cosmology within the noncommutative approach to the standard model of particle physics

the planck scale meeting

wroclaw, 29 june- 4 july 2009

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outline

- motivation

  cosmology

  particle physics
- NonCommutative Geometry (NCG)
- success of the NCG approach to the standard model
- cosmological consequences

  noncommutative corrections to Einstein's equations
  nelson, sakellariadou arXiv:0812.1657

  inflation through the Higgs field
  nelson, sakellariadou arXiv:0903.1520
- conclusions

motivation

cosmology

EU cosmological models can be tested with many very accurate astrophysical data, while high energy experiments (LHC) will test some of the theoretical pillars of these models

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despite the golden era of cosmology, a number of questions:

- origin of DE / DM
- search for natural and well-motivated inflationary model

...

are still awaiting for a definite answer main theoretical approaches upon which cosmological models have been built:

- string theory
- loop quantum gravity

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- string theory
- loop quantum gravity

- Noncommutative geometry

NCG approach to the Standard Model (SM), leading to all detailed structure of SM and implying physical predictions at unification scale

chamseddine, connes, marcolli 2007

particle physics

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$$S_{\text{Einstein–Hilbert}} + S_{\text{Standard Model}}$$

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the difference between these two kinds of symmetries is responsible for not finding a unified theory of all interactions including gravity in addition:

- why the gauge group G is specifically $U(1) \times SU(2) \times SU(3)$?
- why the fermions occupy the particular representations they do?
- why there are three families and why there are 16 fundamental fermions per family?
- what is the theoretical origin of the Higgs mechanism and spontaneous breakdown of gauge symmetries?
- what is the Higgs mass and how to explain all the fermionic masses?

...

to be answered by the ultimate unified theory of all interactions

noncommutative geometry

NCG approach

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as energies approach Planck scale, the quantum nature of ST reveals itself, and $S_{\rm Einstein-Hilbert}$ becomes an approximation

in addition, all forces (including gravity) are unified, so that all interactions correspond to one underlying symmetry

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indirect approach: *search for hidden structure in the functional of gravity coupled to SM of particle physics at present energies*

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self-adjoint operator in $\mathcal{H}$ so that all commutators $[D, a]$ are bounded for $a \in \mathcal{A}$ (inverse of line element)

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a geometry of such a nontrivial noncommutative ST has not yet been considered

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algebra of quaternions

algebra of complex $k \times k$ matrices

quaternion: an element $(a, b, c, d) \in \mathbb{R}^4$

$(\mathbb{H}, +)$ is a commutative group, but $(\mathbb{H}, +, \times)$ is noncommutative

any quaternion can be written as a linear combination of elements of the basis $1, i, j, k$ as $a \cdot 1 + b \cdot i + c \cdot j + d \cdot k$ with a, b, c, d reals

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$k = 4$ is the first value that produces the correct number of fermions in each generation; $k^2 = 16$ in each of 3 generations

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the action functional depends only on the spectrum of the Dirac operator and is of the form:

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test function

fixes the energy scale

f plays a role through its momenta f_0, f_2, f_4

$f_k = \int_0^\infty f(v) v^{k-1} dv$ for $k > 0$ and $f_0 = f(0)$

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these 3 additional real parameters are physically related to the coupling constants at unification, the gravitational constant, and the cosmological constant

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spectral action principal

the action functional depends only on the spectrum of the Dirac operator and is of the form:

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it only accounts for the bosonic part of the model

test function

fixes the energy scale

these 3 additional real parameters are physically related to the coupling constants at unification, the gravitational constant, and the cosmological constant in addition, the empirical data taken as input are:

- there are 16 chiral fermions in each of 3 generations
- the photon is massless
- there are Majorana mass terms for the neutrinos

the full Lagrangian of the SM, minimally coupled to gravity, is obtained as the asymptotic expansion of the spectral action for the product ST:

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$$\mathcal{L}_{SM} = -\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc}f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu -$$
$$\frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu \partial_\nu W^-_\mu -$$
$$W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - igs_w(\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu \partial_\nu W^-_\mu -$$
$$W^-_\mu \partial_\nu W^+_\mu) + A_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \frac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu +$$
$$g^2 c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w(A_\mu Z^0_\nu(W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \frac{1}{2}\partial_\mu H \partial_\mu H - 2M^2\alpha_h H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - \frac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 -$$
$$\beta_h\left(\frac{2M^2}{g^2} + \frac{2M}{g}H + \frac{1}{2}(H^2+\phi^0\phi^0+2\phi^+\phi^-)\right) + \frac{2M^4}{g^2}\alpha_h - g\alpha_h M\left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) -$$
$$\frac{1}{8}g^2\alpha_h\left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2\right) - gMW^+_\mu W^-_\mu H -$$
$$\frac{1}{2}g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \frac{1}{2}ig\left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) +$$
$$\frac{1}{2}g\left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \frac{1}{2}g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) +$$
$$M\,(\frac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_wMA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) -$$
$$ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4}g^2W^+_\mu W^-_\mu\left(H^2+(\phi^0)^2+2\phi^+\phi^-\right) -$$
$$\frac{1}{8}g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu\left(H^2+(\phi^0)^2+2(2s_w^2-1)^2\phi^+\phi^-\right) - \frac{1}{2}g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) -$$
$$\frac{1}{2}ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac{1}{2}g^2s_wA_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) + \frac{1}{2}ig^2s_wA_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) -$$
$$g^2\frac{s_w}{c_w}(2c_w^2-1)Z^0_\mu A_\mu\phi^+\phi^- - g^2s_w^2A_\mu A_\mu\phi^+\phi^- + \frac{1}{2}ig_s\,\lambda^a_{ij}(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar{e}^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar{\nu}^\lambda(\gamma\partial +$$
$$m^\lambda_\nu)\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial+m^\lambda_u)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial+m^\lambda_d)d^\lambda_j + igs_wA_\mu\left(-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \frac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)\right) +$$
$$\frac{ig}{4c_w}Z^0_\mu\{(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2-1-\gamma^5)e^\lambda) + (\bar{d}^\lambda_j\gamma^\mu(\frac{4}{3}s_w^2-1-\gamma^5)d^\lambda_j) + (\bar{u}^\lambda_j\gamma^\mu(1-\frac{8}{3}s_w^2+$$
$$\gamma^5)u^\lambda_j)\} + \frac{ig}{2\sqrt{2}}W^+_\mu\left((\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)U^{lep}{}_{\lambda\kappa}e^\kappa) + (\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) +$$
$$\frac{ig}{2\sqrt{2}}W^-_\mu\left((\bar{e}^\kappa U^{lep\dagger}{}_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) +$$
$$\frac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_e(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1+\gamma^5)e^\kappa\right) +$$
$$\frac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_e(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1-\gamma^5)\nu^\kappa\right) - \frac{g}{2}\frac{m^\lambda_\nu}{M}H(\bar{\nu}^\lambda\nu^\lambda) - \frac{g}{2}\frac{m^\lambda_e}{M}H(\bar{e}^\lambda e^\lambda) +$$
$$\frac{ig}{2}\frac{m^\lambda_\nu}{M}\phi^0(\bar{\nu}^\lambda\gamma^5\nu^\lambda) - \frac{ig}{2}\frac{m^\lambda_e}{M}\phi^0(\bar{e}^\lambda\gamma^5e^\lambda) - \frac{1}{4}\,\bar{\nu}_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat{\nu}_\kappa - \frac{1}{4}\,\overline{\bar{\nu}_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat{\nu}_\kappa} +$$
$$\frac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_d(\bar{u}^\lambda_jC_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar{u}^\lambda_jC_{\lambda\kappa}(1+\gamma^5)d^\kappa_j\right) +$$
$$\frac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_d(\bar{d}^\lambda_jC^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar{d}^\lambda_jC^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j\right) - \frac{g}{2}\frac{m^\lambda_u}{M}H(\bar{u}^\lambda_ju^\lambda_j) - \frac{g}{2}\frac{m^\lambda_d}{M}H(\bar{d}^\lambda_jd^\lambda_j) +$$
$$\frac{ig}{2}\frac{m^\lambda_u}{M}\phi^0(\bar{u}^\lambda_j\gamma^5u^\lambda_j) - \frac{ig}{2}\frac{m^\lambda_d}{M}\phi^0(\bar{d}^\lambda_j\gamma^5d^\lambda_j)$$

phenomenology

relations between gauge coupling constants:

$$g_2^2 = g_3^2 = \frac{5}{3} g_1^2$$

coincide with those obtained in GUTs

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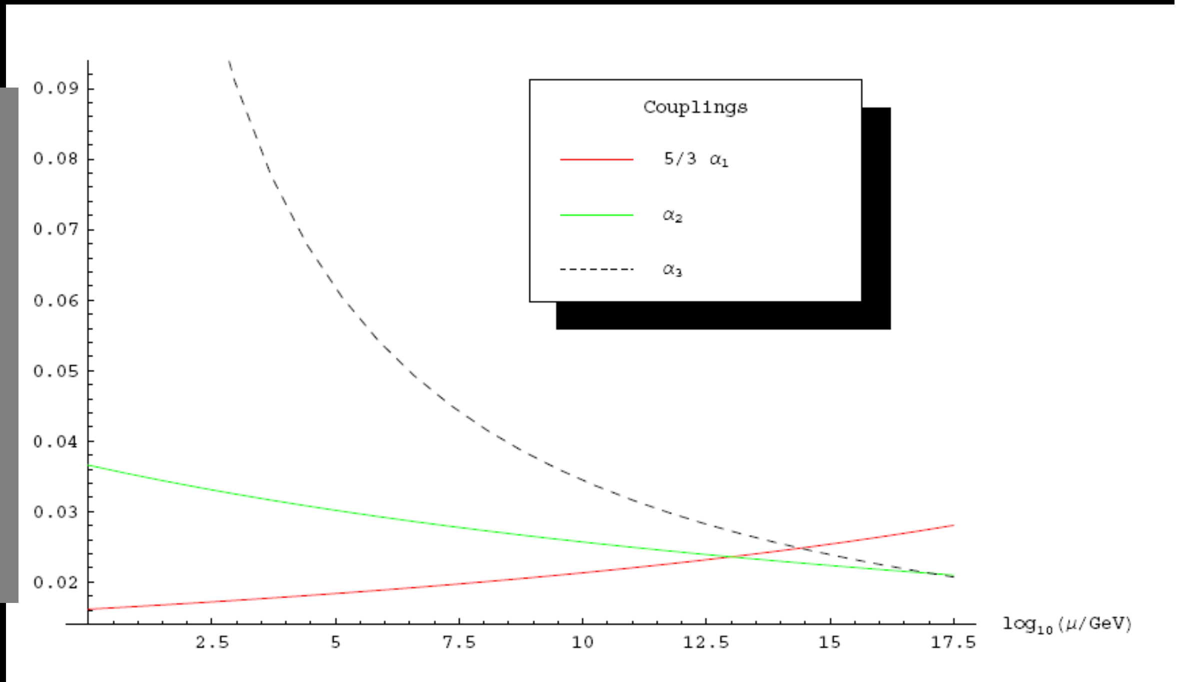
$$\alpha_i = \frac{g_i^2}{4\pi}$$

the graphs of the running of the three constants α_i do not meet exactly, so they do not specify a unique unification energy



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correct representations of the fermions with respect to SU(3)xSU(2)xU(1) are derived

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- no explanation of the number of generations
- no constraints on values of the Yukawa couplings

speculations on the spectrum of the noncommuative space on QG

the small deviation from experimental results of the predictions of the SM derived from spectral action is an indication that the assumption that ST is a product of a continuous 4dim manifold times a discrete space breaks down at energies just below unification scale

at Planckian energies, the structure of ST becomes noncommutative in a nontrivial way, which will change in an intrinsic way the particle spectrum

next steps

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- find the noncommutative space whose limit is the product $\mathcal{M}_4 \times \mathcal{F}$

cosmological consequences

corrections to Einstein's equations

$$\mathcal{L}_{SM} = -\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc}f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu -$$

$$\frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu -$$

$$W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - igs_w(\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu (W^+_\mu \partial_\nu W^-_\mu -$$

$$W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \frac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac{1}{2} g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu +$$

$$g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w (A_\mu Z^0_\nu (W^+_\mu W^-_\nu -$$

$$W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \frac{1}{2}\partial_\mu H \partial_\mu H - 2M^2\alpha_h H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - \frac{1}{2} \partial_\mu\phi^0\partial_\mu\phi^0 -$$

$$\beta_h \left(\frac{2M^2}{g^2} + \frac{2M}{g}H + \frac{1}{2}(H^2+\phi^0\phi^0+2\phi^+\phi^-)\right) + \frac{2M^4}{g^2}\alpha_h - g\alpha_h M \left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) -$$

$$\frac{1}{8} g^2\alpha_h \left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2\right) - gMW^+_\mu W^-_\mu H -$$

$$\frac{1}{2} g\frac{M}{c_w^2} Z^0_\mu Z^0_\mu H - \frac{1}{2} ig \left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) +$$

$$\frac{1}{2} g \left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \frac{1}{2} g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) +$$

$$M\,(\frac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_w MA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) -$$

$$ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4} g^2W^+_\mu W^-_\mu \left(H^2+(\phi^0)^2+2\phi^+\phi^-\right) -$$

$$\frac{1}{8} g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu \left(H^2+(\phi^0)^2+2(2s_w^2-1)^2\phi^+\phi^-\right) - \frac{1}{2} g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) -$$

$$\frac{1}{2} ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac{1}{2} g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) + \frac{1}{2} ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) -$$

$$g^2\frac{s_w}{c_w}(2c_w^2-1)Z^0_\mu A_\mu\phi^+\phi^- - g^2 s_w^2 A_\mu A_\mu\phi^+\phi^- + \frac{1}{2} ig_s\,\lambda^a_{ij}(\bar q^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar e^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar\nu^\lambda(\gamma\partial +$$

$$m^\lambda_\nu)\nu^\lambda - \bar u^\lambda_j(\gamma\partial + m^\lambda_u)u^\lambda_j - \bar d^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + igs_wA_\mu \left(-(\bar e^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar u^\lambda_j\gamma^\mu u^\lambda_j) - \frac{1}{3}(\bar d^\lambda_j\gamma^\mu d^\lambda_j)\right) +$$

$$\frac{ig}{4c_w}Z^0_\mu\{(\bar\nu^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar e^\lambda\gamma^\mu(4s_w^2-1-\gamma^5)e^\lambda) + (\bar d^\lambda_j\gamma^\mu(\frac{4}{3} s_w^2 - 1 - \gamma^5)d^\lambda_j) + (\bar u^\lambda_j\gamma^\mu(1-\frac{8}{3} s_w^2 +$$

$$\gamma^5)u^\lambda_j)\} + \frac{ig}{2\sqrt{2}}W^+_\mu \left((\bar\nu^\lambda\gamma^\mu(1+\gamma^5)U^{lep}{}_{\lambda\kappa}e^\kappa) + (\bar u^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) +$$

$$\frac{ig}{2\sqrt{2}}W^-_\mu\left((\bar e^\kappa U^{lep\dagger}{}_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar d^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) +$$

$$\frac{ig}{2M\sqrt{2}}\phi^+ \left(-m^\kappa_e(\bar\nu^\lambda U^{lep}{}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar\nu^\lambda U^{lep}{}_{\lambda\kappa}(1+\gamma^5)e^\kappa\right) +$$

$$\frac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_e(\bar e^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar e^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1-\gamma^5)\nu^\kappa\right) - \frac{g}{2}\frac{m^\lambda_\nu}{M}H(\bar\nu^\lambda\nu^\lambda) - \frac{g}{2}\frac{m^\lambda_e}{M}H(\bar e^\lambda e^\lambda) +$$

$$\frac{ig}{2}\frac{m^\lambda_\nu}{M}\phi^0(\bar\nu^\lambda\gamma^5\nu^\lambda) - \frac{ig}{2}\frac{m^\lambda_e}{M}\phi^0(\bar e^\lambda\gamma^5 e^\lambda) - \frac{1}{4}\,\bar\nu_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat\nu_\kappa - \frac{1}{4}\,\overline{\bar\nu_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat\nu_\kappa} +$$

$$\frac{ig}{2M\sqrt{2}}\phi^+ \left(-m^\kappa_d(\bar u^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar u^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j\right) +$$

$$\frac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_d(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j\right) - \frac{g}{2}\frac{m^\lambda_u}{M}H(\bar u^\lambda_j u^\lambda_j) - \frac{g}{2}\frac{m^\lambda_d}{M}H(\bar d^\lambda_j d^\lambda_j) +$$

$$\frac{ig}{2}\frac{m^\lambda_u}{M}\phi^0(\bar u^\lambda_j\gamma^5u^\lambda_j) - \frac{ig}{2}\frac{m^\lambda_d}{M}\phi^0(\bar d^\lambda_j\gamma^5 d^\lambda_j)$$

$$\mathcal{S}_{\rm grav} = \int \left(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \tau_0 R^\star R^\star \ - \xi_0 R |\mathbf{H}|^2 \right) \sqrt{g} \mathrm{d}^4 x$$

$g_{\mu\nu}$ the Riemannian metric

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Riemannian curvature term with a contribution from the Weyl curvature

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the action for conformal gravity; the presence of the EH term (and of cosmological constant) explicitly breaks conformal invariance

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Rhiemannian curvature term with a contribution from the Weyl curvature

the action for conformal gravity; the presence of the EH term (and of cosmological constant) explicitly breaks conformal invariance

topological term

$$R^\star R^\star = \frac{1}{4}\epsilon^{\mu\nu\rho\sigma}\epsilon_{\alpha\beta\gamma\delta} R^{\alpha\beta}_{\mu\nu} R^{\gamma\delta}_{\rho\sigma}$$

hence in nondynamical

$g_{\mu\nu}$ the Riemannian metric

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scalar mass term

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$g_{\mu\nu}$ the Riemannian metric

a rescaling $\mathbf{H} = (\sqrt{a f_0}/\pi)\phi$ of the Higgs field ϕ to normalise the kinetic energy

$$\mathcal{S}_{\rm grav} = \int \left(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \tau_0 R^\star R^\star - \xi_0 R|\mathbf{H}|^2 \right) \sqrt{g}\, d^4x$$

$$\frac{1}{\kappa_0^2} = \frac{96 f_2 \Lambda^2 - f_0 c^2}{12\pi^2}$$

$$\alpha_0 = \frac{-3 f_0}{10\pi^2}$$

$$\tau_0 = \frac{11 f_0}{60\pi^2}$$

$$\xi_0 = \frac{1}{12}$$

Λ is the renormalisation cut-off

c is expressed as $c = \mathrm{Tr}(Y_R^\star Y_R)$ which gives the Majorana mass matrix

Y's are used to classify the action of the Dirac operator and give the fermion and lepton masses, as well as lepton mixing e.o.m.

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \alpha_0 \kappa_0^2 \delta(\Lambda) \left[2C^{\mu\lambda\nu\kappa}{}_{;\lambda;\kappa} - C^{\mu\lambda\nu\kappa} R_{\lambda\kappa} \right]$$

$$= \kappa_0^2 \delta(\Lambda) T^{\mu\nu}_{\text{matter}}$$

where

$$\delta(\Lambda) \equiv [1 - 2\kappa_0^2 \xi_0 |\mathbf{H}|^2]^{-1}$$

neglecting the nonminimal coupling between the geometry and the Higgs field, i.e. setting $\phi = 0$ leads to

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \alpha_0 \kappa_0^2 \left[2C^{\mu\lambda\nu\kappa}_{;\lambda;\kappa} - C^{\mu\lambda\nu\kappa} R_{\lambda\kappa} \right] = \kappa_0^2 T^{\mu\nu}_{\text{matter}}$$

$\delta(\Lambda) = 1$

for a general ST with zero spatial curvature and zero cosmological constant, the 4dim metric in conformal time t and Cartesian spatial coordinates (x, y, z)

$$g_{\mu\nu} = \mathrm{diag}\left(\{a(t)\}^2 \left[-(1+\phi(x)), (1+\psi(x)), (1+\psi\,(x)), (1+\psi(x))\right]\right)$$

for a general ST with zero spatial curvature and zero cosmological constant, the 4dim metric in conformal time t and Cartesian spatial coordinates (x, y, z)

$$g_{\mu\nu} = \mathrm{diag}\left(\{a(t)\}^2\left[-(1+\phi(x)),\right.\right.$$
$$\left.\left.(1+\psi(x)),(1+\psi\,(x)),(1+\psi(x))\right]\right)$$

modified Friedmann eq.:

$$-3\left(\frac{\dot{a}}{a}\right)^2 + \left[\nabla^2 - 3\left(\frac{\dot{a}}{a}\right)\right]\psi\,(x)$$
$$+\frac{\alpha_0\kappa_0^2}{3a^2}\nabla^4\left[\psi\,(x) - \phi\,(x)\right] + \mathcal{O}\left(\psi^2, \phi^2, \ldots\right) = \kappa_0^2 T_{00}$$

homogeneous and isotropic case:

$$\phi(x) = \psi(x) = 0$$

⇒ Friedmann eq. reduces to its standard form

any effects of noncommutativity of ST coordinates must disappear in a homogeneous and isotropic ST, all points being equivalent homogeneous and isotropic case:

$$\phi(x) = \psi(x) = 0$$

⇒ Friedmann eq. reduces to its standard form

any effects of noncommutativity of ST coordinates must disappear in a homogeneous and isotropic ST, all points being equivalent

any corrections to the standard cosmological model, due to noncommutative effects, will not occur at the level of the background

nelson, sakellariadou 2008

4dim metric in synchronous gauge:

$$g_{\mu\nu} = \mathrm{diag}\left(\{a(t)\}^2\left[-1, \left(\delta_{ij} + h_{ij}\left(x\right)\right)\right]\right)$$

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$$g_{\mu\nu} = \mathrm{diag}\left(\{a(t)\}^2\left[-1, \left(\delta_{ij} + h_{ij}\left(x\right)\right)\right]\right)$$

modified Friedmann eq.:

$$\begin{aligned}
&-3\left(\frac{\dot a}{a}\right)^2 + \frac{1}{2}\left[4\left(\frac{\dot a}{a}\right)\dot h + 2\ddot h - \nabla^2 h + \nabla_i\nabla_j h^{ij}\right]\\
&-\frac{\alpha_0\kappa_0^2}{6a^2}\left[\partial_t^2\left(\nabla^2 h - 3\nabla_i\nabla_j h^{ij}\right) + \nabla^2\left(\nabla_i\nabla_j h^{ij}\right) - \nabla^4 h\right]\\
&+\mathcal{O}\left(h^2\right) \quad = \quad \kappa_0^2 T_{00}
\end{aligned}$$

$$h \equiv h^i_i$$

for GW (transverse, traceless part of perturbed metric):

$$-3\left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{2}\left[4\left(\frac{\dot{a}}{a}\right)\dot{h} + 2\ddot{h}\right] = \kappa_0^2 T_{00}$$

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noncommutative corrections to Einstein's eqs. do not alter the propagation of gravitational waves

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the corrections to EIsntein's eqs. will be apparent at leading order, only in the case of anisotropic models the corrections to Eisntein's eqs. will be apparent at leading order, only in the case of anisotropic models

Bianchi V

integer

$$g_{\mu\nu} = \mathrm{diag}\left[-1, \{a_1(t)\}^2 e^{-2nz}, \{a_2(t)\}^2 e^{-2nz}, \{a_3(t)\}^2\right]$$

arbitrary functions

$$A_i(t) = \ln a_i(t)$$

$$\begin{aligned}
&\kappa_0^2 T_{00} = \\
&-\dot{A}_3\left(\dot{A}_1+\dot{A}_2\right) - n^2 e^{-2A_3}\left(\dot{A}_1\dot{A}_2 - 3\right) \\
&+\frac{8\alpha_0\kappa_0^2 n^2}{3} e^{-2A_3}\Big[5\left(\dot{A}_1\right)^2 + 5\left(\dot{A}_2\right)^2 - \left(\dot{A}_3\right)^2 \\
&-\dot{A}_1\dot{A}_2 - \dot{A}_2\dot{A}_3 - \dot{A}_3\dot{A}_1 - \ddot{A}_1 - \ddot{A}_2 - \ddot{A}_3 + 3\Big] \\
&-\frac{4\alpha_0\kappa_0^2}{3}\sum_i \Big\{\dot{A}_1\dot{A}_2\dot{A}_3\dot{A}_i \\
&+\dot{A}_i\dot{A}_{i+1}\left(\left(\dot{A}_i - \dot{A}_{i+1}\right)^2 - \dot{A}_i\dot{A}_{i+1}\right) \\
&+\left(\ddot{A}_i + \left(\dot{A}_i\right)^2\right)\Big[-\ddot{A}_i - \left(\dot{A}_i\right)^2 + \frac{1}{2}\left(\ddot{A}_{i+1} + \ddot{A}_{i+2}\right) \\
&+\frac{1}{2}\left(\left(\dot{A}_{i+1}\right)^2 + \left(\dot{A}_{i+2}\right)^2\right)\Big] \\
&+\Big[\dddot{A}_i + 3\dot{A}_i\ddot{A}_i - \left(\ddot{A}_i + \left(\dot{A}_i\right)^2\right)\left(\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}\right)\Big] \\
&\times\left[2\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}\right]\Big\}
\end{aligned}$$

$$\kappa_0^2 T_{00} =$$

$$-\dot{A}_3\left(\dot{A}_1+\dot{A}_2\right) - n^2 e^{-2A_3}\left(\dot{A}_1\dot{A}_2 - 3\right)$$

$$+\frac{8\alpha_0\kappa_0^2 n^2}{3} e^{-2A_3}\left[5\left(\dot{A}_1\right)^2 + 5\left(\dot{A}_2\right)^2 - \left(\dot{A}_3\right)^2\right.$$

$$\left. -\dot{A}_1\dot{A}_2 - \dot{A}_2\dot{A}_3 - \dot{A}_3\dot{A}_1 - \ddot{A}_1 - \ddot{A}_2 - \ddot{A}_3 + 3\right]$$

$$-\frac{4\alpha_0\kappa_0^2}{3}\sum_i\left\{\dot{A}_1\dot{A}_2\dot{A}_3\dot{A}_i\right.$$

$$+\dot{A}_i\dot{A}_{i+1}\left(\left(\dot{A}_i - \dot{A}_{i+1}\right)^2 - \dot{A}_i\dot{A}_{i+1}\right)$$

$$+\left(\ddot{A}_i + \left(\dot{A}_i\right)^2\right)\left[-\ddot{A}_i - \left(\dot{A}_i\right)^2 + \frac{1}{2}\left(\ddot{A}_{i+1} + \ddot{A}_{i+2}\right)\right.$$

$$\left. +\frac{1}{2}\left(\left(\dot{A}_{i+1}\right)^2 + \left(\dot{A}_{i+2}\right)^2\right)\right]$$

$$+\left[\dddot{A}_i + 3\dot{A}_i\ddot{A}_i - \left(\ddot{A}_i + \left(\dot{A}_i\right)^2\right)\left(\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}\right)\right]$$

$$\left. \times\left[2\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}\right]\right\}$$

$$A_i(t) = \ln a_i(t)$$

for slowly varying functions: small corrections

$$\kappa_0^2 T_{00} =$$

$$-\dot{A}_3\left(\dot{A}_1+\dot{A}_2\right) - n^2 e^{-2A_3}\left(\dot{A}_1\dot{A}_2 - 3\right)$$

$$+\frac{8\alpha_0\kappa_0^2 n^2}{3} e^{-2A_3}\left[5\left(\dot{A}_1\right)^2 + 5\left(\dot{A}_2\right)^2 - \left(\dot{A}_3\right)^2 - \dot{A}_1\dot{A}_2 - \dot{A}_2\dot{A}_3 - \dot{A}_3\dot{A}_1 - \ddot{A}_1 - \ddot{A}_2 - \ddot{A}_3 + 3\right]$$

$$-\frac{4\alpha_0\kappa_0^2}{3}\sum_i\Bigg\{\dot{A}_1\dot{A}_2\dot{A}_3\dot{A}_i + \dot{A}_i\dot{A}_{i+1}\left(\left(\dot{A}_i-\dot{A}_{i+1}\right)^2 - \dot{A}_i\dot{A}_{i+1}\right)$$

$$+\left(\ddot{A}_i+\left(\dot{A}_i\right)^2\right)\left[-\ddot{A}_i - \left(\dot{A}_i\right)^2 + \frac{1}{2}\left(\ddot{A}_{i+1}+\ddot{A}_{i+2}\right) + \frac{1}{2}\left(\left(\dot{A}_{i+1}\right)^2 + \left(\dot{A}_{i+2}\right)^2\right)\right]$$

$$+\left[\dddot{A}_i + 3\dot{A}_i\ddot{A}_i - \left(\ddot{A}_i + \left(\dot{A}_i\right)^2\right)\left(\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}\right)\right] \times \left[2\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}\right]\Bigg\}$$

at the same order as standard EH term, but $\propto n^2$ so it vanishes for homogeneous types of Bianchi V

$$A_i(t) = \ln a_i(t)$$

for slowly varying functions: small corrections neglecting the nonminimal coupling between geometry and Higgs field, the noncommutative corrections to Einstein's eqs. are present only in inhomogeneous and anisotropic space-times at energies approaching Higgs scale, the nonminimal coupling of the Higgs field to the curvature cannot be neglected

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \alpha_0 \kappa_0^2 \delta(\Lambda) \left[2C^{\mu\lambda\nu\kappa}{}_{;\lambda;\kappa} - C^{\mu\lambda\nu\kappa} R_{\lambda\kappa} \right] = \kappa_0^2 \delta(\Lambda) T^{\mu\nu}_{\mathrm{matter}}$$

where $$\delta(\Lambda) \equiv [1 - 2\kappa_0^2 \xi_0 |\mathbf{H}|^2]^{-1}$$

for $|\mathbf{H}| \to \sqrt{\mathbf{6}}/\kappa_{\mathbf{0}}$ the correction term dominates

there are corrections even for background geometries

to understand the effects of these corrections, neglect the conformal term, setting $\alpha_0 = 0$

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e.o.m.

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \kappa_0^2 \left[\frac{1}{1 - \kappa_0^2 |\mathbf{H}|^2 / 6} \right] T^{\mu\nu}_{\text{matter}}$$

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$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \kappa_0^2 \left[\frac{1}{1 - \kappa_0^2 |\mathbf{H}|^2/6} \right] T^{\mu\nu}_{\mathrm{matter}}$$

the effect of a nonzero Higgs field is to create an effective gravitational constant

inflation trough the nonminimal coupling between the geometry and the Higgs field

proposal: the scalar field of the SM, the Higgs field, could play the role of the inflaton

but

in the context of the general relativistic cosmology, to get the correct amplitude of density perturbations, the Higgs mass would have to be some 11 orders of magnitude higher than the one required by particle physics proposal: the scalar field of the SM, the Higgs field, could play the role of the inflaton

but

in the context of the general relativistic cosmology, to get the correct amplitude of density perturbations, the Higgs mass would have to be some 11 orders of magnitude higher than the one required by particle physics

re-examine the validity of this statement within cosmological noncommutative geometry study the nonminimal coupling of the geometry to the Higgs field, w.r.t. the possibility of having naturally an inflationary scenario driven by the Higgs field

$$\mathcal{S}_{\rm grav} = \int \Big(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \tau_0 R^\star R^\star$$
$$+\gamma_0 - \xi_0 R|\mathbf{H}|^2 + \frac{1}{2}|D_\mu \mathbf{H}|^2 + V(|\mathbf{H}|) \Big) \sqrt{g}\, \mathrm{d}^4 x$$

Λ^4 -terms

$$V(|\mathbf{H}|) = \lambda_0 |\mathbf{H}|^4 - \mu_0^2 |\mathbf{H}|^2$$

$$\gamma_0 = \frac{1}{\pi^2}(48 f_4 \Lambda^4 - f_2 \Lambda^2 c + \frac{f_0}{4} d)$$

$$\lambda_0 = \frac{\pi^2}{2 f_0} \frac{b}{a^2}$$

$$\mu_0^2 = 2\frac{f_2 \Lambda^2}{f_0} - \frac{e}{a}$$

a, b, c, d, e couplings given through Y's study the nonminimal coupling of the geometry to the Higgs field, w.r.t. the possibility of having naturally an inflationary scenario driven by the Higgs field

$$\mathcal{S}_{\rm grav} = \int \left(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \tau_0 R^\star R^\star + \gamma_0 - \xi_0 R |\mathbf{H}|^2 + \frac{1}{2} |D_\mu \mathbf{H}|^2 + V(|\mathbf{H}|) \right) \sqrt{g}\, \mathrm{d}^4 x$$

$$V(|\mathbf{H}|) = \lambda_0 |\mathbf{H}|^4 - \mu_0^2 |\mathbf{H}|^2$$

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$$\gamma_0 = \frac{1}{\pi^2}(48 f_4 \Lambda^4 - f_2 \Lambda^2 c + \frac{f_0}{4} d)$$

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a, b, c, d, e couplings given through Y's

remark:

in the literature such modifications to EH gravity have been considered by postulating the nonminimal coupling

it was shown that the scale that sets the amplitude of perturbations during Higgs inflation is λ_0/ξ_0^2

this reduction in the amplitude of induced perturbations allows the Higgs field to satisfy the requirements of SM and of inflation

bezrukov, shaposnikov 2007
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conformal transformation of the metric:

$$\left(\frac{1}{2\kappa_0^2} - \xi_0 |\mathbf{H}|^2\right) R \ \rightarrow \ -\frac{1}{2\kappa_0^2}\hat{R}$$

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re-definition of the field:

$$|\mathbf{H}| \;\rightarrow\; |\chi|$$

bezrukov, magnin, shaposnikov 2008

conformal transformation of the metric:

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re-definition of the field: $|\mathbf{H}| \rightarrow |\chi|$

Einstein frame action:

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...kov, magnin, shaposnikov 2008

normalising CMB perturbations to WMAP5 data:

requirement so that Higgs field can produce inflation

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standard Higgs inflation has been recently criticised, arguing that quantum corrections to the semi-classical approximation may no longer be neglected for such exotic inflationary models

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this criticism is not applicable to the noncommuative approach

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this argument does not apply in the noncommutative Higgs field driven inflations, since $\xi_0 = 1/12$

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natural inflation may occur as a consequence of a non-minimal coupling between geometry and the Higgs field