the Max Born supremacy:
relative locality and the quantum-gravity road less traveled

BornXXIX
29·7·2011

Giovanni Amelino-Camelia
University of Rome "La Sapienza"

3 questions

I: how much do we know about the geometry of momentum space?

II: Which laws apply to the limit of quantum gravity obtained by

$$G_N \to 0 \qquad h \to 0 \qquad \text{with } \frac{h}{G_N} \text{ kept fixed}$$

III: Look around; do you "see" spacetime?

the Max Born supremacy:
relative locality and
the quantum-gravity road less traveled

BornXXIX
29·7·2011

Giovanni Amelino-Camelia
University of Rome "La Sapienza"

3 questions

I: how much do we know about the geometry of momentum space?

II: Which laws apply to the limit of quantum gravity obtained by

$$G_N \rightarrow 0 \quad h \rightarrow 0 \qquad \text{with} \quad \frac{h}{G_N} \quad \underline{\text{kept fixed}}$$

III: Look around; do you "see" spacetime?

apparently linked

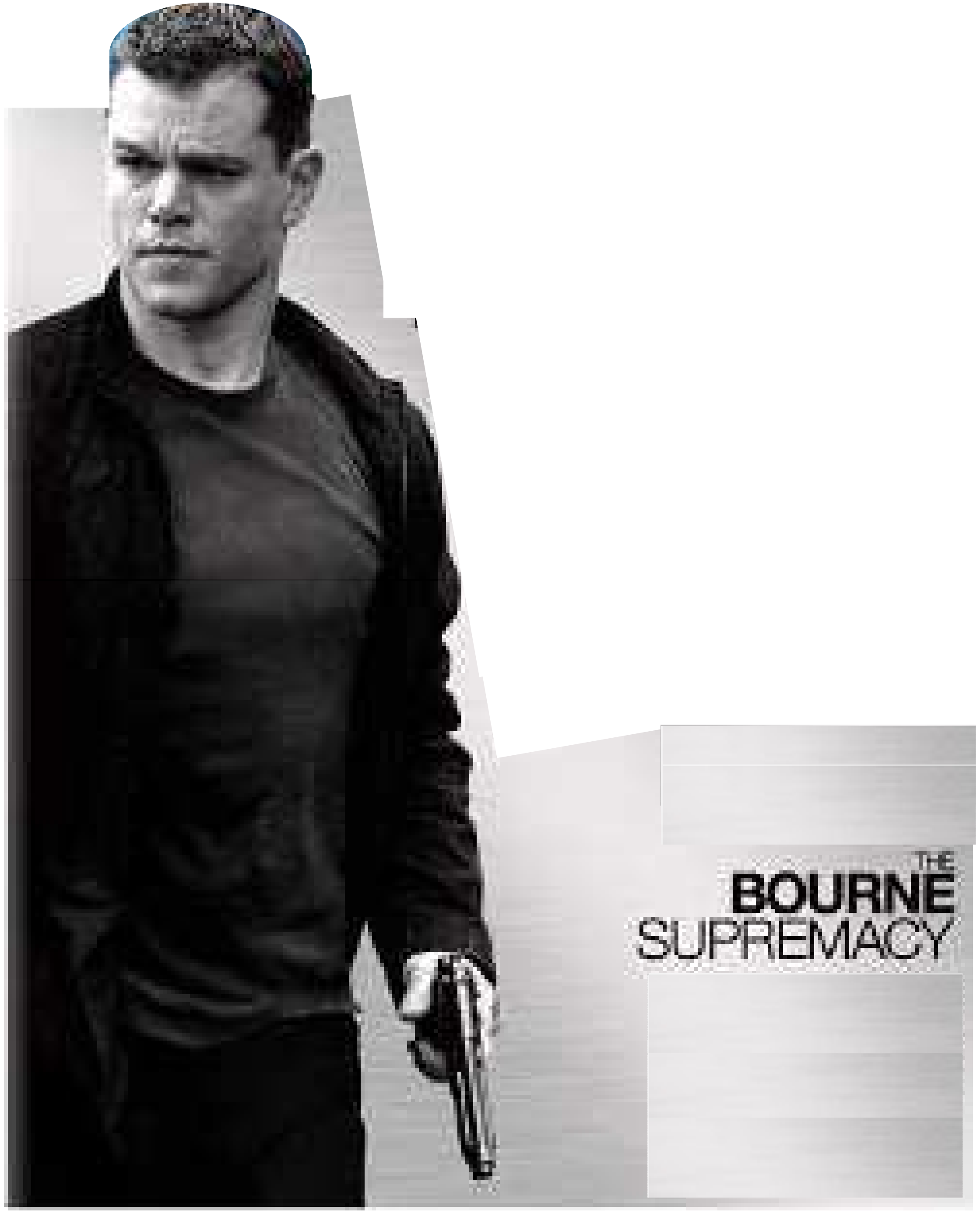
<u>Principle of relative locality</u>
arXiv:1101.0931; arXiv:1104.2019;arXiv:1106.0313;GRG(in press)
with Laurent Freidel, Jerzy Kowalski-Glikman, Lee Smolin

<u>relative locality in kappa-Poincare phase spaces</u>
arXiv:1006.2126; PhysRevLett106, 071301 & arXiv:1102.4637, PhysLettB 700(2011)150
with Niccolo' Loret, Marco Matassa, Flavio Mercati, Giacomo Rosati

<u>relative locality for interacting kappa-Poincare particles</u>
arXiv:1006.????
with Michele Arzano, Jerzy Kowalski-Glikman, Giacomo Rosati, Gabriele Trevisan

BornXXIX
29·7·2011

Giovanni Amelino-Camelia
University of Rome "La Sapienza"



la quiete relativa

in Galilean relativity rest is relative ("quiete relativa"),
so property of being events that occur at the same place at different times is not objective (it is observer dependent)

but absolute time: property of being events that occur at the same time in different (spatial) positions is objective

and every Galileian observer has a clean separation between time and space.
But how do two Galileian observers know that they share the same time and experience the same separation between time and space?
The structureless law of composition of velocities

$$u \oplus v = u + v$$

so do you “see” space?

NO! you “see” spacetime!(you see your past lightcone)

The properties of Lorentz boosts are such that

“space by itself, and time by itself fade away into mere shadows, and only a kind of union of the two [spacetime] preserves an independent reality” (Minkowski 1908)

And what is responsible for this “union” of space and time?
The nonlinearities of the law of composition of velocities

$$\mathrm{v} \oplus \mathrm{u} = \frac{1}{1 + \dfrac{\mathrm{v} \bullet \mathrm{u}}{\mathrm{c}^2}} \left[\mathrm{v} + \frac{1}{\gamma_{\mathrm{v}}} u + \frac{1}{c^2} \frac{\gamma_{\mathrm{v}}}{1 + \gamma_{\mathrm{v}}} (\mathrm{v} \bullet \mathrm{u}) \mathrm{v} \right]$$

the principle of relative locality

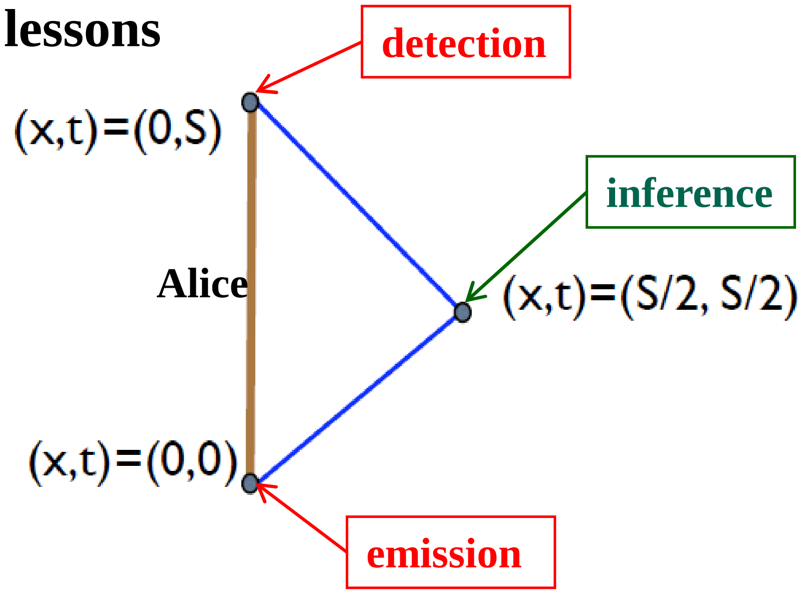
GAC+Freidel+Kowalski-Glikman+Smolin

so do you “see” spacetime?

NO! you “see” (detect) time sequences of particles and then abstract a spacetime by inference!

you are more aware of this when you try to set up a macroscopic spacetime/reference frame (think in particular of the abstraction of a spacetime used to organize logically our inferences for what concerns the observations of distant astros)

This was after all one of Einstein’s key lessons

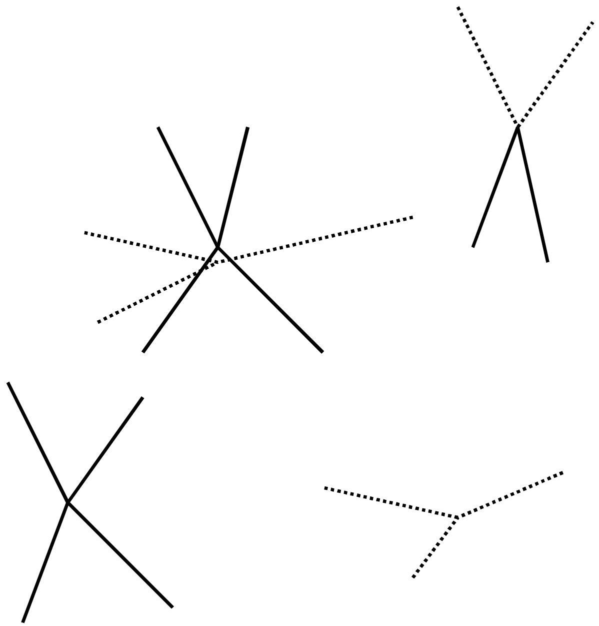


the principle of relative locality

GAC+Freidel+Kowalski-Glikman+Smolin

But do macroscopically-distant observers infer/abstract "the same" spacetime?

What does it even mean to infer "the same" spacetime?
absolute locality: coincidences of events for one Einsteinian observer are also coincidences of events for all other Einstenian observers



the principle of relative locality

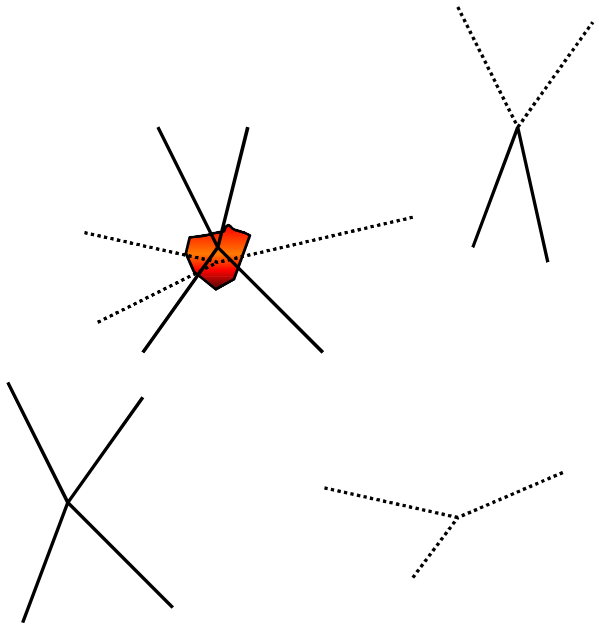
GAC+Freidel+Kowalski-Glikman+Smolin

But do macroscopically-distant observers infer/abstract "the same" spacetime?

What does it even mean to infer "the same" spacetime?
absolute locality: coincidences of events for one Einsteinian observer are also coincidences of events for all other Einstenian observers

And what is it that allows absolute locality?
The structureless (linear) law of composition of momenta

$$p_1 \oplus p_2 \oplus p_3 = p_1 + p_2 + p_3$$



the principle of relative locality

GAC+Freidel+Kowalski-Glikman+Smolin

link from linear conservation of momentum to locality is most familiar nowadays in the context of field theories

$$\int \mathrm{dk}_{\mathrm{j}} \tilde{\Phi}_1(k_1)\tilde{\Phi}_2(k_2)\tilde{\Phi}_3(k_3)\delta(k_1+k_2+k_3) =$$

$$= \int \mathrm{dk}_{\mathrm{j}} d^4x \tilde{\Phi}_1(k_1)\tilde{\Phi}_2(k_2)\tilde{\Phi}_3(k_3) e^{i(k_1+k_2+k_3)x} =$$

$$= \int d^4x \Phi_1(x)\Phi_2(x)\Phi_3(x)$$

Also notice that the conservation law that generates transformations between <u>distant observers</u> are these linear laws of composition of momenta

$$\delta x_I^a = \{\delta x_I^a, b^c \mathcal{P}_c^{tot}\} = b^a \qquad \textbf{with} \qquad \mathcal{P}_c^{tot} = \sum_I p_c^I$$

and the fact that these act on coordinates assigned to the event by one observer in a way that is independent of the details of the event is again responsible for the objectivity of the inferred distant coincidences of events

the principle of relative locality

GAC+Freidel+Kowalski-Glikman+Smolin

evidently <u>distant absolute locality is an inference</u>

notice that this amounts to acquiring the perspective that <u>spacetime is not a "fundamental" entity</u>: whereas we usually tend to perceive (for no good reason) that momenta of particles are introduced on the basis of a spacetime picture, questioning the absoluteness of locality implies that one should <u>introduce first the particles with their momenta (and observers with clocks) and then derive a spacetime picture</u>

can we formalize theories that remove the abstraction of absolute locality while preserving the objectivity of coincidences of events assessed by observers local to the events?

surely we could but it is tough in general and will take time and effort

for now let us focus on the "Planck-scale regime"

$$
\begin{aligned}
G_{Newton} &\rightarrow 0 \\
\hbar &\rightarrow 0 \\
m_p &= \sqrt{\frac{\hbar}{G_{Newton}}} \rightarrow \text{constant}
\end{aligned}
$$

the "planck-scale limit" of quantum gravity

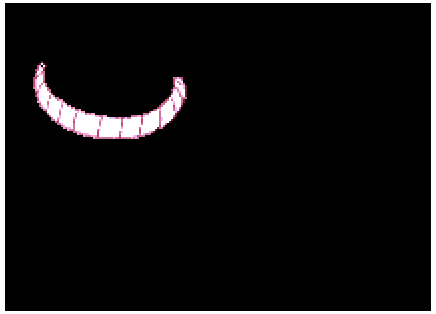
$$G_N \to 0 \qquad \hbar \to 0 \qquad \text{with} \quad \frac{\hbar}{G_N} \approx M_{planck}^2 \quad \text{kept fixed}$$

many QG papers in last 20 years were about Planck scale being a scale of deformation of momentum space....we all failed to notice the dramatic consequences this would have for this limit!!!

in this limit of quantum gravity roughly speaking
quantum mechanics and gravitation are switched off!!

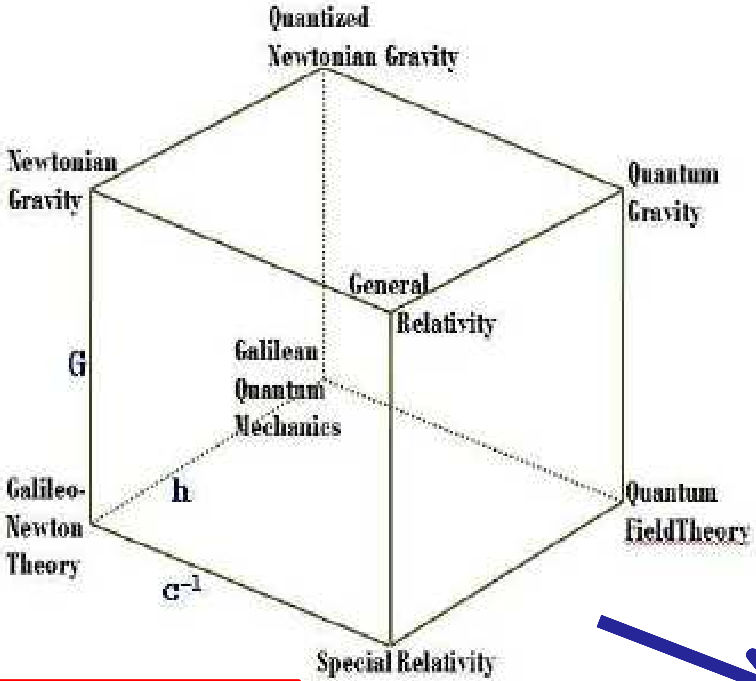
IF the limit is not completely trivial (as implicitly argued by supporters of nonlinearities in momentum space) THEN this limit still contains valuable information about quantum gravity

a sort of Cheshire-cat smile of quantum gravity
described by theories which one should manage to analyze with relatively little effort

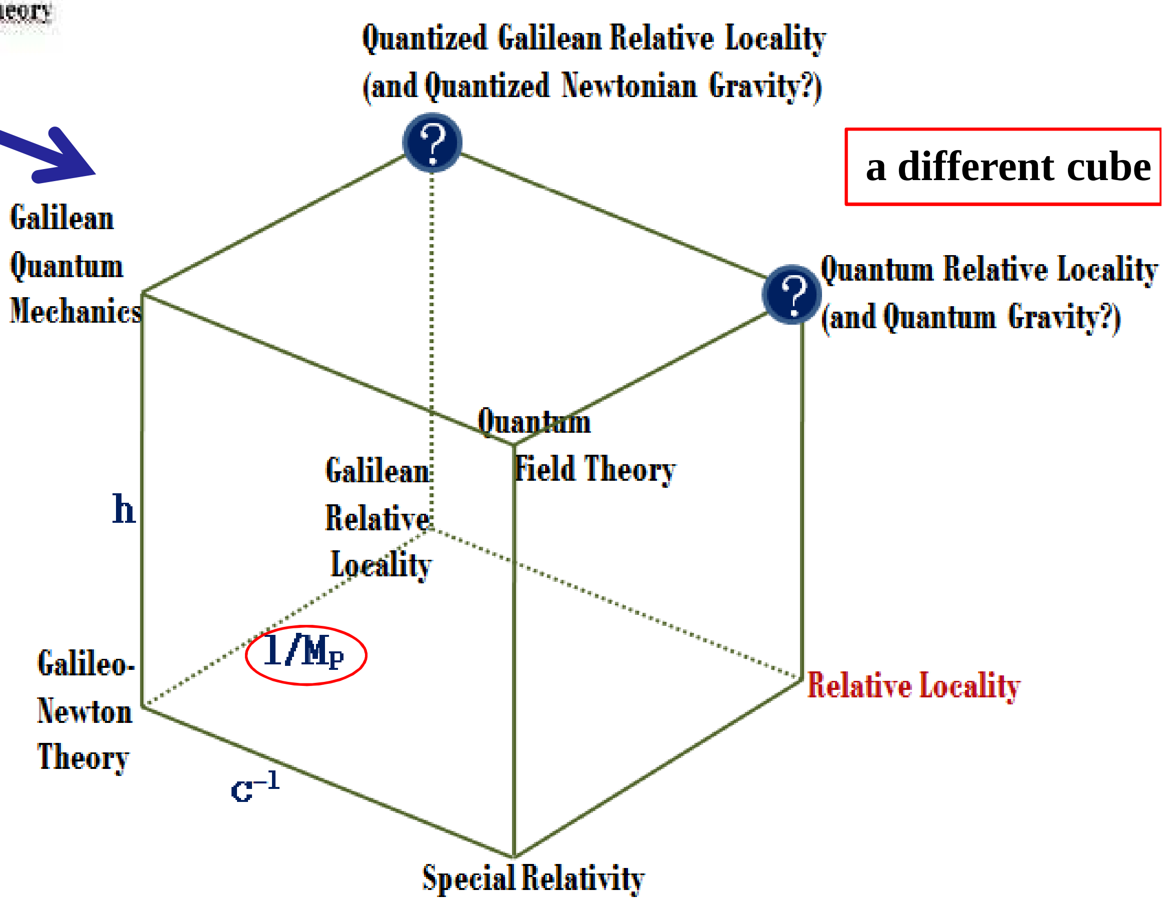


also a change of perspective on how to tackle the quantum-gravity problem

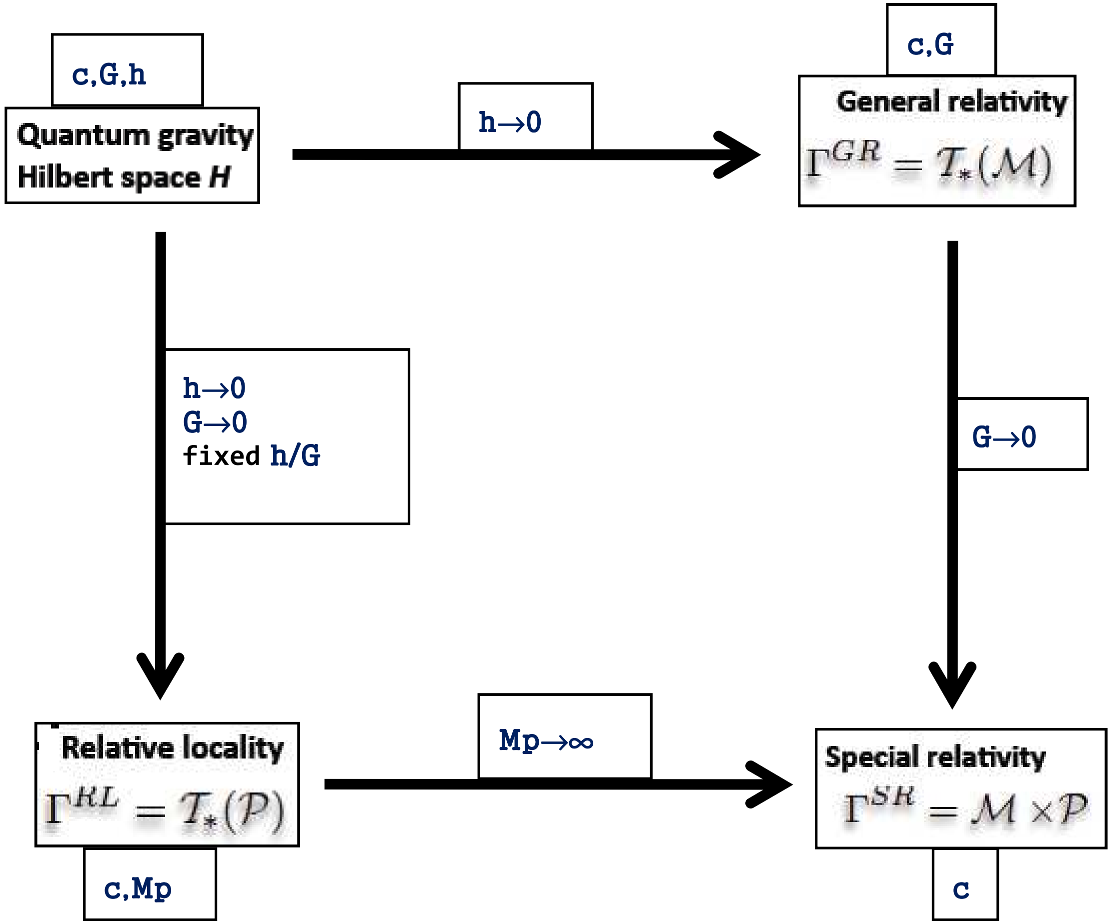
a change of perspective



the Bronstein cube



a different cube



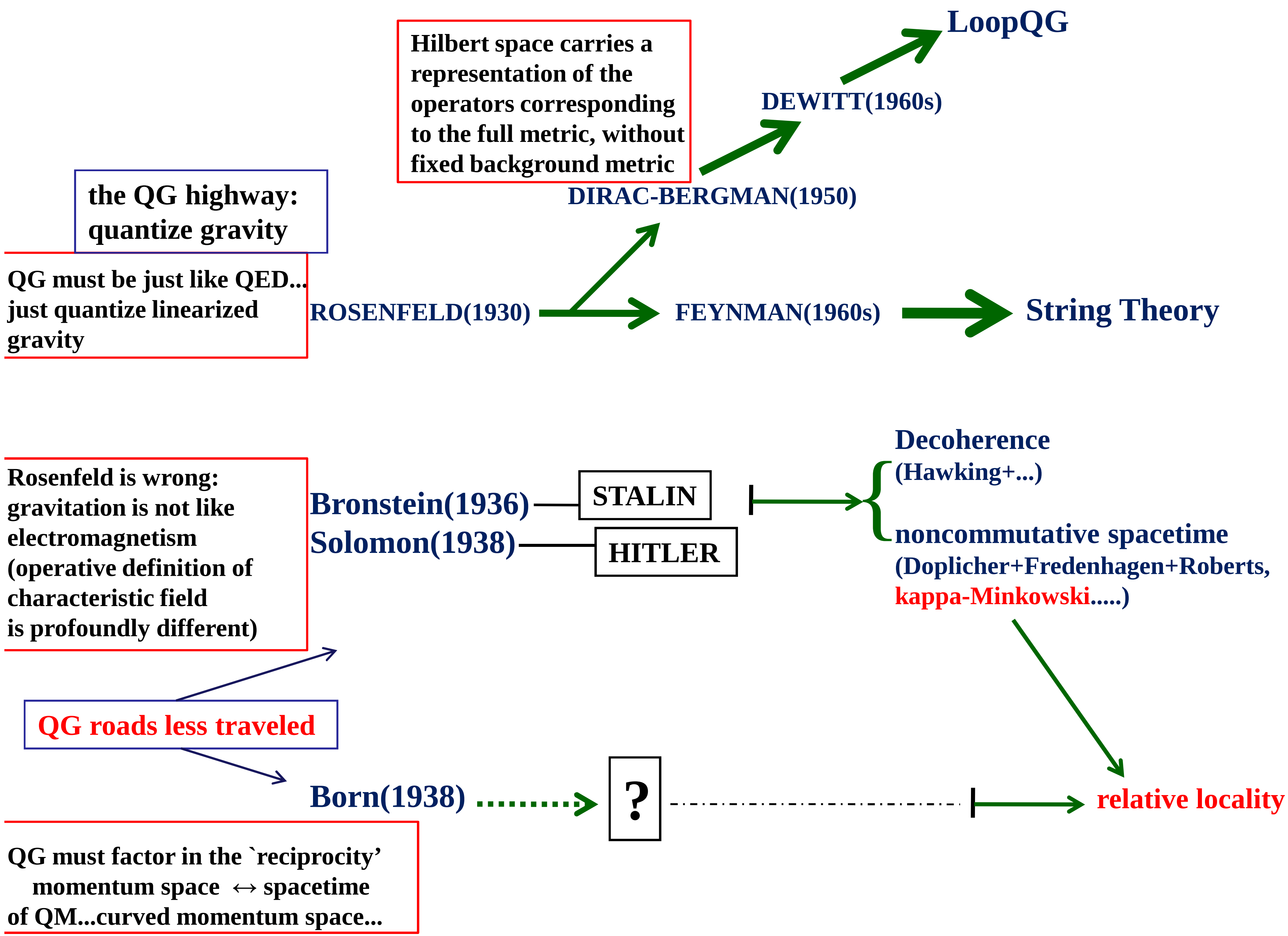
as in Born's intuition for Quantum Gravity (1938) but in dual regimes!!
[also see Majid (1996) GAC+Majid(2000) Kowalski-Glikman(2003)]

In special relativity, the phase space associated with each particle is a product of spacetime and momentum space, *i.e.* $\Gamma^{SR} = \mathcal{M} \times \mathcal{P}$.

In general relativity, the spacetime manifold $\mathcal{M}$ has a curved geometry. The particle phase space is no longer a product. Instead, there is a separate momentum space, $\mathcal{P}_x$ associated to each spacetime point $x \in \mathcal{M}$. This is identified with the cotangent space of $\mathcal{M}$ at x, so that $\mathcal{P}_x = \mathcal{T}_x^*(\mathcal{M})$. The whole phase space is the cotangent bundle of $\mathcal{M}$, *i.e.* $\Gamma^{GR} = \mathcal{T}^*(\mathcal{M})$

Within the framework of relative locality, it is the momentum space $\mathcal{P}$ that is curved. There then must be a separate spacetime, $\mathcal{M}_p$ for each value of momentum, $\mathcal{M}_p = \mathcal{T}_p^*(\mathcal{P})$. The whole phase space is then the cotangent bundle over momentum space, *i.e.* $\Gamma^{RL} = \mathcal{T}^*(\mathcal{P})$.

GAC+Freidel+Kowalski-Glikman+Smolin, arXiv:1106.0313;GRG(in press)



Hilbert space carries a representation of the operators corresponding to the full metric, without fixed background metric
LoopQG
DEWITT(1960s)
the QG highway: quantize gravity
DIRAC-BERGMAN(1950)
QG must be just like QED... just quantize linearized gravity
ROSENFELD(1930)
FEYNMAN(1960s)
String Theory
Decoherence
(Hawking+...)
Rosenfeld is wrong: gravitation is not like electromagnetism (operative definition of characteristic field is profoundly different)
Bronstein(1936)
STALIN
Solomon(1938)
HITLER
noncommutative spacetime
(Doplicher+Fredenhagen+Roberts,
kappa-Minkowski.....)
QG roads less traveled
Born(1938)
?
relative locality
QG must factor in the `reciprocity' momentum space ↔ spacetime of QM...curved momentum space...

strategy aspects of the new perspective:

•momentum space is at the forefront

• geometry of momentum space is proposed as an experimental issue

• we need models to be used as test theories providing guidance to experiments

• many sorts of momentum-space geometry are legitimate candidates

but it is natural to start from a well-known example→

looking at "kappa-Poincare phase spaces" from the relative-locality perspective

Lukierski +Ruegg +Nowicki+Tolstoi +Zakrzewski +Sitarz +Majid +…. …. …. …. ….

kappa-Minkowski $[x_j, x_0] = i\ell x_j$ $\quad [x_j, x_k] = 0$

writing fields in time-to-the-right conventions

$$\Phi(x) = \int d^4k \tilde{\Phi}(k) e^{ik_j x^j} e^{ik_0 x^0}$$

kappa-Poincare' Hopf-algebra symmetries are characterized by

translation generators $P_\mu e^{ik_j x^j} e^{ik_0 x^0} = k_\mu e^{ik_j x^j} e^{ik_0 x^0}$

rotation generators $R_l e^{ik_j x^j} e^{ik_0 x^0} = \epsilon_{lmn} x_m k_n e^{ik_j x^j} e^{ik_0 x^0}$

boost generators

$$\mathcal{N}_l e^{ik_j x^j} e^{ik_0 x^0} = \left[x_0 k_l - x_l \left(\frac{1 - e^{-2\ell k_0}}{2\ell} + \frac{\ell}{2} k_m k^m\right)\right] e^{ik_j x^j} e^{ik_0 x^0}$$

Note that $\ell \equiv 1/\kappa$

page 2

symmetries describe a Hopf algebra
essentially codified in the coproduct;for example for translations

$$P_j\left(e^{ikx}e^{ik_0t}e^{iKx}e^{iK_0t}\right)=P_j\left(e^{i(k+e^{\ell k_0}K)x}e^{i(k_0+K_0)t}\right)$$
$$=\left(k_j+e^{-\ell k_0}K_j\right)\left(e^{ikx}e^{ik_0t}e^{iKx}e^{iK_0t}\right)$$
$$=\left[P_j\left(e^{ikx}e^{ik_0t}\right)\right]\left(e^{iKx}e^{iK_0t}\right)+\left[e^{-\ell P_0}\left(e^{ikx}e^{ik_0t}\right)\right]P_j\left(e^{iKx}e^{iK_0t}\right)$$

Baker
Campbell
Hausdorff

coproduct!!

notice nonlinear
composition of momenta
$$k_j+e^{-\ell k_0}K_j$$

rather unusual form of boost generator due to requirement of closing Hopf algebra and it leads to a deformed mass Casimir

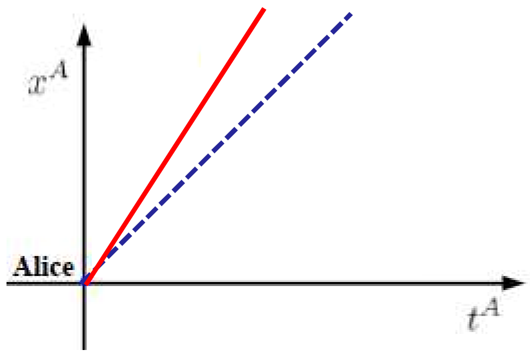
Note that
$\ell \equiv 1/\kappa$

$$\mathcal{C}=\left(\frac{2}{\ell}\right)^2\sinh^2\left(\frac{\ell}{2}P_0\right)-e^{\ell P_0}P_jP^j$$

notice connection
with modified
dispersion relation

wave equation governed by this Casimir operator and the properties of the "kappa-Minkowski noncommutative differential calculus" describes massless waves that propagate at speed

$$v=e^{-\ell|\vec{p}|}\simeq 1-\ell|\vec{p}|$$



GAC +Majid,IntJModPhysA15(2000)4301
GAC +D'Andrea +Mandanici,JCAP0309(2003)06

Lukierski+Nowicki,q-alg/9702003
GAC+Lukierski+Nowicki,hep-th/9706031;
PhysAtomNucl61(1998)1811

same issues also studied in terms of
some "kappa-Poincare phase-space constructions

basically take the commutators on previous
slides and turn them into Poisson brackets:

$$\{x,t\} = -\ell x$$

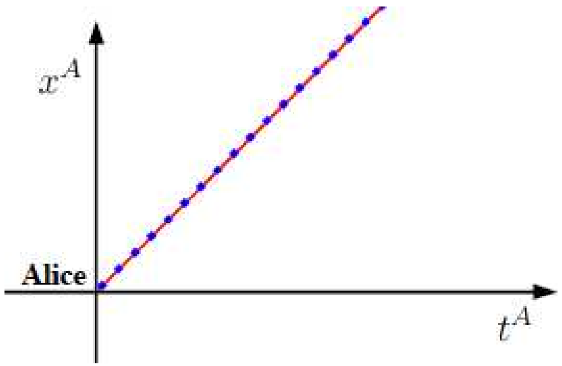
$$\{\Omega,t\} = 1, \qquad \{\Omega,x\} = 0\ ,$$
$$\boxed{\{P,t\} = \ell P,} \qquad \{P,x\} = -1$$

$$\{\Omega,P\} = 0\ , \qquad \{\mathcal{N},\Omega\} = P\ , \qquad \{\mathcal{N},P\} = \Omega + \ell\Omega^2 + \frac{\ell}{2}P^2$$

then derive worldlines of massless particles
within a rather standard Hamiltonian analysis

$$x = x_0 + \left(\frac{p}{\sqrt{p^2+m^2}} - \ell p\left(1 - \frac{p^2}{p^2+m^2}\right)\right)(t-t_0)$$

and for massless particles



kappa-Minkowski also studied in terms of some "kappa-Minkowski phase-space constructions"

basically take the commutators on previous slides and turn them into Poisson brackets:

Note that
$\ell \equiv \lambda$

$$\{x, t\} = -\ell x$$

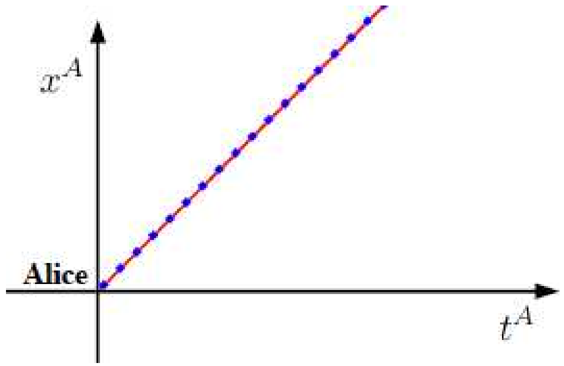
$$\{\Omega, t\} = 1, \qquad \{\Omega, x\} = 0\ ,$$
$$\boxed{\{P, t\} = \ell P,} \qquad \{P, x\} = -1$$

$$\{\Omega, P\} = 0\ , \qquad \{\mathcal{N}, \Omega\} = P\ , \qquad \{\mathcal{N}, P\} = \Omega + \ell\Omega^2 + \frac{\ell}{2}P^2$$

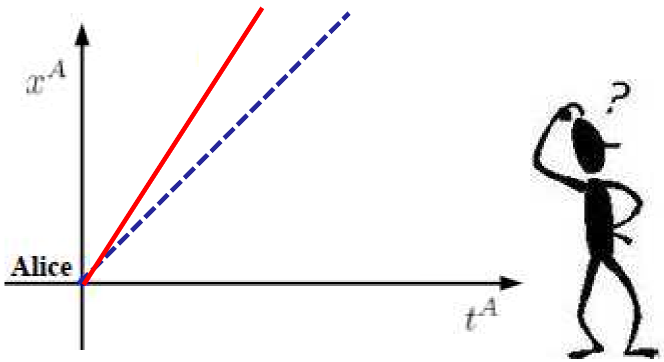
then derive worldlines of massless particles within a rather standard Hamiltonian analysis

$$x = x_0 + \left(\frac{p}{\sqrt{p^2+m^2}} - \ell p\left(1 - \frac{p^2}{p^2+m^2}\right)\right)(t - t_0)$$

and for massless particles



while on previous slide



relativity of locality in “kappa-Poincare phase-spaces”

GAC+Matassa+Mercati+Rosati, arXiv:1006.2126; PhysRevLett106, 071301

Smolin,arXiv:1007.0718

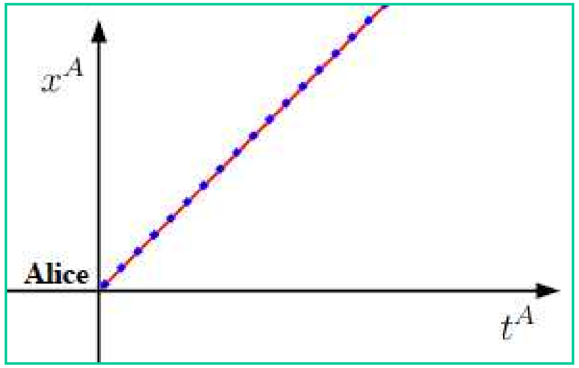
GAC+Loret+Rosati, arXiv:1102.4637;PhysLettB700(2011)150

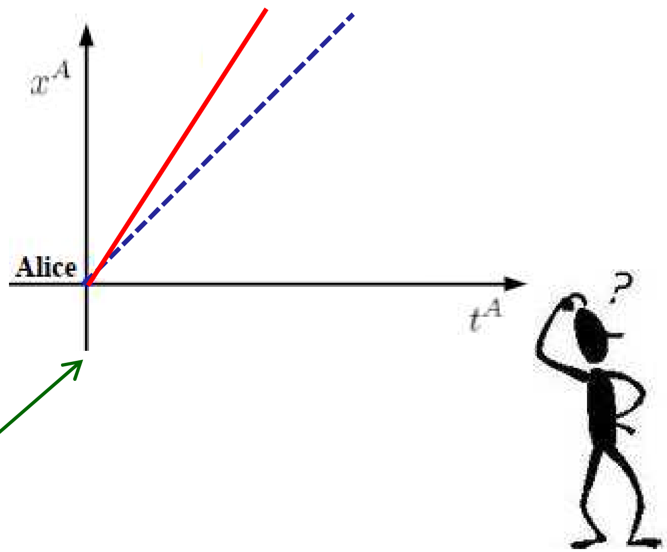
so situation was

$$\{x,t\} = -\ell x$$

$$\{\Omega, P\} = 0\,, \qquad \{\mathcal{N}, \Omega\} = P\,, \qquad \{\mathcal{N}, P\} = \Omega + \ell\Omega^2 + \frac{\ell}{2}P^2$$

$$\{\Omega, t\} = 1, \qquad \{\Omega, x\} = 0\,,$$
$$\{P, t\} = \ell P, \qquad \{P, x\} = -1$$



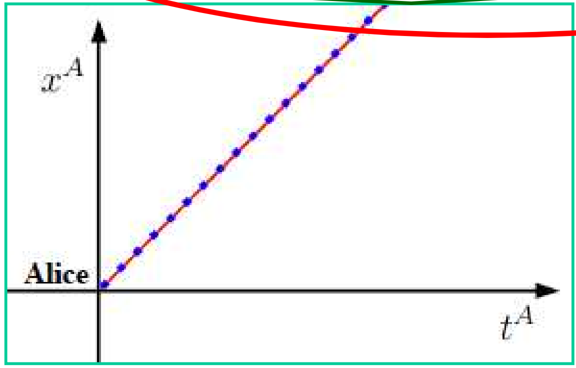


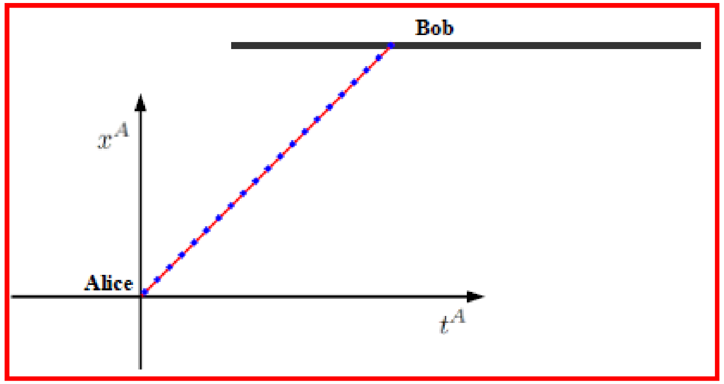
whereas from the “noncommutative Klein-Gordon equation” one would have expected this

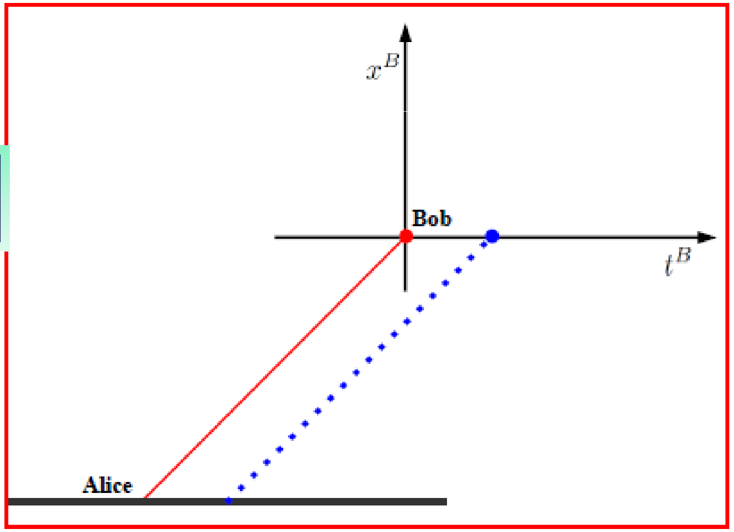
$$\{x,t\} = -\ell x$$

$$\{\Omega, P\} = 0\,, \qquad \{\mathcal{N}, \Omega\} = P\,, \qquad \{\mathcal{N}, P\} = \Omega + \ell\Omega^2 + \frac{\ell}{2}P^2$$

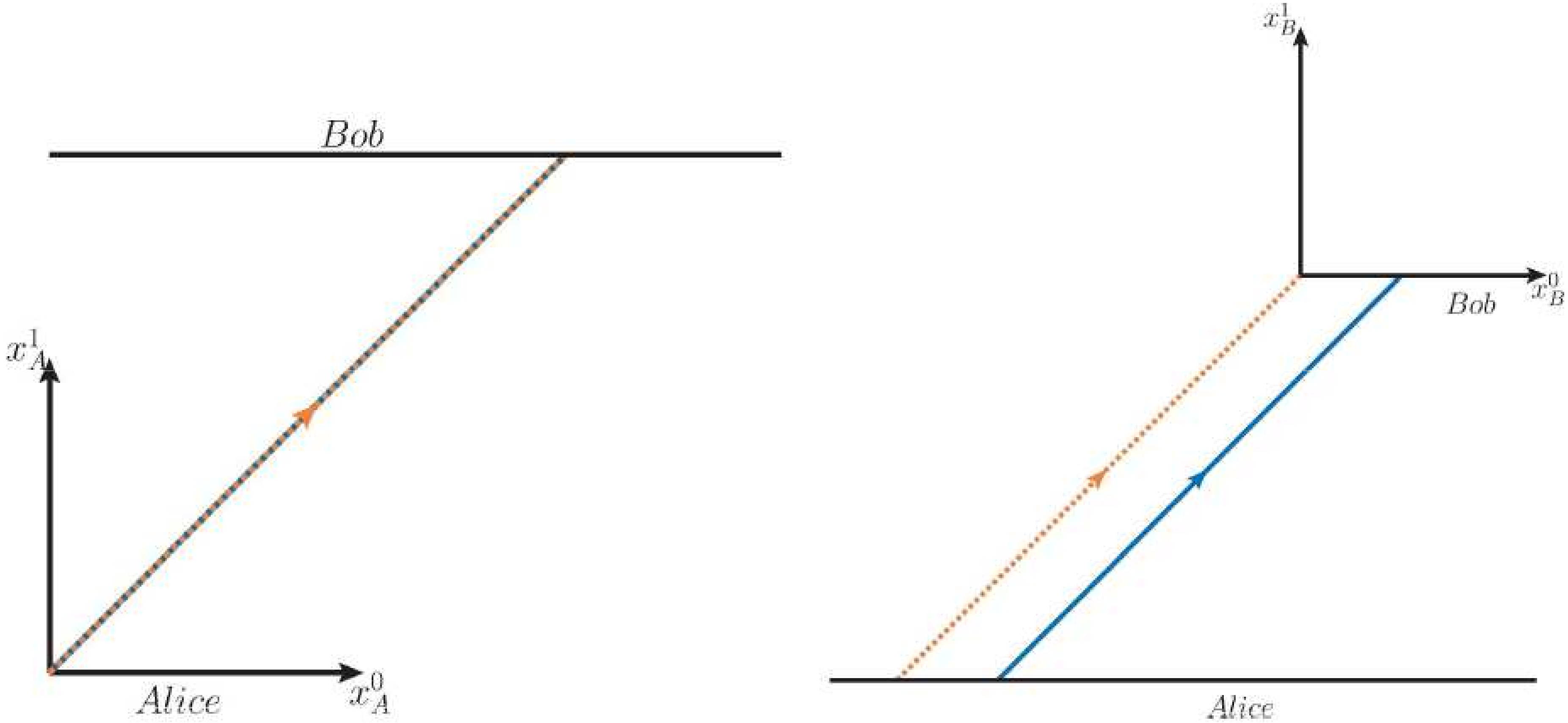
$$\{\Omega, t\} = 1, \qquad \{\Omega, x\} = 0\,,$$
$$\{P, t\} = \ell P, \qquad \{P, x\} = -1$$







what about Bob?



Bob
x_A^1
Alice
x_A^0
x_B^1
Bob
x_B^0
Alice

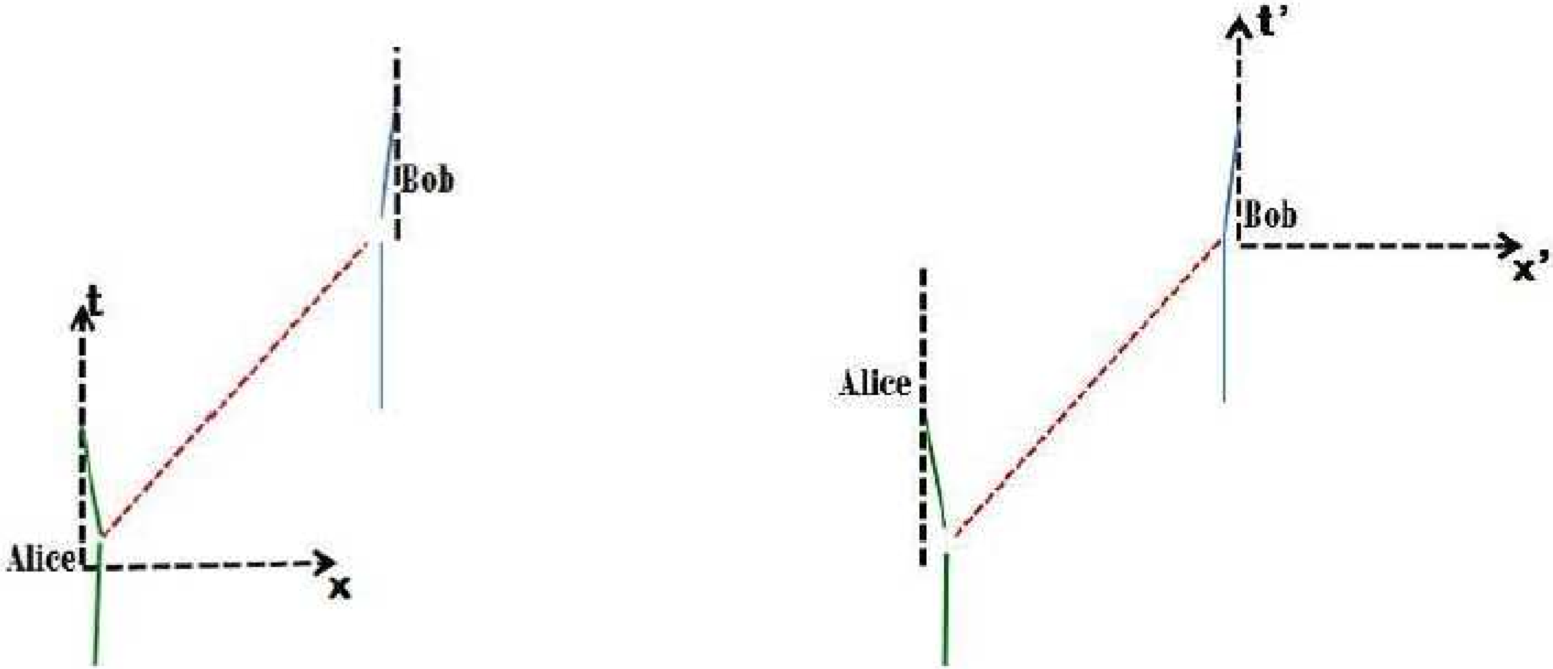
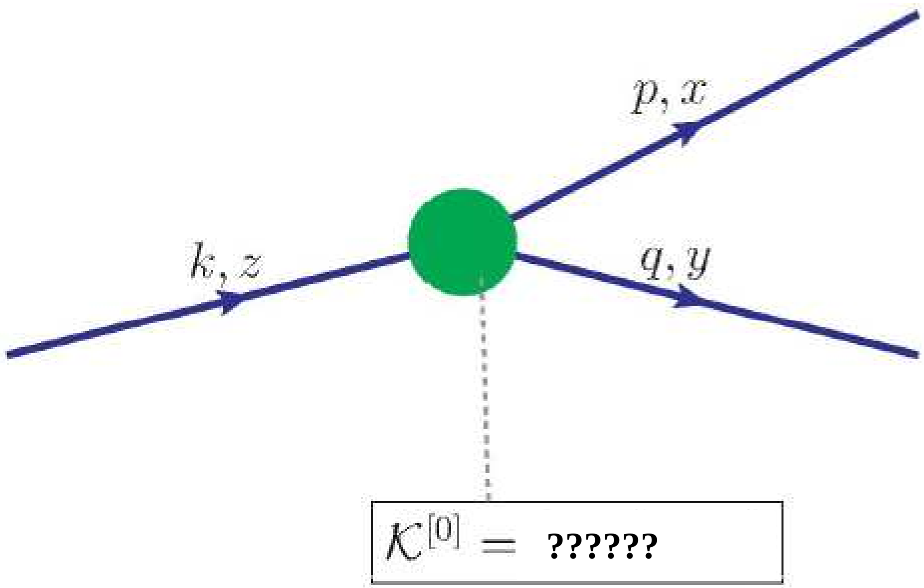


Figure 2. We show the implications of relative locality focusing on the illustrative example of an emission of a photon by a green atom, near Alice, with the absorption of that same photon by a blue atom, near Bob. The causal link between the two processes is still present, and the processes are still local, but the locality of the processes is not manifest in the inferences about distant events of the two observers. According to the coordinates of observer Alice the photon emission by the green atom is indeed a local process but the distant absorption of the photon by the blue atom appears to be a nonlocal process. In reverse, according to the coordinates of observer Bob the photon absorption by the blue atom is indeed a local process but the distant emission of the photon by the green atom appears to be a nonlocal process.

the principle of relative locality

GAC+Freidel+Kowalski-Glikman+Smolin

let us apply our proposal for interacting particles with relative locality to the case of kappa-Poincare
THIS PRODUCES THE FIRST MODEL OF INTERACTING KAPPA-P PARTICLES!!!
Same equations of motion as in traditional kappa-P phase space constructions.
But now we add interactions (see Jurek's talk) via some boundary terms, at endpoints of worldlines, enforcing the conservation laws



$$\mathcal{K}^{[0]} = k \oplus (\ominus q) \oplus (\ominus p)$$

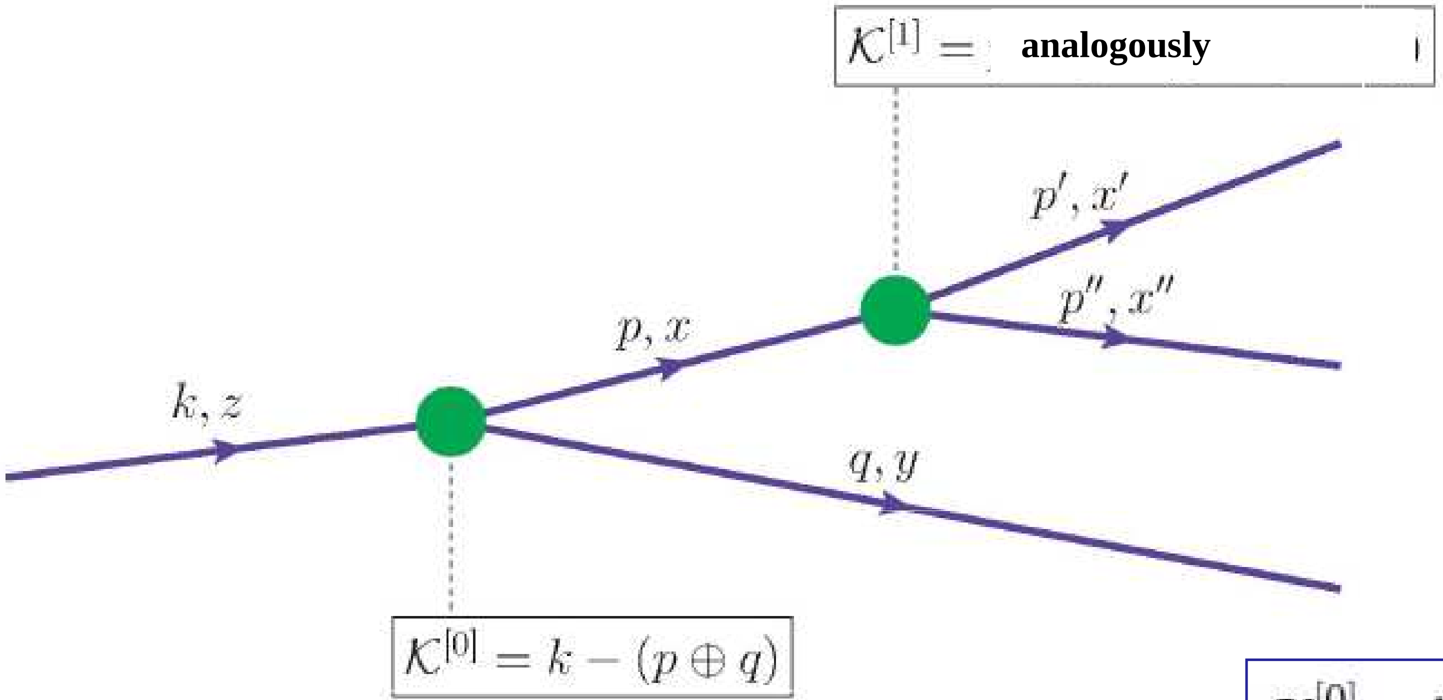
$$\mathcal{K}^{[0]} = (\ominus q) \oplus (\ominus p) \oplus k$$

$$\mathcal{K}^{[0]} = k - (p \oplus q)$$

GAC+Arzano+Kowalski-Glikman+Rosati+Trevisan, arXiv:1106.????

GAC+Arzano+Kowalski-Glikman+Rosati+Trevisan, arXiv:1106.????

but if we look at the case of finite wordlines (worldlines with 2 endpoints)

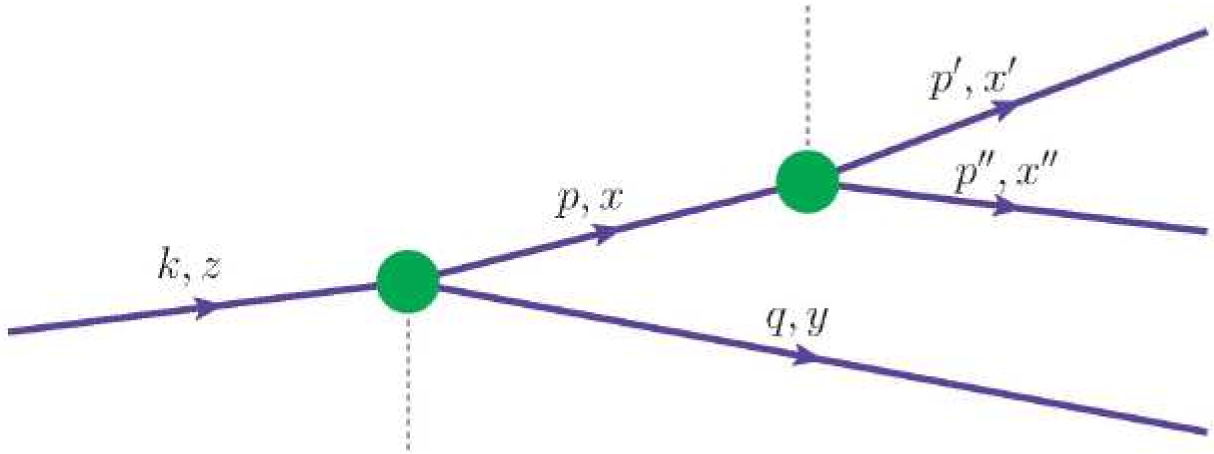


$$\mathcal{K}^{[0]} = k \oplus (\ominus q) \oplus (\ominus p)$$

$$\mathcal{K}^{[0]} = (\ominus q) \oplus (\ominus p) \oplus k$$

$$\mathcal{K}^{[0]} = k - (p \oplus q)$$

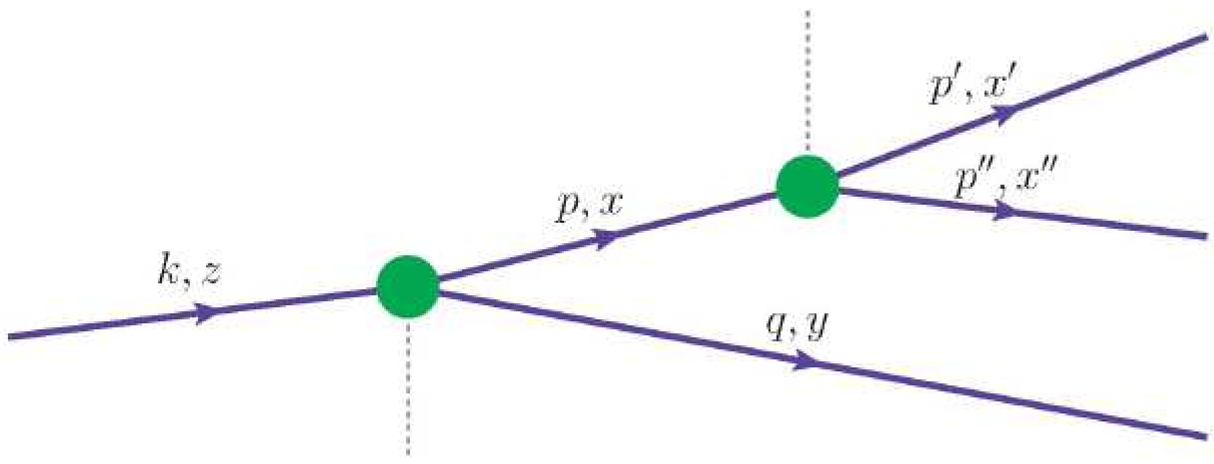
following this prescription the action is invariant under translations generated by "total momentum" (with "sum" performed via $\oplus$)



$$
\begin{aligned}
\mathcal{S}^{\kappa(2)} = \mathcal{S}^{\kappa(2)}_{bulk} + \mathcal{S}^{\kappa(2)}_{int} = & \int_{-\infty}^{s_0} ds \left(z^\mu \dot{k}_\mu + \ell z^1 k_1 \dot{k}_0 + \mathcal{N}_k \mathcal{C}_\kappa[k]\right) + \int_{s_0}^{s_1} ds \left(x^\mu \dot{p}_\mu + \ell x^1 p_1 \dot{p}_0 + \mathcal{N}_p \mathcal{C}_\kappa[p]\right) \\
& + \int_{s_1}^{+\infty} ds \left(x'^\mu \dot{p}'_\mu + \ell x'^1 p'_1 \dot{p}'_0 + \mathcal{N}_{p'} \mathcal{C}_\kappa[p']\right) + \int_{s_1}^{+\infty} ds \left(x''^\mu \dot{p}''_\mu + \ell x''^1 p''_1 \dot{p}''_0 + \mathcal{N}_{p''} \mathcal{C}_\kappa[p'']\right) \\
& + \int_{s_0}^{+\infty} ds \left(y^\mu \dot{q}_\mu + \ell y^1 q_1 \dot{q}_0 + \mathcal{N}_q \mathcal{C}_\kappa[q]\right) \\
& - \xi^\mu_{[0]} \mathcal{K}^{[0]}_\mu(s_0) - \xi^\mu_{[1]} \mathcal{K}^{[1]}_\mu(s_1)
\end{aligned}
$$

where indeed with $\mathcal{K}^{[0]}$ and $\mathcal{K}^{[1]}$ we take respectively $k - (p \oplus q)$ and $(p \oplus q) - (p' \oplus p'' \oplus q)$.

GAC+Arzano+Kowalski-Glikman+Rosati+Trevisan, arXiv:1106.????



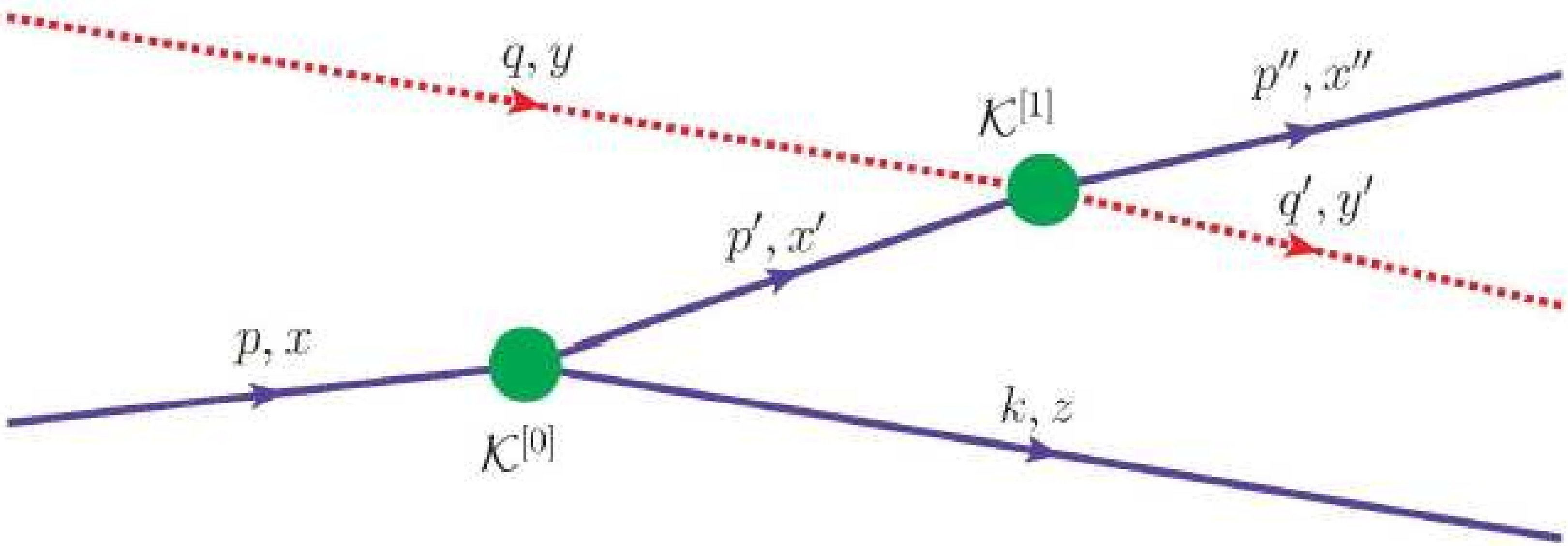
bulk action;free propagation

$$
\begin{aligned}
\mathcal{S}^{\kappa(2)} = \mathcal{S}^{\kappa(2)}_{bulk} + \mathcal{S}^{\kappa(2)}_{int} = & \int_{-\infty}^{s_0} ds \left(z^\mu \dot{k}_\mu + \ell z^1 k_1 \dot{k}_0 + \mathcal{N}_k \mathcal{C}_\kappa[k] \right) + \int_{s_0}^{s_1} ds \left(x^\mu \dot{p}_\mu + \ell x^1 p_1 \dot{p}_0 + \mathcal{N}_p \mathcal{C}_\kappa[p] \right) \\
& + \int_{s_1}^{+\infty} ds \left(x'^\mu \dot{p}'_\mu + \ell x'^1 p'_1 \dot{p}'_0 + \mathcal{N}_{p'} \mathcal{C}_\kappa[p'] \right) + \int_{s_1}^{+\infty} ds \left(x''^\mu \dot{p}''_\mu + \ell x''^1 p''_1 \dot{p}''_0 + \mathcal{N}_{p''} \mathcal{C}_\kappa[p''] \right) \\
& + \int_{s_0}^{+\infty} ds \left(y^\mu \dot{q}_\mu + \ell y^1 q_1 \dot{q}_0 + \mathcal{N}_q \mathcal{C}_\kappa[q] \right) \\
& - \xi^\mu_{[0]} \mathcal{K}^{[0]}_\mu(s_0) - \xi^\mu_{[1]} \mathcal{K}^{[1]}_\mu(s_1)
\end{aligned}
$$

boundary terms; interactions

where indeed with $\mathcal{K}^{[0]}$ and $\mathcal{K}^{[1]}$ we take respectively $k - (p \oplus q)$ and $(p \oplus q) - (p' \oplus p'' \oplus q)$.

GAC+Arzano+Kowalski-Glikman+Rosati+Trevisan, arXiv:1106.????



$$\mathcal{K}_\mu^{[0]}(s_0) = (q \oplus p)_\mu - (q \oplus p' \oplus k)_\mu \qquad \mathcal{K}_\mu^{[0]} = (p \oplus q)_\mu - (k \oplus p' \oplus q)_\mu$$

the torsion of the kappaP-momentum-space geometry can be measured this way: different times of arrival!!!

GAC+Ellis+Mavromatos+Nanopoulos+Sarkar, Nature393,763(1998)
GAC,Nature408,661 (2000)

kappa-P inspires a momentum-space geometry with nonmetricity and torsion

***nonmetricity→time delays are produced**

Freidel+Smolin,arXiv:1103.5626

***torsion+nonmetricity→ "nonsystematic" time delays are produced**

GAC+Arzano+Kowalski-Glikman+Rosati+Trevisan, arXiv:1106.????

effects are indeed completely negligible on terrestrial scales

but Nature provides a nearly ideal laboratory: GRBs

GAC+Ellis+Mavromatos+Nanopoulos+Sarkar, Nature393,763(1998)

GAC,Nature408,661 (2000)

- **photons observed up to ~100 GeV**
- **emission of photons nearly simultaneous on a time scale of seconds**
- **distance established (redshift>1 !!!) for many such highenergy GRBs**

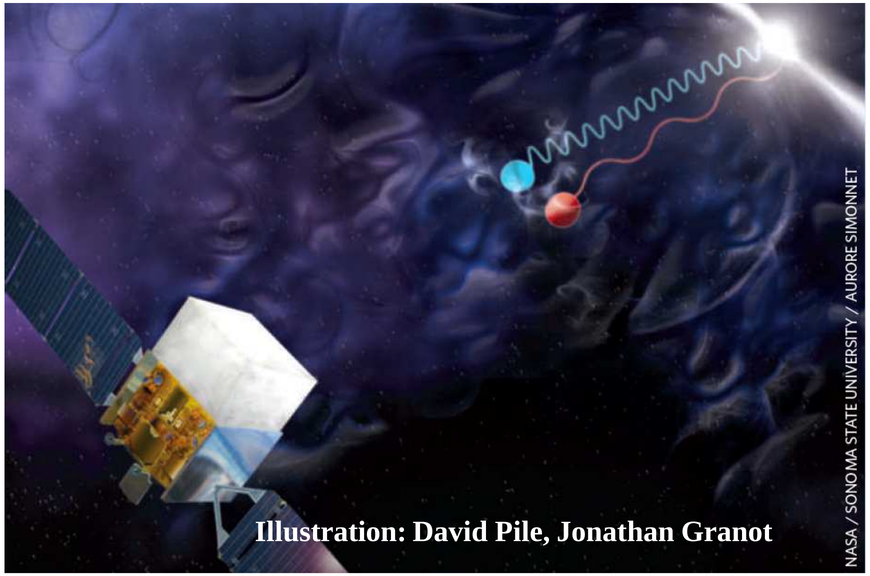


Illustration: David Pile, Jonathan Granot

other developments

distinguishing between “longitudinal” and “transverse” relative locality:

GAC+Barcaroli+Loret, arXiv:1106.????

“dual-gravity lensing”
Freidel+Smolin,arXiv:1103.5626

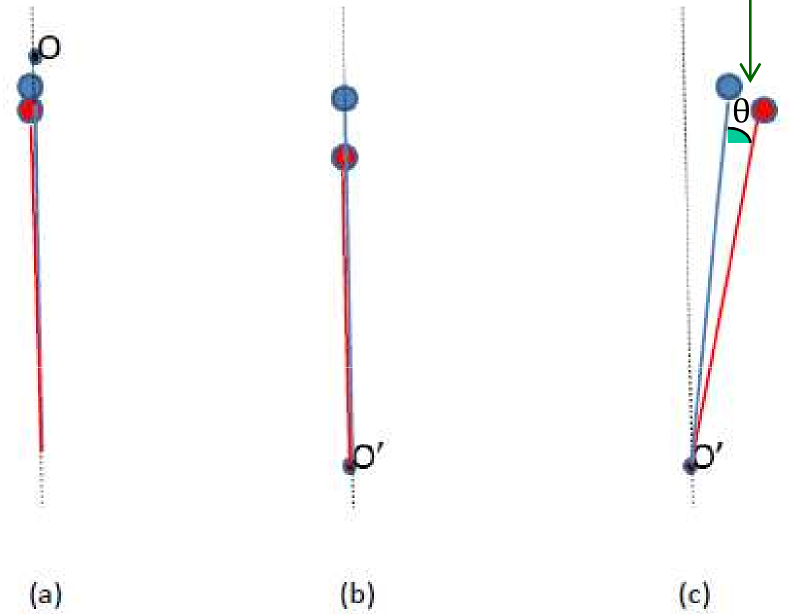


Figure 1. With relative locality a pair of events established to be coincident by a nearby observer (panel (a)) may be described as events which are not coincident in the coordinates of distant observers. With longitudinal relative locality the distant observer describes the events as non-coincident along the direction connecting the observer to the events (panel (b)). With transverse relative locality the distant observer describes the events as non-coincident along a direction orthogonal to the direction connecting the observer to the events (panel (c)). With transverse relative locality it becomes possible for particles emitted at coincident events along parallel directions to be detected at a distant observer as coming from different directions, a feature which one may qualify as “dual-gravity lensing" (see red and blue lines in panel (c)).

other developments

Gubitosi+Mercati, arXiv:1106.5710

GAC, arXiv:1107.????

boosts on relative-locality momentum spaces:

if momentum space is “kappa-Poincare inspired” the kappa-Poincare Hopf algebra provides needed guidance

for more general choices of metric and connection for momentum space proceed analogously, following relativistic criteria codified in the doubly-special-relativity programme

kappa-Poincare continues to be for me
“like a box of chocolates”, filled with
a never-ending collection of
wonderful things, that keep surprising me

thanks Jurek!!

happy birthday!!!!