Spontaneous Dimensional Reduction in Short-Distance Quantum Gravity?

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"The Planck Scale"
Wroclaw, June 2009

Accumulating bits of evidence that quantum gravity simplifies at short distances

- Causal dynamical triangulations
- Exact renormalization group/asymptotic safety
- Loop quantum gravity area spectrum
- Anisotropic scaling models (Hořava)

$\boxed{\text{Are these hints telling us something important?}}$

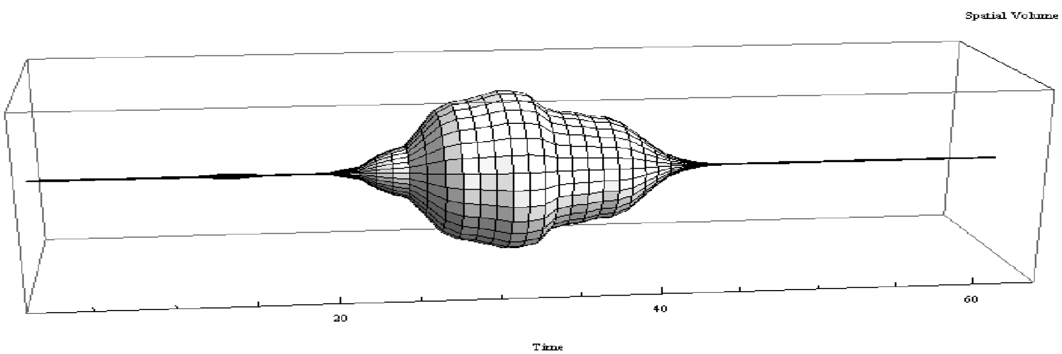
Causal dynamical triangulations

Approximate path integral by sum over discrete triangulated manifolds

$$\int [dg] e^{iI_{EH}[g]} \Rightarrow \sum e^{iI_{Regge}[\Delta]}$$

New ingredient (Ambjørn, Jurkiewicz, Loll):

– fixed time slicing, no topology change/baby universes



How do you “measure” the dimension of spacetime?

Spectral dimension d_S: dimension of spacetime seen by random walker

Heat kernel $K(x,x';s)$: $\left(\frac{\partial}{\partial s} - \Delta_x\right) K(x,x';s) = 0$

$$K(x,x';s) \sim (4\pi s)^{-d_S/2} e^{-\sigma(x,x')/2s} (1 + \dots)$$
$$K(x,x;s) \sim (4\pi s)^{-d_S/2}$$

Ambjørn, Jurkiewicz, and Loll:

- $d_S(\sigma \gg 0) = 4$,
- $d_S(\sigma \to 0) \approx 2$

Propagator $G(x,x') \sim \int_0^\infty ds\, K(x,x';s) \sim \begin{cases} \sigma^{-1}(x,x') & \sigma \text{ large} \\ \log|\sigma(x,x')| & \sigma \text{ small} \end{cases}$

Short distances: characteristic behavior of a propagator in two dimensions

Renormalization group

Lauscher, Reuter, Niedermaier, etc.:

Look at renormalization group flow for Einstein gravity plus higher derivative terms

- Define scale-dependent effective action (integrate out momenta $> k$)
- Truncate: keep only finitely many terms
- Evaluate functional ("exact") renormalization group flow
- Find evidence for non-Gaussian fixed point, "asymptotic safety"

At fixed point:

- anomalous dimensions $\Leftrightarrow$ two-dimensional field theory
- propagators $\sim \log|x - x'|$
- spectral dimension $d_S \sim 2$

General argument: non-Gaussian fixed point $\Rightarrow$ large anomalous dimensions
$\Rightarrow$ 2-d propagators

Loop quantum gravity

Area spectrum $A \sim \sqrt{\ell_j^2(\ell_j^2+\ell_p^2)}$ with $\ell_j = \sqrt{j}\ell_p$

– large areas: $A \sim \ell_j^2$

– small areas: $A \sim \ell_j \ell_p$

Modesto: assume metric scales as areas under $\ell \longrightarrow \lambda\ell$;

then reproduce causal dynamical triangulations result for spectral dimension

Anisotropic scaling models

Hořava: new class of renormalizable (but not diffeo-invariant) models of gravity:

– short distances: nonrelativistic gravitons (broken Lorentz symmetry)

– large distances: approaches Einstein gravity

Anisotropic scaling ($\mathrm{x} \longrightarrow b\mathrm{x},\ t \longrightarrow b^3 t$)

Then diffusion described by operator $\sim \Delta^2 \Rightarrow d_S = 2$ at short distances

Short distance approximation

Wheeler-DeWitt equation:

$$\left\{16\pi\ell_p^2 G_{ijkl}\frac{\delta}{\delta g_{ij}}\frac{\delta}{\delta g_{kl}} - \frac{1}{16\pi\ell_p^2}\sqrt{g}\,{}^{(3)}R\right\}\Psi[g] = 0$$

"strong coupling" ($G \longrightarrow \infty$) $\Leftrightarrow$ "small distance" ($\ell_p \longrightarrow \infty$)

$\Leftrightarrow$ "ultralocal" (no spatial derivatives)

Classical solution:

– Kasner at each point if $\ell_p \longrightarrow \infty$

– normally BKL/Mixmaster if ℓ_p large but finite

(Kasner eras with bounces in which axes change)

Any signs of "dimensional reduction"?

Which dimension is picked out?

Geodesics in Kasner

$$ds^2 = dt^2 - t^{2p_1}dx^2 - t^{2p_2}dy^2 - t^{2p_3}dz^2$$
$$\textit{with } -\frac{1}{3} < p_1 < 0 < p_2 < p_3, \quad p_1 + p_2 + p_3 = 1 = p_1^2 + p_2^2 + p_3^2$$

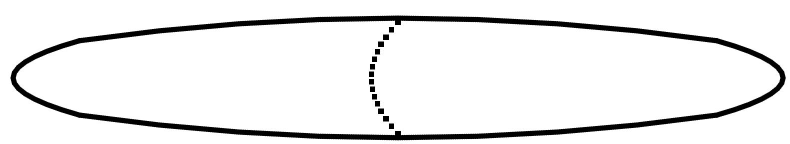
Start timelike geodesic at $t = t_0$, $\mathrm{x} = 0$ with random initial velocity

Look at proper distance along each axis at time t:

small t: $\begin{cases} s_x = t^{p_1}x \sim t^{p_1} \\ s_y = t^{p_2}y \sim 0 \\ s_z = t^{p_3}z \sim 0 \end{cases}$ large t: $\begin{cases} s_x = t^{p_1}x \sim t \\ s_y = t^{p_2}y \sim t^{\max(p_2, 1+p_1-p_2)} \\ s_z = t^{p_3}z \sim t^{p_3} \end{cases}$

Geodesics explore a nearly one-dimensional space. . .

Particle horizon approaches line as $t \to 0$



Heat kernel for Kasner space

Various approximations (Futamase, Berkin):

$$K(x,x;s) \sim \frac{1}{(4\pi s)^2}(1+Qs) \quad \text{with } Q \sim \frac{1}{t^2}$$

Small t: Q term dominates, $d_S \sim 2$

HaMiDeW coefficients:

$$K(x,x;s) \sim \frac{1}{(4\pi s)^2}(1+[a_1]s+[a_2]s^2+\dots)$$

$[a_1]$ gives $\ln \sigma(x,x')$ term in propagator

Asymptotic silence?

Cosmology near generic spacelike singularity:

- Asymptotic silence: light cones shrink to timelike lines
- asymptotic locality: inhomogeneities fall outside shrinking horizons faster than they grow

$\Rightarrow$ spatial points decouple; BKL behavior

Underlying physics: extreme focusing near initial singularity

Is this also true at very short distances?

Raychaudhuri equation:

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma_\alpha{}^\beta \sigma_\beta{}^\alpha + \omega_{\alpha\beta}\omega^{\alpha\beta} - R_{\alpha\beta}k^\alpha k^\beta$$

Quantum fluctuations:

$$\langle\theta^2\rangle = \langle\theta\rangle^2 + (\Delta\theta)^2$$
$$\langle\sigma^2\rangle = \langle\sigma\rangle^2 + (\Delta\sigma)^2$$

Fluctuations focus null geodesics: large effect at Planck scale?

Note that θ is conjugate to area A: $\Delta\theta\Delta A \sim \ell_p$

$\Delta A \sim \ell_p^2$ at short distances $\Rightarrow \Delta\theta \sim 1/\ell_p$

Some further hints from renormalization group analysis:

$R \sim \ell_p^{-2}$ near Planck scale

Does spacetime foam focus geodesics?

Short-distance picture:

- short distance asymptotic silence
- "random" direction at each point in space
  - not changing *too* rapidly in space, but regions of size $\gg \ell_p$ fairly independent
  - evolving in time; "bouncing," axes rotating, etc.
- effective two-dimensional behavior:

  dynamics concentrated along preferred direction
- space threaded by lines; spacetime foliated by not-very-smooth two-surfaces
- product wave function "$\prod_x \Psi[a, \beta]$"

Can we use this?

't Hooft, Verlinde and Verlinde, Kabat and Ortiz: eikonal approximation

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta + h_{ij} dy^i dy^j$$

with different natural scales for the two metrics