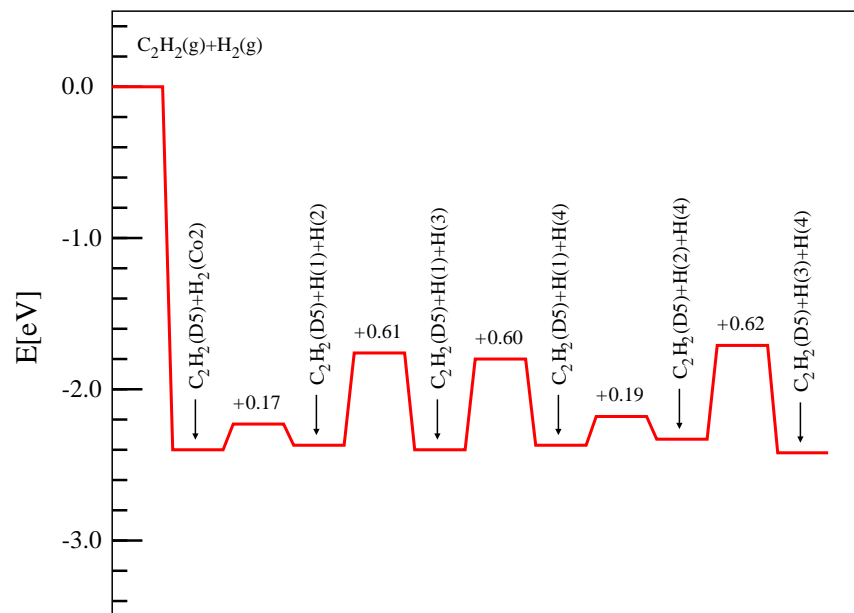
## H<sub>2</sub> dissociation



# Acetylene hydrogenation on the surface of o-Al<sub>13</sub>Co<sub>4</sub>



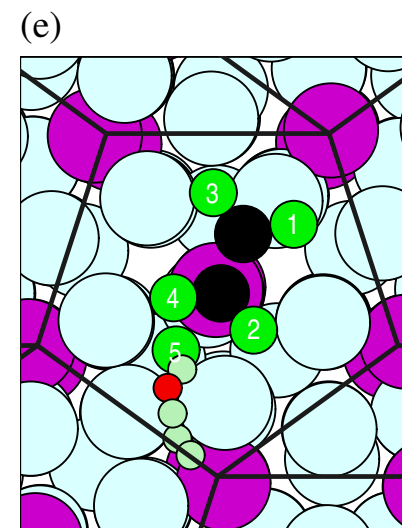
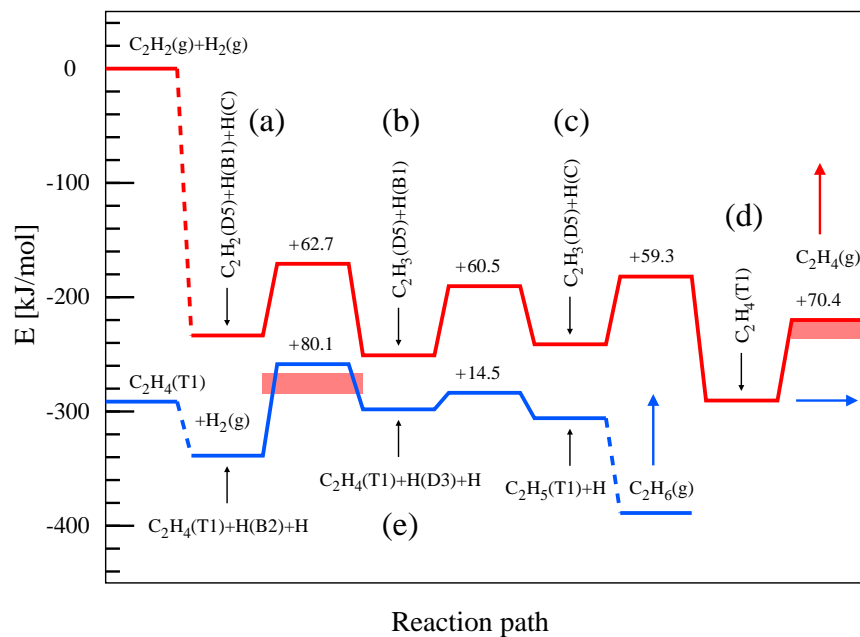
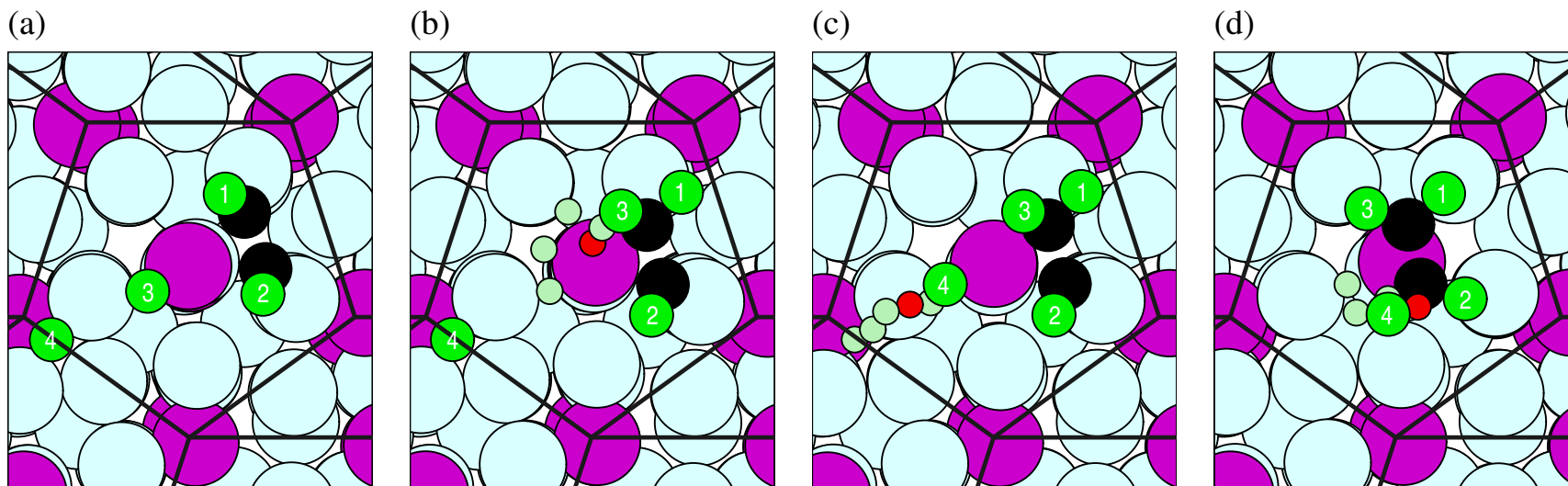
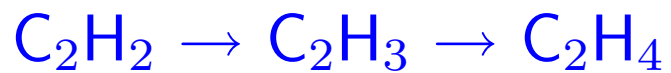
structural model



energy profile

## Diffusion jumps of H atoms

Activation energy barriers  $E_a \leq 0.63 \text{ eV} = 60 \text{ kJ/mol}$   
 1 eV = 96.5 kJ/mol



# Comparison with Pd catalyst

Activation energy barriers

	$C_2H_2 \rightarrow C_2H_3$ [kJ/mol]	$C_2H_3 \rightarrow C_2H_4$ [kJ/mol]	desorp. [kJ/mol]	$C_2H_4 \rightarrow C_2H_6$ [kJ/mol]
Pd *	66	74	82	72
$Al_{13}Co_4$	63	61	70	80

\* D. Mei et al.: J. Catal. **268**, 181 (2009)

Surface of  $Al_{13}Co_4$  - better catalyst than Pd  
Higher activity - better selectivity

# Short summary

- Surfaces of quasicrystals - various adsorption sites for molecules
- Surface of  $\text{Al}_{13}\text{Co}_4$  - peculiar surface - complex morphology
- Good catalyst: Activity and selectivity
- Acetylene hydrogenation:  $\text{C}_2\text{H}_2 \rightarrow \text{C}_2\text{H}_4 \rightarrow \text{C}_2\text{H}_6$
- Active site -  $\text{Al}_5\text{Co}$  complex
- Surface of  $\text{Al}_{13}\text{Co}_4$  - good catalyst - better than Pd

# Methods

- TB-LMTO (Linear Muffin-Tin Orbital Method) - DFT
- VASP (Vienna Ab-initio Simulation Package) - DFT
  - Equilibrium structure - relaxation by interatomic forces
  - Structural stability - clean surface and adsorbants
  - Charge density distribution - chemical bonding
  - Simulated STM images
  - Simulated cleavage - formation of the surface
  - Adsorption energies
  - Activation energy barriers

Thank you for your attention!