# Constrained BF theory as gravity

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# Content of the talk

1. MacDowell-Mansouri gravity
2. BF theory reformulation
3. Supergravity
4. Canonical analysis
5. Gravitational Noether charges

# General Relativity

VARIABLE: $g_{\mu\nu}$ metric

*Hilbert-Einstein Action*

ACTION: $$S = \frac{1}{16\pi G}\int \mathrm{d}^4x\, \sqrt{-g}\, (R - 2\Lambda)$$

$$T^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu} = 0, \qquad R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$$

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VARIABLE: *so*$(1,3)$*-connection* $\omega^{ab}_{\mu}$ and *tetrad* $e^a_\mu$

*Palatini Action*

ACTION: $$S = \frac{1}{64\pi G}\int d^4x\, \epsilon^{abcd}\big(R_{\mu\nu\, ab}\, e_{\rho\, k}\, e_{\sigma\, d} - \frac{\Lambda}{3} e_{\mu\, a}\, e_{\nu\, b}\, e_{\rho\, c}\, e_{\sigma\, d}\big)\epsilon^{\mu\nu\rho\sigma}$$

$$R_{\mu\nu}{}^{ab} = \partial_\mu \omega_\nu{}^{ab} - \partial_\nu \omega_\mu{}^{ab} + \omega_\mu{}^{a}{}_{c}\, \omega_\nu{}^{cb} - \omega_\nu{}^{a}{}_{c}\, \omega_\mu{}^{cb}, \qquad R^a{}_b = d\omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b$$

$$T_{\mu\nu}{}^{a} = \mathcal{D}^{\omega}_{\mu}\, e_\nu{}^a - \mathcal{D}^{\omega}_{\nu}\, e_\mu{}^a, \qquad T^a = D^\omega e^a = de^a + \omega^a{}_b \wedge e^b$$

# $A_\mu^{IJ}$ connection of the $SO(2,3)$

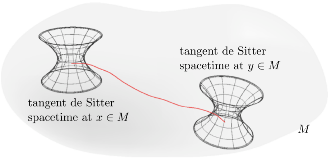
MacDowell and Mansouri proposal

$$A_\mu^{IJ} \to \begin{cases} A_\mu^{ab} = \omega_\mu^{ab} \\ A_\mu^{a5} = \frac{1}{\ell} e_\mu^a \end{cases} \quad \text{with } \frac{\Lambda}{3} = -\frac{1}{\ell^2}, \quad \begin{array}{l} a,b = (1,2,3,4) \\ I,J = (1,2,3,4,5) \end{array}$$

$$\mathbb{A}_\mu = \frac{1}{2}\omega_\mu^{ab} M_{ab} + \frac{1}{\ell} e_\mu^a P_a = \frac{1}{2} A_\mu{}^{IJ} M_{IJ} \qquad M_{a5} = P_a$$

For de Sitter group the geometric interpretation of this construction:

$SO(1,4)$ connection $A = (\omega, e)$ encodes the geometry of the spacetime $\mathcal{M}$ by "rolling de Sitter manifold along $\mathcal{M}$" [Wise]



# MacDowell-Mansouri 1977

Connection 1-form $A^{IJ}$

$$A^{IJ} = \begin{pmatrix} \omega^{ab} & \frac{1}{\ell} e^a \\ -\frac{1}{\ell} e^b & 0 \end{pmatrix}$$

Curvature 2-form $F^{IJ} = dA^{IJ} + A^{IK} \wedge A_K{}^J$

$$F^{IJ} = \begin{pmatrix} R^{ab} + \frac{1}{\ell^2} e^a \wedge e^b & \frac{1}{\ell} T^a \\ -\frac{1}{\ell} T^b & 0 \end{pmatrix}$$

Bianchi identity: $D^A F^{IJ} = 0$

# MacDowell-Mansouri 1977

$$F^{IJ} \to \hat{F}^{IJ} = F^{ab}$$

General Relativity as gauge symmetry breaking theory

$$S_{MM}(A) = \frac{\ell^2}{64\pi G} \int tr\big(\hat{F} \wedge \star \hat{F}\big)$$

$$S_{MM}(A) = \frac{\ell^2}{64\pi G} \int \left(R^{ab} + \frac{1}{\ell^2} e^a \wedge e^b\right) \wedge \left(R^{cd} + \frac{1}{\ell^2} e^c \wedge e^d\right) \epsilon_{abcd}$$

$$32\pi G\, S_{MM} = \int \boxed{\text{Palatini}} + \frac{1}{2\ell^2} \int \boxed{\text{cosmological}} + \frac{\ell^2}{2} \int \boxed{Euler_4}$$

**Equations of motion :** $\left(R^{ab} \wedge e^c + \frac{1}{2\ell^2} e^a \wedge e^b \wedge e^c\right) \epsilon_{abcd} = 0\,, \qquad T^a = 0$

# Perturbed $BF$ theory

Introduce independent $so(2,3)$-valued 2-form $B$ to the action

$$\mathcal{S} = \int_{\mathcal{M}} tr\left(B \wedge F - \frac{G\Lambda}{6}\hat{B} \wedge *\hat{B}\right)$$

$$\delta B_{a5} : \ F^{a5} = 0, \qquad \delta \hat{B}_{ab} : \ F^{ab} = \frac{G\Lambda}{3}\epsilon^{abcd}B_{cd}$$

- In this form Macdowell-Mansouri gravity has the appearance of the deformation of a topological gauge theory.
- The symmetry breaking occurs in the last term with dimensionless coefficient proportional to: $G\Lambda \sim 10^{-120}$.
- In some sense general relativity is "not too far pertubatively" from a topological field theory.

# Holst action 1996

Ashtekar connection

Phase space of gravity can be described by self and antiselfdual connections provided by projector $(1 \mp i\star)$

$$ {}^{\pm}\omega_i^a = \omega_i^{0a} \mp \frac{i}{2}\epsilon^{abc}\omega_{i\,bc} $$

Real Barbero-Immirzi connection

$$ {}^{\gamma}\omega_i^a = \omega_i^{0a} + \frac{\gamma}{2}\epsilon^{0abc}\omega_{i\,bc}, \qquad \mathcal{P}_a^i = \frac{4}{G}\epsilon_{abc}\epsilon^{ijk}e_j^b\,e_k^c $$

$$ \{{}^{\gamma}\omega_i^a(x), \mathcal{P}_b^j(y)\} = \gamma\delta(x-y)\delta_i^j\delta_b^a $$

Holst Action

$$ S = \frac{1}{32\pi G}\int \epsilon_{abcd}\,R^{ab}\wedge e^c\wedge e^d + \frac{2}{32\pi G\gamma}\int R^{ab}\wedge e_a\wedge e_b $$

# Starodubtsev and Freidel 2005

1-form connection $A$ and independent $so(2,3)$-valued 2-form $B$

$$S = \int_{\mathcal{M}} tr\left(B \wedge F - \frac{\beta}{2} B \wedge B - \frac{\alpha}{4} \hat{B} \wedge * \hat{B}\right)$$

Action proposed by Starodubtsev and Freidel

$$S = \int_{\mathcal{M}} \left(B_{IJ} \wedge F^{IJ} - \frac{\beta}{2} B_{IJ} \wedge B^{IJ} - \frac{\alpha}{4} \epsilon_{abcd5} B^{ab} \wedge B^{cd}\right)$$

with constants $(\alpha, \beta\, \ell) \to (G, \Lambda, \gamma)$:

$$\gamma = \frac{\beta}{\alpha}, \quad \Lambda = -\frac{3}{\ell^2}, \quad where \quad \alpha = \frac{G\Lambda}{3(1+\gamma^2)}, \quad \beta = \frac{\gamma G \Lambda}{3(1+\gamma^2)}$$

# Constrained BF theory

$$\mathcal{S} = \int d^4x\, \epsilon^{\mu\nu\lambda\rho} \left( B_{\mu\nu IJ} F^{IJ}_{\lambda\rho} - \frac{\beta}{2} B_{\mu\nu IJ} B^{IJ}_{\lambda\rho} - \frac{\alpha}{4} \epsilon_{IJKL5} B^{IJ}_{\mu\nu} B^{KL}_{\lambda\rho} \right)$$

Structure standing behind the constrained $BF$ model

$$\begin{aligned} 32\pi G\, S \quad = & \quad \int R^{ab} \wedge e^c \wedge e^d\, \epsilon_{abcd} + \frac{1}{2\ell^2} \int e^a \wedge e^b \wedge e^c \wedge e^d\, \epsilon_{abcd} \\ + & \quad \frac{\ell^2}{2} \int R^{ab} \wedge R^{cd}\, \epsilon_{abcd} + \frac{2}{\gamma} \int R^{ab} \wedge e_a \wedge e_b \\ - & \quad \ell^2 \gamma \int R^{ab} \wedge R_{ab} + 2\frac{\gamma^2+1}{\gamma} \int (T^a \wedge T_a - R^{ab} \wedge e_a \wedge e_b) \end{aligned}$$

$$32\pi G\, S = \int \boxed{\text{Palatini}} + \frac{1}{2\ell^2} \int \boxed{\text{cosmological}} + \frac{\ell^2}{2} \int \boxed{\text{Euler}} + \frac{2}{\gamma} \int \boxed{\text{Holst}} - \ell^2\gamma \int \boxed{\text{Pontryagin}} + 2\frac{\gamma^2+1}{\gamma} \int \boxed{\text{Nieh/Yan}}$$

# Supergravity $\mathcal{N} = 1$

*Adding* gravitino $\psi_\mu^\alpha$ *spin-3/2 field*

$$\mathbb{A}_\mu = \frac{1}{2}\omega_\mu{}^{ab} M_{ab} + \frac{1}{\ell} e_\mu{}^a P_a + \kappa \bar{\psi}_\mu^\alpha Q_\alpha$$

*Super-curvature*

$$\mathbb{F}_{\mu\nu} = \partial_\mu \mathbb{A}_\nu - \partial_\nu \mathbb{A}_\mu - i[\mathbb{A}_\mu, \mathbb{A}_\nu] = \frac{1}{2} F^{(s)IJ}_{\mu\nu} M_{IJ} + \bar{\mathcal{F}}^\alpha_{\mu\nu} Q_\alpha$$

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**Supergravity as a constrained *BF* theory**

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# Supergravity $\mathcal{N} = 1$

## *Supergravity action*

$$S^{sugra} = \frac{1}{64\pi G}\int \epsilon^{\mu\nu\rho\sigma}\,\epsilon_{abcd}\left(R^{ab}_{\mu\nu}\, e_\rho{}^c\, e_\sigma{}^d - \frac{\Lambda}{3}\, e_\mu{}^a\, e_\nu{}^a\, e_\rho{}^c\, e_\sigma{}^d\right) d^4x$$

$$-\frac{1}{64\pi G}\int \epsilon^{\mu\nu\rho\sigma}\left(\frac{1}{2}\,\bar{\psi}_\mu\,\gamma_5\,\gamma_a\, e^a_\nu\, D^\omega_\rho\,\psi_\sigma \;-\; \frac{i}{4\ell}\,\bar{\psi}_\mu\,\gamma_5\,\gamma_{ab}\, e^a_\nu\, e^b_\rho\,\psi_\sigma\right) d^4x$$

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**Cosmological constant in supergravity**

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We construct an extension of pure supergravity which contains a cosmological term and a masslike term for the spin-3/2 field. Unlike another recent model which incorporates these features, that presented here is constructed from the usual spin-2, spin-3/2 fields alone.

The action

$$I=\int\left(-\frac{1}{4\kappa^2}\, e e^{a\mu} e^{b\nu} R_{\mu\nu ab} + \frac{3\lambda^2 e}{2} - \tfrac{1}{2}\,\epsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda\gamma_5\gamma_\mu D_\nu\psi_\rho \;-\lambda\kappa e\bar{\psi}_\lambda\,\sigma^{\lambda\rho}\,\psi_\rho\right) d^4x$$

# Canonical analysis

$$S = \int dt \int d^3x \, \mathcal{L}_{BF}, \qquad \mathcal{H} = p\,\dot{q} - \mathcal{L}$$

$$\mathcal{L}_{BF} = \mathcal{P}^i_{IJ} \dot{A}_i^{IJ} + A_0^{IJ} \boxed{D_i^A \mathcal{P}^i_{IJ}} + B_{0i}{}^{IJ} \boxed{2\epsilon^{ijk} \, F_{jk\,IJ} - \beta\, \mathcal{P}^i_{IJ} - \frac{\alpha}{2}\, \epsilon_{IJKL4}\, \mathcal{P}^{i\,KL}}$$

$$\{A_i^{IJ}(x), P^j_{KL}\} = \delta^{IJ}_{KL} \delta(x-y) \delta^j_i \,, \qquad \text{where } \mathcal{P}^{i\,IJ} \equiv 2\epsilon^{ijk} \, B_{jk}{}^{IJ}$$

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**Hamiltonian analysis of SO(4, 1) constrained BF theory**

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(Dated: March 12, 2010)

# Constraints

Complicated explicit constraints

$$\begin{aligned}
\Phi^i_\alpha &= \mathcal{P}^i_\alpha - \frac{4}{\ell\beta}\epsilon^{ijk}\,\mathcal{D}^\omega_j\, e_{k\ \alpha} \approx 0 \\
\Phi^i_{\alpha\beta} &= \mathcal{P}^i_{\alpha\beta} - M_{\alpha\beta}{}^{\gamma\delta}\, F_{jk\ \gamma\delta}\,\epsilon^{ijk} \approx 0 \\
\Pi_{\alpha\beta} &= \frac{2}{\ell^2}\,\epsilon^{ijk}\mathcal{D}^\omega_i\left(K_{\alpha\beta}{}^{\gamma\delta}\ e_{j\ \gamma}\, e_{k\ \delta}\right) \approx 0 \\
\Pi_\alpha &= \frac{1}{\ell}\,\epsilon^{ijk}\, K_{\alpha\beta}{}^{\gamma\delta}\ e_i{}^\beta\ R_{jk\ \gamma\delta} - \frac{2\alpha}{(\alpha^2+\beta^2)\ell^3}\,\epsilon^{ijk}\,\epsilon_{\alpha\beta\gamma\delta}\ e_i{}^\beta\ e_j{}^\gamma\ e_k{}^\delta \approx 0
\end{aligned}$$

where

$$M^{\alpha\beta}{}_{\gamma\delta} \equiv \frac{\alpha}{(\alpha^2+\beta^2)}(\gamma\,\delta^{\alpha\beta}_{\gamma\delta} - \epsilon^{\alpha\beta}{}_{\gamma\delta}),$$

$$K^{\alpha\beta}{}_{\gamma\delta} \equiv \frac{\alpha}{(\alpha^2+\beta^2)}(\tfrac{1}{\gamma}\,\delta^{\alpha\beta}_{\gamma\delta} + \epsilon^{\alpha\beta}{}_{\gamma\delta})$$

# Topological terms in the action

$$32\pi\, S_{topological} = \frac{\gamma^2+1}{\gamma\, G}\boxed{NY_4} + \frac{3\gamma}{2G\Lambda}\boxed{P_4} - \frac{3}{4G\Lambda}\boxed{E_4}$$

**They all can be expressed as a total derivative!**

$$32\pi\, S_{topological} = \int \partial_\mu(\ something\ )\, d^4x$$

For constant time surfaces on a manifold $\Sigma \times \mathrm{R}$ without spacial boundary ($\partial\Sigma = 0$) all terms with spacial derivatives drop out, which means

$$32\pi\, S_{topological} = \int \partial_0\, W(e,\omega)\,, \qquad \text{where}$$

$$W = \frac{4}{\beta\ell^2}\int_\Sigma \epsilon^{ijk}\left(e_{i\,a}\mathcal{D}^\omega_j\, e^a_k\right) + \frac{2\alpha}{(\alpha^2+\beta^2)}\int_\Sigma \Big((\gamma - i)\mathcal{L}_{CS}({}^+\omega) + (\gamma + i)\mathcal{L}_{CS}({}^-\omega)\Big)$$

(functional of torsion and self and anti-self dual Chern-Simons forms $\mathcal{L}_{CS}$)

# Canonical transformation

**Canonical transformation**

$$\mathbf{P}\cdot\dot{\mathbf{Q}} - K(\mathbf{Q},\mathbf{P},t) + \frac{dW}{dt} = [\mathbf{p}\cdot\dot{\mathbf{q}} - H(\mathbf{q},\mathbf{p},t)]$$

**New momenta**

$$\boxed{\mathscr{P}^i_a} = \mathcal{P}^i_a + \{\mathcal{P}^i_a, W(\omega, e)\}, \quad \boxed{\mathscr{P}^i_{ab}} = \mathcal{P}^i_{ab} + \{\mathcal{P}^i_{ab}, W(\omega, e)\}$$

Canonical analysis of constrained BF theory
for manifold without boundary

is equivalent to

analysis of Holst action
with shifted definition of the momenta

# Black hole thermodynamics

| *1st law of thermodynamics* | *1st law of black hole dynamics* |
|---|---|
| $dE = TdS + dW$ | $dM = \frac{\kappa}{8\pi G}\, dA + \Omega\, dJ + \Phi\, dQ$ |

We identify

- the surface gravity of a black hole with temperature
- the area of the event horizon with the entropy

Surface gravity $\kappa$=acceleration needed to keep an object at horizon.
$\kappa^2 = -\frac{1}{2}\nabla_\nu \xi^\mu\, \nabla^\nu \xi_\mu$ (for example for Schwarzchild $\kappa = \frac{c^4}{4GM}$).

Beckenstein and Hawking black hole entropy ($l_p = \sqrt{G\hbar/c^3}$)

$$Entropy = \frac{Area}{4l_p^2}, \qquad Temperature = \frac{\kappa}{2\pi}$$

# R. Wald 1993

*Emmy Noether's theorem*

Any differentiable (smooth) symmetry of the action of a physical system has a corresponding conservation law.

Killing vectors as generators of diffeomorphism symmetry

*Gravitational Noether charges*

$$\begin{cases} Q[\xi_t]_\infty \\ Q[\xi_\varphi]_\infty \\ Q[\xi_t + \Omega\xi_\varphi]_H \end{cases} \rightarrow \begin{cases} Mass \\ Angular\ momentum \\ Temperature \cdot Entropy \end{cases}$$

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**Gravity as a constrained BF theory: Noether charges and Immirzi parameter**

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# Euler term as boundary term

$$32\pi G\, S = \int \boxed{\text{Palatini}} + \frac{1}{2\ell^2} \int \boxed{\text{cosmological}} + \rho \int \boxed{Euler_4}$$

Noether charge for Schwarzschild-AdS black hole

$$Mass = Q(\partial_t) = \frac{M}{2}\left(1 + \frac{2}{\ell^2}\,\rho\right) + \lim_{r\to\infty} \frac{\pi r^3}{4G\ell^2}\left(1 - \frac{2}{\ell^2}\,\rho\right)$$

Factor $\rho$ should be equal to the factor from the MacDowell-Mansouri model!

Adding boundary condition (AdS asymptotics) at the infinity:

$$\left(R^{ab}(\omega) - \frac{1}{\ell^2}\, e^a \wedge e^b\right)\Big|_\infty = 0$$

provides differentiability of the action! (it's due to $\Theta = \delta\omega_{ab} \wedge F^{ab}$):

$$\delta(Palatini + \Lambda + Euler) = \int_M (f.e.)_a \delta e^a + \int_M (f.e.)_{ab}\, \delta\omega^{ab} + \int_M d\Theta = 0$$

# Generalized Noether charge

Structure behind BF theory

$$\boxed{(\text{Palatini} + \Lambda) + \tfrac{\ell^2}{2}\text{Euler}} + \gamma \boxed{(\text{Holst+Nieh/Yan}) + \tfrac{\ell^2}{2}\text{Pontryagin}}$$

Rewritten form of our action with $M^{ab}{}_{cd} = \frac{\alpha}{(\alpha^2+\beta^2)}(\gamma\,\delta^{ab}{}_{cd} - \epsilon^{ab}{}_{cd})$:

$$S_{BF}(\omega, e) = \frac{1}{16\pi}\int_M \left(\frac{1}{4}M^{abcd}F_{ab}\wedge F_{cd} - \frac{1}{\beta\ell^2}\,T^a\wedge T_a\right)$$

Noether charge from Wald's approach generalized to the case of first order gravity with Immirzi parameter:

$$Q[\xi] = \int_{\partial\Sigma}(\xi^\mu A^{IJ}_\mu)\frac{\delta\mathcal{L}}{\delta F^{IJ}} = \frac{1}{32\pi}\int_{\partial\Sigma}(\xi^\mu\omega^{ab}_\mu)M_{abcd}F^{cd}$$

*Generalized Noether charge*

$$Q[\xi] = \frac{\ell^2}{32\pi G}\int_{\partial\Sigma}(\xi^\mu\omega_{\mu\,ab})\left(\epsilon^{ab}{}_{cd}F^{cd}_{\;jk} - 2\gamma F^{ab}_{\;jk}\right)dx^j\wedge dx^k\,.$$

# Schwarzschild–AdS spacetime

$$ds^2 = -f(r)^2 dt^2 + f(r)^{-2} dr^2 + r^2(d\theta^2 + \sin^2\theta \, d\varphi^2)$$

$$f(r)^2 = (1 - \frac{2GM}{r} + \frac{r^2}{\ell^2})$$

There is no Immirzi parameter in the black hole thermodynamics

- $Q[\xi_t]_\infty = M$
- $Q[\xi_\varphi]_\infty = 0$
- $Q[\xi_t + \Omega\xi_\varphi]_H = \frac{\kappa}{2\pi} \cdot \frac{4\pi(r_H^2 + \ell^2)}{4G}$

- Mass
- Angular momentum
- Temperature · Entropy

*Entropy shifted by a constant!*

$$Entropy = \frac{Area}{4G} + \frac{4\pi\ell^2}{4G}$$

# Immirzi parameter

Entropy calculated in LQG framework

$$\mathrm{S}_{LQG} = \frac{\log 2}{\gamma\pi\sqrt{3}} \frac{Area}{4G}$$

Immirzi parameter might be present in black hole thermodynamics

$$\int_{\partial\Sigma} \partial_\theta g_{t\varphi} \neq 0$$

Under investigation: AdS–Taub–NUT metric with NUT charge

$$ds^2 = -f(r)^2(dt + 4n\sin^2\frac{\theta}{2}d\varphi)^2 + \frac{dr^2}{f(r)^2} + (n^2 + r^2)(r^2 d\theta^2 + r^2 \sin\theta\, d\varphi^2)$$

$$\text{with} \qquad f(r)^2 = \frac{r^2 - 2GMr - n^2 + (r^4 - 3n^4 + 6n^2r^2)\ell^{-2}}{n^2 + r^2}$$

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