The Group Field Theory approach to Quantum Gravity: some recent results

Daniele Oriti

Max Planck Institute for Gravitational Physics (Albert Einstein Institute)
Golm, Germany, EU

XV Max Born Symposium, Wroclaw, 02/07/2009

OUTLINE

- the GFT formalism
  - the field and the GFT kinematics
  - classical GFT dynamics
  - quantum GFT: Feynman diagrams and Feynman amplitudes
  - interpretation and link with other approaches
  - examples: 3d Riemannian quantum gravity
- some recent results
  - perturbative GFT renormalization - the 3d case
  - effective non-commutative matter fields from GFT
  - new GFT models for 4d gravity
- conclusions

The Group Field Theory formalism

L. Freidel, hep-th/0505016; DO, gr-qc/0512103; DO, gr-qc/0607032

A QFT for Quantum Gravity?

- QFT is best formalism we have for quantum microscopic physics and many-particle physics (particle physics, atomic physics, condensed matter,...)
- A QFT on which spacetime?
  - a QFT of gravitons on some background doesn't work
  - QG should explain origin and properties of spacetime itself (geometry *and* topology?)
  - background independence!
  - it can be only[1] be a QFT on some auxiliary, internal or "meta-space"
  - only background structures in GR: Lorentz group, i.e. local gauge group - equivalence principle; space of geometries - superspace; spacetime topology (?)
- a QFT of what? what are the fundamental quanta?
  - gravitons do not work
  - quanta of space itself! fundamental excitations of space around the vacuum (nothing = not even space)
  - we are not starting from scratch: ideas and results from LQG, matrix models, simplicial QG,...

[1] Unless......asymptotic safety....

A QFT FOR QUANTUM GRAVITY?

- QFT is best formalism we have for quantum microscopic physics and many-particle physics (particle physics, atomic physics, condensed matter,...)
- A QFT on which spacetime?
  - a QFT of gravitons on some background doesn't work
  - QG should explain origin and properties of spacetime itself (geometry *and* topology?)
  - background independence!
  - it can be only[1] be a QFT on some auxiliary, internal or "meta-space"
  - only background structures in GR: Lorentz group, i.e. local gauge group - equivalence principle; space of geometries - superspace; spacetime topology (?)
- a QFT of what? what are the fundamental quanta?
  - gravitons do not work
  - quanta of space itself! fundamental excitations of space around the vacuum (nothing = not even space)
  - we are not starting from scratch: ideas and results from LQG, matrix models, simplicial QG,...

[1] Unless......asymptotic safety....

A QFT FOR QUANTUM GRAVITY?

- QFT is best formalism we have for quantum microscopic physics and many-particle physics (particle physics, atomic physics, condensed matter,...)
- A QFT on which spacetime?
  - a QFT of gravitons on some background doesn't work
  - QG should explain origin and properties of spacetime itself (geometry *and* topology?)
  - background independence!
  - it can be only[1] be a QFT on some auxiliary, internal or "meta-space"
  - only background structures in GR: Lorentz group, i.e. local gauge group - equivalence principle; space of geometries - superspace; spacetime topology (?)
- a QFT of what? what are the fundamental quanta?
  - gravitons do not work
  - quanta of space itself! fundamental excitations of space around the vacuum (nothing = not even space)
  - we are not starting from scratch: ideas and results from LQG, matrix models, simplicial QG,...

[1] Unless......asymptotic safety....

A QFT FOR QUANTUM GRAVITY?

- QFT is best formalism we have for quantum microscopic physics and many-particle physics (particle physics, atomic physics, condensed matter,...)
- A QFT on which spacetime?
  - a QFT of gravitons on some background doesn't work
  - QG should explain origin and properties of spacetime itself (geometry *and* topology?)
  - background independence!
  - it can be only[1] be a QFT on some auxiliary, internal or "meta-space"
  - only background structures in GR: Lorentz group, i.e. local gauge group - equivalence principle; space of geometries - superspace; spacetime topology (?)
- a QFT of what? what are the fundamental quanta?
  - gravitons do not work
  - quanta of space itself! fundamental excitations of space around the vacuum (nothing = not even space)
  - we are not starting from scratch: ideas and results from LQG, matrix models, simplicial QG,...

[1] Unless......asymptotic safety....

A QFT FOR QUANTUM GRAVITY?

- QFT is best formalism we have for quantum microscopic physics and many-particle physics (particle physics, atomic physics, condensed matter,...)
- A QFT on which spacetime?
  - a QFT of gravitons on some background doesn't work
  - QG should explain origin and properties of spacetime itself (geometry *and* topology?)
  - background independence!
  - it can be only[1] be a QFT on some auxiliary, internal or "meta-space"
  - only background structures in GR: Lorentz group, i.e. local gauge group - equivalence principle; space of geometries - superspace; spacetime topology (?)
- a QFT of what? what are the fundamental quanta?
  - gravitons do not work
  - quanta of space itself! fundamental excitations of space around the vacuum (nothing = not even space)
  - we are not starting from scratch: ideas and results from LQG, matrix models, simplicial QG,...

[1] Unless......asymptotic safety....

A QFT FOR QUANTUM GRAVITY?

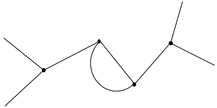
- QFT is best formalism we have for quantum microscopic physics and many-particle physics (particle physics, atomic physics, condensed matter,...)
- A QFT on which spacetime?
  - a QFT of gravitons on some background doesn't work
  - QG should explain origin and properties of spacetime itself (geometry *and* topology?)
  - background independence!
  - it can be only[1] be a QFT on some auxiliary, internal or "meta-space"
  - only background structures in GR: Lorentz group, i.e. local gauge group - equivalence principle; space of geometries - superspace; spacetime topology (?)
- a QFT of what? what are the fundamental quanta?
  - gravitons do not work
  - quanta of space itself! fundamental excitations of space around the vacuum (nothing = not even space)
  - we are not starting from scratch: ideas and results from LQG, matrix models, simplicial QG,...

[1] Unless......asymptotic safety....

From point particles to fields, from matrices/tensors to GFT

point particles $\longrightarrow$ fields

$$S(X) = \frac{1}{2}X^2 + \frac{\lambda}{3}X^3$$

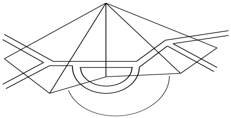


$$S(\phi) = \frac{1}{2}\int dx\, \phi(x)^2 + \frac{\lambda}{3}\int dx\, \phi(x)^3$$

$\downarrow$ $\downarrow$

matrices $\longrightarrow$ Group Field Theory

$$S(M) = \frac{1}{2}M_{ij}M_{ji} + \frac{\lambda}{3}M_{ij}M_{jk}M_{ki}$$

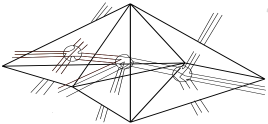


$$S(\phi) = \frac{1}{2}\int [dg]\phi(g_1,g_2)\phi(g_2,g_1) + \frac{\lambda}{3!}\int [dg]\,\phi(g_1,g_2)\phi(g_2,g_3)\phi(g_3,g_1)$$

$\downarrow$ $\downarrow$

tensors $\longrightarrow$ Group Field Theory

$$S(T) = \frac{1}{2}T_{ijk}T_{kji} + \frac{\lambda}{3}T_{ijk}T_{klm}T_{mni}T_{nlj}$$



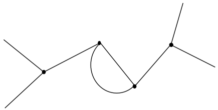
$$S(\phi) = \frac{1}{2}\int [dg]\phi(g_1,g_2,g_3)\phi(g_3,g_2,g_1) + \frac{\lambda}{4!}\int [dg]\,\phi(g_1,g_2,g_3)\phi(g_3,g_4,g_5)\phi(g_5,g_6,g_1)\phi(g_6,g_4,g_2)$$

$\downarrow$

FROM POINT PARTICLES TO FIELDS, FROM MATRICES/TENSORS TO GFT

point particles $\longrightarrow$ fields

$$S(X) = \frac{1}{2}X^2 + \frac{\lambda}{3}X^3$$

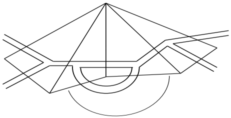


$$S(\phi) = \frac{1}{2}\int dx\,\phi(x)^2 + \frac{\lambda}{3}\int dx\,\phi(x)^3$$

$\downarrow$ $\downarrow$

matrices $\longrightarrow$ Group Field Theory

$$S(M) = \frac{1}{2}M_{ij}M_{ji} + \frac{\lambda}{3}M_{ij}M_{jk}M_{ki}$$

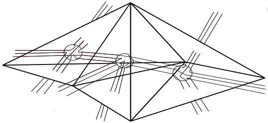


$$S(\phi) = \frac{1}{2}\int [dg]\phi(g_1,g_2)\phi(g_2,g_1) + \frac{\lambda}{3!}\int [dg]\,\phi(g_1,g_2)\phi(g_2,g_3)\phi(g_3,g_1)$$

$\downarrow$ $\downarrow$

tensors $\longrightarrow$ Group Field Theory

$$S(T) = \frac{1}{2}T_{ijk}T_{kji} + \frac{\lambda}{3}T_{ijk}T_{klm}T_{mni}T_{nlj}$$



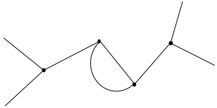
$$S(\phi) = \frac{1}{2}\int [dg]\phi(g_1,g_2,g_3)\phi(g_3,g_2,g_1) + \frac{\lambda}{4!}\int [dg]\,\phi(g_1,g_2,g_3)\phi(g_3,g_4,g_5)\phi(g_5,g_6,g_1)\phi(g_6,g_4,g_2)$$

$\downarrow$

FROM POINT PARTICLES TO FIELDS, FROM MATRICES/TENSORS TO GFT

point particles $\longrightarrow$ fields

$$S(X) = \frac{1}{2}X^2 + \frac{\lambda}{3}X^3$$

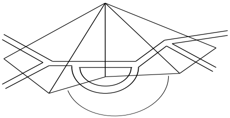


$$S(\phi) = \frac{1}{2}\int dx\, \phi(x)^2 + \frac{\lambda}{3}\int dx\, \phi(x)^3$$

$\downarrow$ $\downarrow$

matrices $\longrightarrow$ Group Field Theory

$$S(M) = \frac{1}{2} M_{ij}M_{ji} + + \frac{\lambda}{3} M_{ij}M_{jk}M_{ki}$$

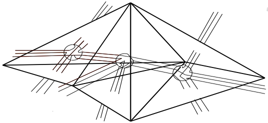


$$S(\phi) = \frac{1}{2}\int [dg]\phi(g_1,g_2)\phi(g_2,g_1) + + \frac{\lambda}{3!}\int [dg]\, \phi(g_1,g_2)\phi(g_2,g_3)\phi(g_3,g_1)$$

$\downarrow$ $\downarrow$

tensors $\longrightarrow$ Group Field Theory

$$S(T) = \frac{1}{2} T_{ijk}T_{kji} + + \frac{\lambda}{3} T_{ijk}T_{klm}T_{mni}T_{nlj}$$



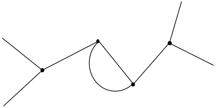
$$S(\phi) = \frac{1}{2}\int [dg]\phi(g_1,g_2,g_3)\phi(g_3,g_2,g_1) + + \frac{\lambda}{4!}\int [dg]\, \phi(g_1,g_2,g_3)\phi(g_3,g_4,g_5) \phi(g_5,g_6,g_1)\phi(g_6,g_4,g_2)$$

$\downarrow$

From point particles to fields, from matrices/tensors to GFT

point particles $\longrightarrow$ fields

$$S(X) = \frac{1}{2}X^2 + \frac{\lambda}{3}X^3$$

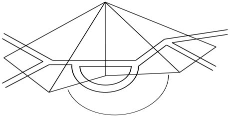


$$S(\phi) = \frac{1}{2}\int dx\,\phi(x)^2 + \frac{\lambda}{3}\int dx\,\phi(x)^3$$

$\downarrow$ $\downarrow$

matrices $\longrightarrow$ Group Field Theory

$$S(M) = \frac{1}{2}M_{ij}M_{ji} + \frac{\lambda}{3}M_{ij}M_{jk}M_{ki}$$

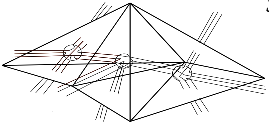


$$S(\phi) = \frac{1}{2}\int [dg]\phi(g_1,g_2)\phi(g_2,g_1) + \frac{\lambda}{3!}\int [dg]\,\phi(g_1,g_2)\phi(g_2,g_3)\phi(g_3,g_1)$$

$\downarrow$ $\downarrow$

tensors $\longrightarrow$ Group Field Theory

$$S(T) = \frac{1}{2}T_{ijk}T_{kji} + \frac{\lambda}{3}T_{ijk}T_{klm}T_{mni}T_{nlj}$$

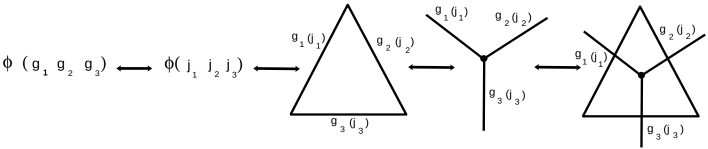


$$S(\phi) = \frac{1}{2}\int [dg]\phi(g_1,g_2,g_3)\phi(g_3,g_2,g_1) + \frac{\lambda}{4!}\int [dg]\,\phi(g_1,g_2,g_3)\phi(g_3,g_4,g_5)\phi(g_5,g_6,g_1)\phi(g_6,g_4,g_2)$$

$\downarrow$

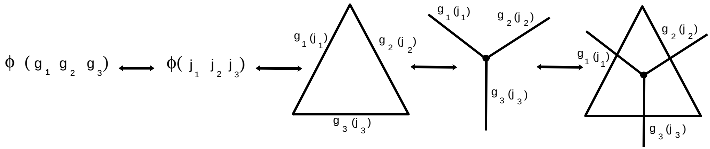
GFT KINEMATICS

- consider a complex field ϕ over D copies of a group manifold G (e.g. Lorentz group, for QG): $\phi(g_1, ..., g_D) : \underbrace{G \times ... \times G}_{D} \rightarrow \mathbb{C}$
- field can be expanded in modes (Peter-Weyl/Plancherel decomposition into unitary irreducible representations): $\phi(g_1, ..., g_D) = \sum_{J_1,...,J_D} \phi^{J_1...J_D}_{k_1 l_1,...,k_D l_D} D^{J_1}_{k_1 l_1}(g_1)...D^{J_D}_{k_D l_D}(g_D)$
- $\phi \simeq$ fundamental building block of quantum geometry - represented as (D-1)-simplex or vertex of dual graph (spin network vertex)
- $g_i \simeq$ gravity holonomies; $J_i \simeq$ (D-2)-volumes (justified by quantum GFT amplitudes and 1st quantization of simplicial geometry)



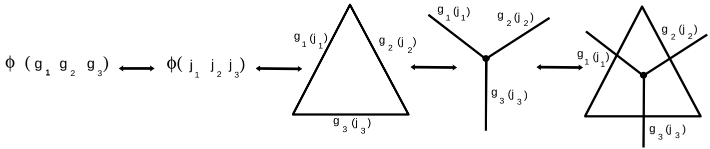
GFT KINEMATICS

- consider a complex field ϕ over D copies of a group manifold G (e.g. Lorentz group, for QG): $\phi(g_1, ..., g_D) : \underbrace{G \times ... \times G}_{D} \to \mathbb{C}$
- field can be expanded in modes (Peter-Weyl/Plancherel decomposition into unitary irreducible representations): $\phi(g_1, ..., g_D) = \sum_{J_1,...,J_D} \phi^{J_1...J_D}_{k_1 l_1,...,k_D l_D} D^{J_1}_{k_1 l_1}(g_1)...D^{J_D}_{k_D l_D}(g_D)$
- $\phi \simeq$ fundamental building block of quantum geometry - represented as (D-1)-simplex or vertex of dual graph (spin network vertex)
- $g_i \simeq$ gravity holonomies; $J_i \simeq$ (D-2)-volumes (justified by quantum GFT amplitudes and 1st quantization of simplicial geometry)



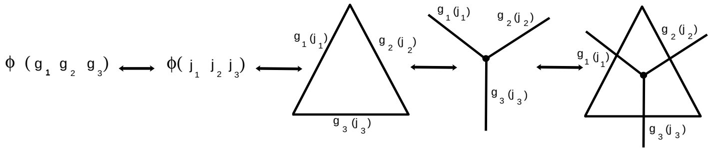
GFT KINEMATICS

- consider a complex field ϕ over D copies of a group manifold G (e.g. Lorentz group, for QG): $\phi(g_1, ..., g_D) : \underbrace{G \times ... \times G}_{D} \rightarrow \mathbb{C}$
- field can be expanded in modes (Peter-Weyl/Plancherel decomposition into unitary irreducible representations):
  $\phi(g_1, ..., g_D) = \sum_{J_1,...,J_D} \phi^{J_1...J_D}_{k_1 l_1,...,k_D l_D} D^{J_1}_{k_1 l_1}(g_1)...D^{J_D}_{k_D l_D}(g_D)$
- $\phi \simeq$ fundamental building block of quantum geometry - represented as (D-1)-simplex or vertex of dual graph (spin network vertex)
- $g_i \simeq$ gravity holonomies; $J_i \simeq$ (D-2)-volumes (justified by quantum GFT amplitudes and 1st quantization of simplicial geometry)



GFT KINEMATICS

- consider a complex field ϕ over D copies of a group manifold G (e.g. Lorentz group, for QG): $\phi(g_1, ..., g_D) : \underbrace{G \times ... \times G}_{D} \rightarrow \mathbb{C}$
- field can be expanded in modes (Peter-Weyl/Plancherel decomposition into unitary irreducible representations):
  $\phi(g_1, ..., g_D) = \sum_{J_1,...,J_D} \phi^{J_1...J_D}_{k_1 l_1,..,k_D l_D} D^{J_1}_{k_1 l_1}(g_1)...D^{J_D}_{k_D l_D}(g_D)$
- $\phi \simeq$ fundamental building block of quantum geometry - represented as (D-1)-simplex or vertex of dual graph (spin network vertex)
- $g_i \simeq$ gravity holonomies; $J_i \simeq$ (D-2)-volumes (justified by quantum GFT amplitudes and 1st quantization of simplicial geometry)



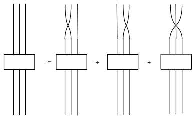
GFT CLASSICAL DYNAMICS

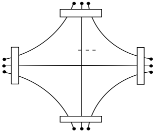
- field action:

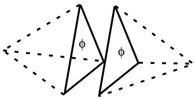
$$S_D(\phi,\lambda) \;=\; \int dg_i d\tilde{g}_i \phi(g_i)\mathcal{K}(g_i,\tilde{g}_i^{-1})\phi(\tilde{g}_i) + \lambda \int dg_{ij}\phi(g_{1j})...\phi(g_{D+1j})\,\mathcal{V}(g_{ij})$$

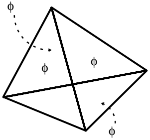
exact choice of the $\mathcal{K}$ and $\mathcal{V}$ defines the model

- combinatorics of arguments in $\mathcal{V}$ reflects gluing of (D-2)-faces in a D-simplex, interaction of (D-1)-simplices to form D-simplices
  $\mathcal{K} \longrightarrow$ gluing of D-simplices along (D-1)-simplices









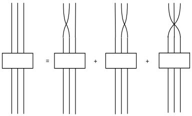
GFT CLASSICAL DYNAMICS

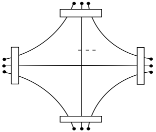
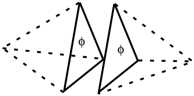
- field action:

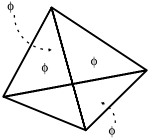
$$S_D(\phi,\lambda) \;=\; \int dg_i d\tilde{g}_i \phi(g_i)\mathcal{K}(g_i,\tilde{g}_i^{-1})\phi(\tilde{g}_i) + \lambda \int dg_{ij}\phi(g_{1j})...\phi(g_{D+1j})\,\mathcal{V}(g_{ij})$$

  exact choice of the $\mathcal{K}$ and $\mathcal{V}$ defines the model

- combinatorics of arguments in $\mathcal{V}$ reflects gluing of (D-2)-faces in a D-simplex, interaction of (D-1)-simplices to form D-simplices
  $\mathcal{K} \longrightarrow$ gluing of D-simplices along (D-1)-simplices



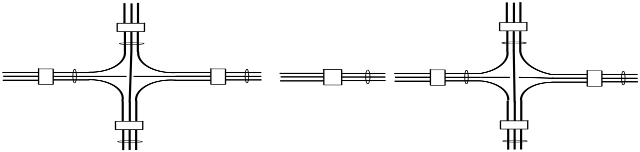


GFT QUANTUM DYNAMICS

- the quantum theory is defined by the partition function, in terms of its Feynman expansion:

$$Z = \int \mathcal{D}\phi \, e^{iS[\phi]} = \sum_{\Gamma} \frac{\lambda^{N_\Gamma}}{sym[\Gamma]} Z(\Gamma)$$

- building blocks of FD are:
  - lines of propagation, with D labelled strands (dual to(D-1)-simplices),
  - vertices of interaction (made of (D+1)× D labelled strands re-routed following the combinatorics of a D-simplex) (thus dual to D-simplices),



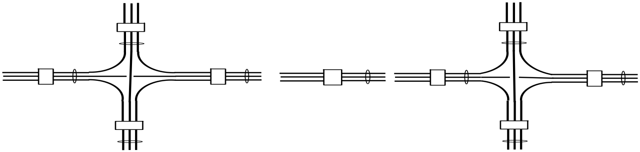
- Feynman graphs Γ are fat graphs/cellular complexes topologically dual to D-dimensional triangulated (pseudo-)manifolds of ALL topologies
- Quantum Gravity formulated as a sum over simplicial complexes of all topologies, as interaction processes

GFT QUANTUM DYNAMICS

- the quantum theory is defined by the partition function, in terms of its Feynman expansion:

$$Z = \int \mathcal{D}\phi \, e^{iS[\phi]} = \sum_{\Gamma} \frac{\lambda^{N_\Gamma}}{sym[\Gamma]} Z(\Gamma)$$

- building blocks of FD are:
  - lines of propagation, with D labelled strands (dual to(D-1)-simplices),
  - vertices of interaction (made of (D+1)$\times$ D labelled strands re-routed following the combinatorics of a D-simplex) (thus dual to D-simplices),



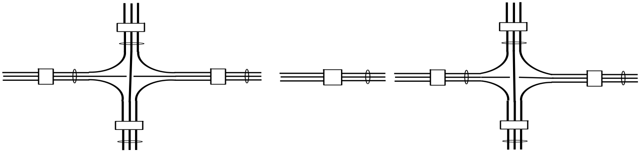
- Feynman graphs Γ are fat graphs/cellular complexes topologically dual to D-dimensional triangulated (pseudo-)manifolds of ALL topologies
- Quantum Gravity formulated as a sum over simplicial complexes of all topologies, as interaction processes

GFT QUANTUM DYNAMICS

- the quantum theory is defined by the partition function, in terms of its Feynman expansion:

$$Z = \int \mathcal{D}\phi \, e^{iS[\phi]} = \sum_{\Gamma} \frac{\lambda^{N_\Gamma}}{sym[\Gamma]} Z(\Gamma)$$

- building blocks of FD are:
  - lines of propagation, with D labelled strands (dual to(D-1)-simplices),
  - vertices of interaction (made of (D+1)$\times$ D labelled strands re-routed following the combinatorics of a D-simplex) (thus dual to D-simplices),



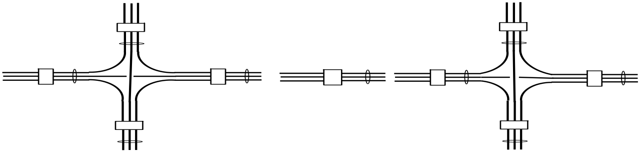
- Feynman graphs Γ are fat graphs/cellular complexes topologically dual to D-dimensional triangulated (pseudo-)manifolds of ALL topologies
- Quantum Gravity formulated as a sum over simplicial complexes of all topologies, as interaction processes

GFT QUANTUM DYNAMICS

- the quantum theory is defined by the partition function, in terms of its Feynman expansion:

$$Z = \int \mathcal{D}\phi \, e^{iS[\phi]} = \sum_{\Gamma} \frac{\lambda^{N_\Gamma}}{sym[\Gamma]} Z(\Gamma)$$

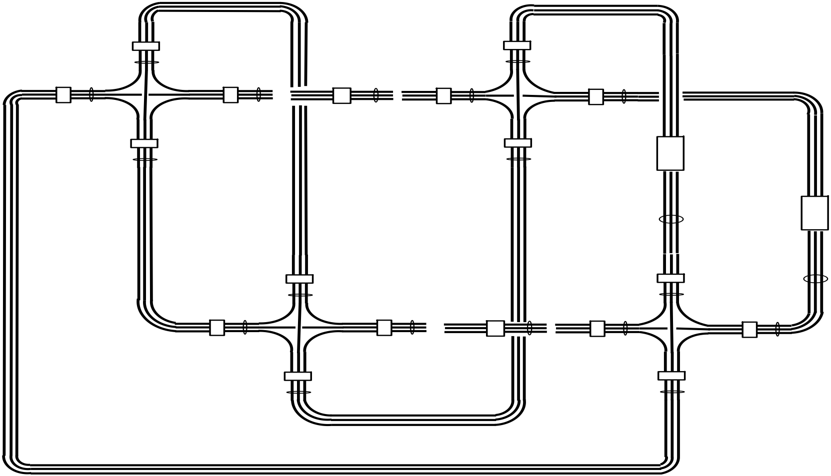
- building blocks of FD are:
  - lines of propagation, with D labelled strands (dual to(D-1)-simplices),
  - vertices of interaction (made of (D+1)$\times$ D labelled strands re-routed following the combinatorics of a D-simplex) (thus dual to D-simplices),



- Feynman graphs Γ are fat graphs/cellular complexes topologically dual to D-dimensional triangulated (pseudo-)manifolds of ALL topologies
- Quantum Gravity formulated as a sum over simplicial complexes of all topologies, as interaction processes

GFT QUANTUM DYNAMICS

An example: $D = 3$



GFT QUANTUM DYNAMICS

- Feynman amplitudes can be written in both configuration (g_i) and momentum (J_i) space

$$Z(\Gamma) = \prod \int dg_i \, A(g_i) = \prod \sum_{J_i} A(J_i)$$

- $Z(\Gamma)$ are, in momentum space, Spin Foam models (labelled 2-complexes), histories of spin networks

$$\langle \Psi_1 \mid \Psi_2 \rangle = \int \mathcal{D}\phi \, O_{\Psi_1}(\phi) \, O_{\Psi_2}(\phi) \, e^{iS(\phi)} = \sum_{\Gamma / \partial\Gamma = \gamma_{\Psi_1} \cup \gamma_{\Psi_2}} \frac{\lambda^{N_\Gamma}}{sym[\Gamma]} Z(\Gamma)$$

- $Z(\Gamma) \simeq$ discrete and algebraic QG path integral $\rightarrow$ $''\int \mathcal{D}g_\Delta \; e^{i\, S_\Delta(g)}\, ''$
- $Z(\Gamma)$ are derived/constructed/motivated from discretizaton of classical gravity theories, and, in some models, take the explicit form of simplicial gravity path integrals

GFT QUANTUM DYNAMICS

- Feynman amplitudes can be written in both configuration (g_i) and momentum (J_i) space

$$Z(\Gamma) = \prod \int dg_i\, A(g_i) = \prod \sum_{J_i} A(J_i)$$

- $Z(\Gamma)$ are, in momentum space, Spin Foam models (labelled 2-complexes), histories of spin networks

$$\langle \Psi_1 \mid \Psi_2 \rangle = \int \mathcal{D}\phi\, O_{\Psi_1}(\phi)\, O_{\Psi_2}(\phi)\, e^{iS(\phi)} = \sum_{\Gamma/\partial\Gamma = \gamma_{\Psi_1} \cup \gamma_{\Psi_2}} \frac{\lambda^{N_\Gamma}}{sym[\Gamma]}\, Z(\Gamma)$$

- $Z(\Gamma) \simeq$ discrete and algebraic QG path integral $\rightarrow$ $'' \int \mathcal{D}g_\Delta\, e^{i S_\Delta(g)} ''$
- $Z(\Gamma)$ are derived/constructed/motivated from discretizaton of classical gravity theories, and, in some models, take the explicit form of simplicial gravity path integrals

GFT QUANTUM DYNAMICS

- Feynman amplitudes can be written in both configuration (g_i) and momentum (J_i) space

$$Z(\Gamma) \;=\; \prod \int dg_i \, A(g_i) = \prod \sum_{J_i} A(J_i)$$

- $Z(\Gamma)$ are, in momentum space, Spin Foam models (labelled 2-complexes), histories of spin networks

$$\langle \Psi_1 \mid \Psi_2 \rangle = \int \mathcal{D}\phi \, O_{\Psi_1}(\phi) \, O_{\Psi_2}(\phi) \, e^{iS(\phi)} = \sum_{\Gamma/\partial\Gamma = \gamma_{\Psi_1} \cup \gamma_{\Psi_2}} \frac{\lambda^{N_\Gamma}}{sym[\Gamma]} \, Z(\Gamma)$$

- $Z(\Gamma) \simeq$ discrete and algebraic QG path integral $\rightarrow$ $'' \int \mathcal{D}g_\Delta \; e^{i\, S_\Delta(g)} \, ''$
- $Z(\Gamma)$ are derived/constructed/motivated from discretizaton of classical gravity theories, and, in some models, take the explicit form of simplicial gravity path integrals

GFT QUANTUM DYNAMICS

- Feynman amplitudes can be written in both configuration (g_i) and momentum (J_i) space

$$Z(\Gamma) \;=\; \prod \int dg_i \, A(g_i) = \prod \sum_{J_i} A(J_i)$$

- $Z(\Gamma)$ are, in momentum space, Spin Foam models (labelled 2-complexes), histories of spin networks

$$\langle \Psi_1 \mid \Psi_2 \rangle = \int \mathcal{D}\phi \, O_{\Psi_1}(\phi) \, O_{\Psi_2}(\phi) \, e^{iS(\phi)} = \sum_{\Gamma/\partial\Gamma = \gamma_{\Psi_1} \cup \gamma_{\Psi_2}} \frac{\lambda^{N_\Gamma}}{sym[\Gamma]} \, Z(\Gamma)$$

- $Z(\Gamma) \simeq$ discrete and algebraic QG path integral $\rightarrow$ $'' \int \mathcal{D}g_\Delta \; e^{i\, S_\Delta(g)} \; ''$
- $Z(\Gamma)$ are derived/constructed/motivated from discretizaton of classical gravity theories, and, in some models, take the explicit form of simplicial gravity path integrals

EXAMPLE: 3D RIEMANNIAN QUANTUM GRAVITY

$G = SU(2)$, Real field: $\phi(g_1, g_2, g_3) = \phi(g_1 g, g_2 g, g_3 g) : \frac{SU(2)^{\times 3}}{SU(2)} \to \mathbb{R}$

$$S[\phi] = \frac{1}{2} \int dg_i \, \phi(g_1, g_2, g_3)^2 + \frac{\lambda}{4!} \int dg_j \, \phi(g_1, g_2, g_3)\phi(g_3, g_4, g_5)\phi(g_5, g_6, g_1)\phi(g_6, g_4, g_2)$$

The Feynman amplitudes for this model are:

$$Z(\Gamma) = \prod_{e* \in \Gamma} \int dg_{e*} \prod_{f*} \delta\Big(\prod_{e* \in \partial f*} g_{e*}\Big)$$

If expanded in $SU(2)$ representations, it gives Ponzano-Regge spin foam model

Same result from path integral quantization of 3d BF theory $\approx$ 3d Riemannian gravity in 1st order form on triangulation Δ dual to Γ

$$S_{\mathcal{M}}(e, \omega) = \int_{\mathcal{M}} tr(e \wedge F(\omega)) \to S_{\Delta}(X_e, g_{e*}) = \sum_e tr\left(X_e G_e(g_{e*})\right)$$

$$Z(\Gamma) = \prod_e \int_{su(2)} dX_e \prod_{e*} \int_{SU(2)} dg_{e*} \; e^{i \sum_e tr(X_e G_e(g_{e*}))}$$

EXAMPLE: 3D RIEMANNIAN QUANTUM GRAVITY

$G = SU(2)$, Real field: $\phi(g_1, g_2, g_3) = \phi(g_1 g, g_2 g, g_3 g) : \frac{SU(2)^{\times 3}}{SU(2)} \to \mathbb{R}$

$$S[\phi] = \frac{1}{2}\int dg_i\, \phi(g_1, g_2, g_3)^2 + \frac{\lambda}{4!}\int dg_j\, \phi(g_1, g_2, g_3)\phi(g_3, g_4, g_5)\phi(g_5, g_6, g_1)\phi(g_6, g_4, g_2)$$

The Feynman amplitudes for this model are:

$$Z(\Gamma) = \prod_{e* \in \Gamma} \int dg_{e*} \prod_{f*} \delta\Big(\prod_{e* \in \partial f*} g_{e*} \Big)$$

If expanded in $SU(2)$ representations, it gives Ponzano-Regge spin foam model

Same result from path integral quantization of 3d BF theory $\approx$ 3d Riemannian gravity in 1st order form on triangulation Δ dual to Γ

$$S_{\mathcal{M}}(e, \omega) = \int_{\mathcal{M}} tr(e \wedge F(\omega)) \to S_{\Delta}(X_e, g_{e*}) = \sum_e tr\left(X_e G_e(g_{e*})\right)$$

$$Z(\Gamma) = \prod_e \int_{su(2)} dX_e \prod_{e*} \int_{SU(2)} dg_{e*}\; e^{i \sum_e tr(X_e G_e(g_{e*}))}$$

EXAMPLE: 3D RIEMANNIAN QUANTUM GRAVITY

$G = SU(2)$, Real field: $\phi(g_1, g_2, g_3) = \phi(g_1 g, g_2 g, g_3 g) : \frac{SU(2)^{\times 3}}{SU(2)} \longrightarrow \mathbb{R}$

$$S[\phi] = \frac{1}{2} \int dg_i \, \phi(g_1, g_2, g_3)^2 + \frac{\lambda}{4!} \int dg_j \, \phi(g_1, g_2, g_3)\phi(g_3, g_4, g_5)\phi(g_5, g_6, g_1)\phi(g_6, g_4, g_2)$$

The Feynman amplitudes for this model are:

$$Z(\Gamma) = \prod_{e* \in \Gamma} \int dg_{e*} \prod_{f*} \delta(\prod_{e* \in \partial f*} g_{e*})$$

If expanded in $SU(2)$ representations, it gives Ponzano-Regge spin foam model

Same result from path integral quantization of 3d BF theory $\approx$ 3d Riemannian gravity in 1st order form on triangulation Δ dual to Γ

$$S_{\mathcal{M}}(e, \omega) = \int_{\mathcal{M}} tr(e \wedge F(\omega)) \rightarrow S_{\Delta}(X_e, g_{e*}) = \sum_e tr\left(X_e G_e(g_{e*})\right)$$

$$Z(\Gamma) = \prod_e \int_{su(2)} dX_e \prod_{e*} \int_{SU(2)} dg_{e*} \; e^{i \sum_e tr(X_e G_e(g_{e*}))}$$

CONVERGENCE OF APPROACHES...

- GFT can be common framework for various approaches:
  - Loop Quantum Gravity and spin foam models:
    - GFT states are Spin Networks, GFT perturbative expansion defines physical scalar product (dynamics)
    - GFT Feynman amplitudes are Spin Foam models (sum over histories of spin networks)
  - Quantum Regge Calculus: GFT defines simplicial QG path integral, with unique (for given GFT) measure
  - Dynamical Triangulations: GFT describes QG (perturbatively) as sum over triangulations, but contains also non-perturbative info
- At the same time, GFTs offer new tools, and a new perspective on continuum/classical approximation

CONVERGENCE OF APPROACHES...

- GFT can be common framework for various approaches:
  - Loop Quantum Gravity and spin foam models:
    - GFT states are Spin Networks, GFT perturbative expansion defines physical scalar product (dynamics)
    - GFT Feynman amplitudes are Spin Foam models (sum over histories of spin networks)
  - Quantum Regge Calculus: GFT defines simplicial QG path integral, with unique (for given GFT) measure
  - Dynamical Triangulations: GFT describes QG (perturbatively) as sum over triangulations, but contains also non-perturbative info
- At the same time, GFTs offer new tools, and a new perspective on continuum/classical approximation

CONVERGENCE OF APPROACHES...

- GFT can be common framework for various approaches:
  - Loop Quantum Gravity and spin foam models:
    - GFT states are Spin Networks, GFT perturbative expansion defines physical scalar product (dynamics)
    - GFT Feynman amplitudes are Spin Foam models (sum over histories of spin networks)
  - Quantum Regge Calculus: GFT defines simplicial QG path integral, with unique (for given GFT) measure
  - Dynamical Triangulations: GFT describes QG (perturbatively) as sum over triangulations, but contains also non-perturbative info
- At the same time, GFTs offer new tools, and a new perspective on continuum/classical approximation

CONVERGENCE OF APPROACHES...

- GFT can be common framework for various approaches:
  - Loop Quantum Gravity and spin foam models:
    - GFT states are Spin Networks, GFT perturbative expansion defines physical scalar product (dynamics)
    - GFT Feynman amplitudes are Spin Foam models (sum over histories of spin networks)
  - Quantum Regge Calculus: GFT defines simplicial QG path integral, with unique (for given GFT) measure
  - Dynamical Triangulations: GFT describes QG (perturbatively) as sum over triangulations, but contains also non-perturbative info
- At the same time, GFTs offer new tools, and a new perspective on continuum/classical approximation

CONVERGENCE OF APPROACHES...

- GFT can be common framework for various approaches:
  - Loop Quantum Gravity and spin foam models:
    - GFT states are Spin Networks, GFT perturbative expansion defines physical scalar product (dynamics)
    - GFT Feynman amplitudes are Spin Foam models (sum over histories of spin networks)
  - Quantum Regge Calculus: GFT defines simplicial QG path integral, with unique (for given GFT) measure
  - Dynamical Triangulations: GFT describes QG (perturbatively) as sum over triangulations, but contains also non-perturbative info
- At the same time, GFTs offer new tools, and a new perspective on continuum/classical approximation

GFT: some recent results

- GFT perturbative renormalization
- From GFT to matter non-commutative QFT
- GFT model(s) for 4D quantum gravity

PERTURBATIVE GFT RENORMALIZATION - THE 3D CASE

L. Freidel, R. Gurau, DO, 0905.3772 [hep-th]

(newest developments: J. Magnen, K. Noui, V. Rivasseau, M. Smerlak, 0906.5477 [hep-th])

Question: can you control the perturbative GFT sum over Feynman diagrams?

$$S = \frac{1}{2}\int \left(\phi(g_1, g_2, g_3)\right)^2 + \frac{\lambda}{4!}\int \phi(g_1, g_2, g_3)\phi(g_3, g_4, g_5)\phi(g_5, g_6, g_1)\phi(g_6, g_4, g_2)$$

$$Z(\Gamma) = \prod_{e*\in\Gamma}\int dg_{e*}\prod_{f*}\delta\Big(\prod_{e*\in\partial f*} g_{e*}\Big)$$

FD are cellular complexes Γ dual to 3d triangulations

- divergences associated to bubbles (3-cells in FDs)
- highly involved combinatorial structure, all topologies included and pseudo-manifolds $\rightarrow$ difficult to isolate divergences, unclear which subclass of FD needs renormalization
- results:
  - algorithm for identifying bubbles in FD, and their boundary triangulations
  - identification of subclass of FD, natural generalization of 2d planar graphs, allowing for contraction procedure
  - exact power counting of divergences for this class of FD
  - conjecture: these are the only relevant FD in generalized scaling limit
  - conjecture: these FD all corresponds to -manifolds- of -trivial topology-

PERTURBATIVE GFT RENORMALIZATION - THE 3D CASE

L. Freidel, R. Gurau, DO, 0905.3772 [hep-th]

(newest developments: J. Magnen, K. Noui, V. Rivasseau, M. Smerlak, 0906.5477 [hep-th])

Question: can you control the perturbative GFT sum over Feynman diagrams?

$$S = \frac{1}{2}\int (\phi(g_1,g_2,g_3))^2 + \frac{\lambda}{4!}\int \phi(g_1,g_2,g_3)\phi(g_3,g_4,g_5)\phi(g_5,g_6,g_1)\phi(g_6,g_4,g_2)$$

$$Z(\Gamma) = \prod_{e*\in\Gamma}\int dg_{e*}\prod_{f*}\delta(\prod_{e*\in\partial f*} g_{e*})$$

FD are cellular complexes Γ dual to 3d triangulations

- divergences associated to bubbles (3-cells in FDs)
- highly involved combinatorial structure, all topologies included and pseudo-manifolds $\rightarrow$ difficult to isolate divergences, unclear which subclass of FD needs renormalization
- results:
  - algorithm for identifying bubbles in FD, and their boundary triangulations
  - identification of subclass of FD, natural generalization of 2d planar graphs, allowing for contraction procedure
  - exact power counting of divergences for this class of FD
  - conjecture: these are the only relevant FD in generalized scaling limit
  - conjecture: these FD all corresponds to -manifolds- of -trivial topology-

PERTURBATIVE GFT RENORMALIZATION - THE 3D CASE

L. Freidel, R. Gurau, DO, 0905.3772 [hep-th]

(newest developments: J. Magnen, K. Noui, V. Rivasseau, M. Smerlak, 0906.5477 [hep-th])

Question: can you control the perturbative GFT sum over Feynman diagrams?

$$S = \frac{1}{2}\int (\phi(g_1,g_2,g_3))^2 + \frac{\lambda}{4!}\int \phi(g_1,g_2,g_3)\phi(g_3,g_4,g_5)\phi(g_5,g_6,g_1)\phi(g_6,g_4,g_2)$$

$$Z(\Gamma) = \prod_{e*\in\Gamma}\int dg_{e*}\prod_{f*}\delta(\prod_{e*\in\partial f*} g_{e*})$$

FD are cellular complexes Γ dual to 3d triangulations

- divergences associated to bubbles (3-cells in FDs)
- highly involved combinatorial structure, all topologies included and pseudo-manifolds $\rightarrow$ difficult to isolate divergences, unclear which subclass of FD needs renormalization
- results:
  - algorithm for identifying bubbles in FD, and their boundary triangulations
  - identification of subclass of FD, natural generalization of 2d planar graphs, allowing for contraction procedure
  - exact power counting of divergences for this class of FD
  - conjecture: these are the only relevant FD in generalized scaling limit
  - conjecture: these FD all corresponds to -manifolds- of -trivial topology-

PERTURBATIVE GFT RENORMALIZATION - THE 3D CASE

L. Freidel, R. Gurau, DO, 0905.3772 [hep-th]

(newest developments: J. Magnen, K. Noui, V. Rivasseau, M. Smerlak, 0906.5477 [hep-th])

Question: can you control the perturbative GFT sum over Feynman diagrams?

$$S = \frac{1}{2}\int (\phi(g_1,g_2,g_3))^2 + \frac{\lambda}{4!}\int \phi(g_1,g_2,g_3)\phi(g_3,g_4,g_5)\phi(g_5,g_6,g_1)\phi(g_6,g_4,g_2)$$

$$Z(\Gamma) = \prod_{e*\in\Gamma}\int dg_{e*}\prod_{f*}\delta(\prod_{e*\in\partial f*} g_{e*})$$

FD are cellular complexes Γ dual to 3d triangulations

- divergences associated to bubbles (3-cells in FDs)
- highly involved combinatorial structure, all topologies included and pseudo-manifolds $\rightarrow$ difficult to isolate divergences, unclear which subclass of FD needs renormalization
- results:
  - algorithm for identifying bubbles in FD, and their boundary triangulations
  - identification of subclass of FD, natural generalization of 2d planar graphs, allowing for contraction procedure
  - exact power counting of divergences for this class of FD
  - conjecture: these are the only relevant FD in generalized scaling limit
  - conjecture: these FD all corresponds to -manifolds- of -trivial topology-

PERTURBATIVE GFT RENORMALIZATION - THE 3D CASE

L. Freidel, R. Gurau, DO, 0905.3772 [hep-th]

(newest developments: J. Magnen, K. Noui, V. Rivasseau, M. Smerlak, 0906.5477 [hep-th])

Question: can you control the perturbative GFT sum over Feynman diagrams?

$$S = \frac{1}{2}\int (\phi(g_1,g_2,g_3))^2 + \frac{\lambda}{4!}\int \phi(g_1,g_2,g_3)\phi(g_3,g_4,g_5)\phi(g_5,g_6,g_1)\phi(g_6,g_4,g_2)$$

$$Z(\Gamma) = \prod_{e*\in\Gamma}\int dg_{e*}\prod_{f*}\delta\Big(\prod_{e*\in\partial f*} g_{e*}\Big)$$

FD are cellular complexes Γ dual to 3d triangulations

- divergences associated to bubbles (3-cells in FDs)
- highly involved combinatorial structure, all topologies included and pseudo-manifolds $\rightarrow$ difficult to isolate divergences, unclear which subclass of FD needs renormalization
- results:
  - algorithm for identifying bubbles in FD, and their boundary triangulations
  - identification of subclass of FD, natural generalization of 2d planar graphs, allowing for contraction procedure
  - exact power counting of divergences for this class of FD
  - conjecture: these are the only relevant FD in generalized scaling limit
  - conjecture: these FD all corresponds to -manifolds- of -trivial topology-

PERTURBATIVE GFT RENORMALIZATION - THE 3D CASE

L. Freidel, R. Gurau, DO, 0905.3772 [hep-th]

(newest developments: J. Magnen, K. Noui, V. Rivasseau, M. Smerlak, 0906.5477 [hep-th])

Question: can you control the perturbative GFT sum over Feynman diagrams?

$$S = \frac{1}{2}\int (\phi(g_1,g_2,g_3))^2 + \frac{\lambda}{4!}\int \phi(g_1,g_2,g_3)\phi(g_3,g_4,g_5)\phi(g_5,g_6,g_1)\phi(g_6,g_4,g_2)$$

$$Z(\Gamma) = \prod_{e*\in\Gamma}\int dg_{e*}\prod_{f*}\delta(\prod_{e*\in\partial f*} g_{e*})$$

FD are cellular complexes Γ dual to 3d triangulations

- divergences associated to bubbles (3-cells in FDs)
- highly involved combinatorial structure, all topologies included and pseudo-manifolds $\rightarrow$ difficult to isolate divergences, unclear which subclass of FD needs renormalization
- results:
  - algorithm for identifying bubbles in FD, and their boundary triangulations
  - identification of subclass of FD, natural generalization of 2d planar graphs, allowing for contraction procedure
  - exact power counting of divergences for this class of FD
  - conjecture: these are the only relevant FD in generalized scaling limit
  - conjecture: these FD all corresponds to -manifolds- of -trivial topology-

PERTURBATIVE GFT RENORMALIZATION - THE 3D CASE

L. Freidel, R. Gurau, DO, 0905.3772 [hep-th]

(newest developments: J. Magnen, K. Noui, V. Rivasseau, M. Smerlak, 0906.5477 [hep-th])

Question: can you control the perturbative GFT sum over Feynman diagrams?

$$S = \frac{1}{2}\int (\phi(g_1,g_2,g_3))^2 + \frac{\lambda}{4!}\int \phi(g_1,g_2,g_3)\phi(g_3,g_4,g_5)\phi(g_5,g_6,g_1)\phi(g_6,g_4,g_2)$$

$$Z(\Gamma) = \prod_{e*\in\Gamma}\int dg_{e*}\prod_{f*}\delta(\prod_{e*\in\partial f*} g_{e*})$$

FD are cellular complexes Γ dual to 3d triangulations

- divergences associated to bubbles (3-cells in FDs)
- highly involved combinatorial structure, all topologies included and pseudo-manifolds $\rightarrow$ difficult to isolate divergences, unclear which subclass of FD needs renormalization
- results:
  - algorithm for identifying bubbles in FD, and their boundary triangulations
  - identification of subclass of FD, natural generalization of 2d planar graphs, allowing for contraction procedure
  - exact power counting of divergences for this class of FD
  - conjecture: these are the only relevant FD in generalized scaling limit
  - conjecture: these FD all corresponds to -manifolds- of -trivial topology-

PERTURBATIVE GFT RENORMALIZATION - THE 3D CASE

L. Freidel, R. Gurau, DO, 0905.3772 [hep-th]

(newest developments: J. Magnen, K. Noui, V. Rivasseau, M. Smerlak, 0906.5477 [hep-th])

Question: can you control the perturbative GFT sum over Feynman diagrams?

$$S = \frac{1}{2}\int (\phi(g_1,g_2,g_3))^2 + \frac{\lambda}{4!}\int \phi(g_1,g_2,g_3)\phi(g_3,g_4,g_5)\phi(g_5,g_6,g_1)\phi(g_6,g_4,g_2)$$

$$Z(\Gamma) = \prod_{e*\in\Gamma}\int dg_{e*}\prod_{f*}\delta(\prod_{e*\in\partial f*} g_{e*})$$

FD are cellular complexes Γ dual to 3d triangulations

- divergences associated to bubbles (3-cells in FDs)
- highly involved combinatorial structure, all topologies included and pseudo-manifolds $\rightarrow$ difficult to isolate divergences, unclear which subclass of FD needs renormalization
- results:
  - algorithm for identifying bubbles in FD, and their boundary triangulations
  - identification of subclass of FD, natural generalization of 2d planar graphs, allowing for contraction procedure
  - exact power counting of divergences for this class of FD
  - conjecture: these are the only relevant FD in generalized scaling limit
  - conjecture: these FD all corresponds to -manifolds- of -trivial topology-

PERTURBATIVE GFT RENORMALIZATION - THE 3D CASE

L. Freidel, R. Gurau, DO, 0905.3772 [hep-th]

(newest developments: J. Magnen, K. Noui, V. Rivasseau, M. Smerlak, 0906.5477 [hep-th])

Question: can you control the perturbative GFT sum over Feynman diagrams?

$$S = \frac{1}{2}\int (\phi(g_1,g_2,g_3))^2 + \frac{\lambda}{4!}\int \phi(g_1,g_2,g_3)\phi(g_3,g_4,g_5)\phi(g_5,g_6,g_1)\phi(g_6,g_4,g_2)$$

$$Z(\Gamma) = \prod_{e*\in\Gamma}\int dg_{e*}\prod_{f*}\delta(\prod_{e*\in\partial f*} g_{e*})$$

FD are cellular complexes Γ dual to 3d triangulations

- divergences associated to bubbles (3-cells in FDs)
- highly involved combinatorial structure, all topologies included and pseudo-manifolds $\rightarrow$ difficult to isolate divergences, unclear which subclass of FD needs renormalization
- results:
  - algorithm for identifying bubbles in FD, and their boundary triangulations
  - identification of subclass of FD, natural generalization of 2d planar graphs, allowing for contraction procedure
  - exact power counting of divergences for this class of FD
  - conjecture: these are the only relevant FD in generalized scaling limit
  - conjecture: these FD all corresponds to -manifolds- of -trivial topology-

EFFECTIVE NON-COMMUTATIVE MATTER FIELDS FROM GFT

- Insights from analog gravity models in condensed matter
  (C. Barcelo, S. Liberati, M. Visser, gr-qc/0505065, G. Volovik, gr-qc/0005091):
  - effective metric and its dynamics from hydrodynamics of microscopic system $\Rightarrow$ (modified) GR from GFT hydrodynamics?
  - quasi-particles (perturbations around ground state) see only effective metric $\Rightarrow$ effective QFT for matter in emergent metric from GFT? which effective matter QFT? incorporate QG effects? (DO, 0903.3970 [hep-th])
- General hypothesis
  - effective QG spacetime is non-commutative with Lie algebra structure: $[x_\mu, x_\nu] = C^\alpha_{\mu\nu} x_\alpha$
  - by duality, corresponding momentum space is a (curved) group manifold (Majid)
  - symmetry group of such NC spacetimes is Hopf algebra
  - suggests QFT in such NC spacetime, in momentum space, is a GFT

These NC spacetimes are arena for much current QG phenomenology (G. Amelino-Camelia, 0806.0339 [gr-qc]), especially in the context of Deformed Special Relativity (DSR) (J. Kowalski-Glikman, hep-th/0405273)

Task: derive such NC field theories from more fundamental GFT models
3d Riemannian: W. Fairbairn, E. Livine, gr-qc/0702125
4d Lorentzian: DSR on κ-Minkowski: F.Girelli, E.Livine, DO, 0903.3475 [gr-qc]

EFFECTIVE NON-COMMUTATIVE MATTER FIELDS FROM GFT

- Insights from analog gravity models in condensed matter
(C. Barcelo, S. Liberati, M. Visser, gr-qc/0505065, G. Volovik, gr-qc/0005091):
  - effective metric and its dynamics from hydrodynamics of microscopic system $\Rightarrow$ (modified) GR from GFT hydrodynamics?
  - quasi-particles (perturbations around ground state) see only effective metric $\Rightarrow$ effective QFT for matter in emergent metric from GFT? which effective matter QFT? incorporate QG effects? (DO, 0903.3970 [hep-th])
- General hypothesis
  - effective QG spacetime is non-commutative with Lie algebra structure: $[x_\mu, x_\nu] = C^\alpha_{\mu\nu} x_\alpha$
  - by duality, corresponding momentum space is a (curved) group manifold (Majid)
  - symmetry group of such NC spacetimes is Hopf algebra
  - suggests QFT in such NC spacetime, in momentum space, is a GFT

These NC spacetimes are arena for much current QG phenomenology (G. Amelino-Camelia, 0806.0339 [gr-qc]), especially in the context of Deformed Special Relativity (DSR) (J. Kowalski-Glikman, hep-th/0405273)

Task: derive such NC field theories from more fundamental GFT models
3d Riemannian: W. Fairbairn, E. Livine, gr-qc/0702125
4d Lorentzian: DSR on κ-Minkowski: F.Girelli, E.Livine, DO, 0903.3475 [gr-qc]

EFFECTIVE NON-COMMUTATIVE MATTER FIELDS FROM GFT

- Insights from analog gravity models in condensed matter (C. Barcelo, S. Liberati, M. Visser, gr-qc/0505065, G. Volovik, gr-qc/0005091):
  - effective metric and its dynamics from hydrodynamics of microscopic system $\Rightarrow$ (modified) GR from GFT hydrodynamics?
  - quasi-particles (perturbations around ground state) see only effective metric $\Rightarrow$ effective QFT for matter in emergent metric from GFT? which effective matter QFT? incorporate QG effects? (DO, 0903.3970 [hep-th])
- General hypothesis
  - effective QG spacetime is non-commutative with Lie algebra structure: $[x_\mu, x_\nu] = \mathcal{C}^\alpha_{\mu\nu} x_\alpha$
  - by duality, corresponding momentum space is a (curved) group manifold (Majid)
  - symmetry group of such NC spacetimes is Hopf algebra
  - suggests QFT in such NC spacetime, in momentum space, is a GFT

These NC spacetimes are arena for much current QG phenomenology (G. Amelino-Camelia, 0806.0339 [gr-qc]), especially in the context of Deformed Special Relativity (DSR) (J. Kowalski-Glikman, hep-th/0405273)

Task: derive such NC field theories from more fundamental GFT models
3d Riemannian: W. Fairbairn, E. Livine, gr-qc/0702125
4d Lorentzian: DSR on κ-Minkowski: F.Girelli, E.Livine, DO, 0903.3475 [gr-qc]

EFFECTIVE NON-COMMUTATIVE MATTER FIELDS FROM GFT

- Insights from analog gravity models in condensed matter (C. Barcelo, S. Liberati, M. Visser, gr-qc/0505065, G. Volovik, gr-qc/0005091):
  - effective metric and its dynamics from hydrodynamics of microscopic system $\Rightarrow$ (modified) GR from GFT hydrodynamics?
  - quasi-particles (perturbations around ground state) see only effective metric $\Rightarrow$ effective QFT for matter in emergent metric from GFT? which effective matter QFT? incorporate QG effects? (DO, 0903.3970 [hep-th])
- General hypothesis
  - effective QG spacetime is non-commutative with Lie algebra structure: $[x_\mu, x_\nu] = \mathcal{C}^{\alpha}_{\mu\nu} x_\alpha$
  - by duality, corresponding momentum space is a (curved) group manifold (Majid)
  - symmetry group of such NC spacetimes is Hopf algebra
  - suggests QFT in such NC spacetime, in momentum space, is a GFT

These NC spacetimes are arena for much current QG phenomenology (G. Amelino-Camelia, 0806.0339 [gr-qc]), especially in the context of Deformed Special Relativity (DSR) (J. Kowalski-Glikman, hep-th/0405273)

Task: derive such NC field theories from more fundamental GFT models
3d Riemannian: W. Fairbairn, E. Livine, gr-qc/0702125
4d Lorentzian: DSR on κ-Minkowski: F.Girelli, E.Livine, DO, 0903.3475 [gr-qc]

EFFECTIVE NON-COMMUTATIVE MATTER FIELDS FROM GFT

- Insights from analog gravity models in condensed matter (C. Barcelo, S. Liberati, M. Visser, gr-qc/0505065, G. Volovik, gr-qc/0005091):
  - effective metric and its dynamics from hydrodynamics of microscopic system $\Rightarrow$ (modified) GR from GFT hydrodynamics?
  - quasi-particles (perturbations around ground state) see only effective metric $\Rightarrow$ effective QFT for matter in emergent metric from GFT? which effective matter QFT? incorporate QG effects? (DO, 0903.3970 [hep-th])
- General hypothesis
  - effective QG spacetime is non-commutative with Lie algebra structure: $[x_\mu, x_\nu] = \mathcal{C}^\alpha_{\mu\nu} x_\alpha$
  - by duality, corresponding momentum space is a (curved) group manifold (Majid)
  - symmetry group of such NC spacetimes is Hopf algebra
  - suggests QFT in such NC spacetime, in momentum space, is a GFT

These NC spacetimes are arena for much current QG phenomenology (G. Amelino-Camelia, 0806.0339 [gr-qc]), especially in the context of Deformed Special Relativity (DSR) (J. Kowalski-Glikman, hep-th/0405273)

Task: derive such NC field theories from more fundamental GFT models
3d Riemannian: W. Fairbairn, E. Livine, gr-qc/0702125
4d Lorentzian: DSR on κ-Minkowski: F.Girelli, E.Livine, DO, 0903.3475 [gr-qc]

EFFECTIVE NON-COMMUTATIVE MATTER FIELDS FROM GFT

- Insights from analog gravity models in condensed matter (C. Barcelo, S. Liberati, M. Visser, gr-qc/0505065, G. Volovik, gr-qc/0005091):
  - effective metric and its dynamics from hydrodynamics of microscopic system ⇒ (modified) GR from GFT hydrodynamics?
  - quasi-particles (perturbations around ground state) see only effective metric ⇒ effective QFT for matter in emergent metric from GFT? which effective matter QFT? incorporate QG effects? (DO, 0903.3970 [hep-th])
- General hypothesis
  - effective QG spacetime is non-commutative with Lie algebra structure: $[x_\mu, x_\nu] = \mathcal{C}^{\alpha}_{\mu\nu} x_\alpha$
  - by duality, corresponding momentum space is a (curved) group manifold (Majid)
  - symmetry group of such NC spacetimes is Hopf algebra
  - suggests QFT in such NC spacetime, in momentum space, is a GFT

These NC spacetimes are arena for much current QG phenomenology (G. Amelino-Camelia, 0806.0339 [gr-qc]), especially in the context of Deformed Special Relativity (DSR) (J. Kowalski-Glikman, hep-th/0405273)

Task: derive such NC field theories from more fundamental GFT models
3d Riemannian: W. Fairbairn, E. Livine, gr-qc/0702125
4d Lorentzian: DSR on κ-Minkowski: F.Girelli, E.Livine, DO, 0903.3475 [gr-qc]

EFFECTIVE NON-COMMUTATIVE MATTER FIELDS FROM GFT

- Insights from analog gravity models in condensed matter (C. Barcelo, S. Liberati, M. Visser, gr-qc/0505065, G. Volovik, gr-qc/0005091):
  - effective metric and its dynamics from hydrodynamics of microscopic system $\Rightarrow$ (modified) GR from GFT hydrodynamics?
  - quasi-particles (perturbations around ground state) see only effective metric $\Rightarrow$ effective QFT for matter in emergent metric from GFT? which effective matter QFT? incorporate QG effects? (DO, 0903.3970 [hep-th])
- General hypothesis
  - effective QG spacetime is non-commutative with Lie algebra structure: $[x_\mu, x_\nu] = \mathcal{C}^\alpha_{\mu\nu} x_\alpha$
  - by duality, corresponding momentum space is a (curved) group manifold (Majid)
  - symmetry group of such NC spacetimes is Hopf algebra
  - suggests QFT in such NC spacetime, in momentum space, is a GFT

These NC spacetimes are arena for much current QG phenomenology (G. Amelino-Camelia, 0806.0339 [gr-qc]), especially in the context of Deformed Special Relativity (DSR) (J. Kowalski-Glikman, hep-th/0405273)

Task: derive such NC field theories from more fundamental GFT models
3d Riemannian: W. Fairbairn, E. Livine, gr-qc/0702125
4d Lorentzian: DSR on κ-Minkowski: F.Girelli, E.Livine, DO, 0903.3475 [gr-qc]

Effective non-commutative matter fields from GFT

- Insights from analog gravity models in condensed matter (C. Barcelo, S. Liberati, M. Visser, gr-qc/0505065, G. Volovik, gr-qc/0005091):
  - effective metric and its dynamics from hydrodynamics of microscopic system $\Rightarrow$ (modified) GR from GFT hydrodynamics?
  - quasi-particles (perturbations around ground state) see only effective metric $\Rightarrow$ effective QFT for matter in emergent metric from GFT? which effective matter QFT? incorporate QG effects? (DO, 0903.3970 [hep-th])
- General hypothesis
  - effective QG spacetime is non-commutative with Lie algebra structure: $[x_\mu, x_\nu] = \mathcal{C}^{\alpha}_{\mu\nu} x_\alpha$
  - by duality, corresponding momentum space is a (curved) group manifold (Majid)
  - symmetry group of such NC spacetimes is Hopf algebra
  - suggests QFT in such NC spacetime, in momentum space, is a GFT

These NC spacetimes are arena for much current QG phenomenology (G. Amelino-Camelia, 0806.0339 [gr-qc]), especially in the context of Deformed Special Relativity (DSR) (J. Kowalski-Glikman, hep-th/0405273)

Task: derive such NC field theories from more fundamental GFT models
3d Riemannian: W. Fairbairn, E. Livine, gr-qc/0702125
4d Lorentzian: DSR on κ-Minkowski: F.Girelli, E.Livine, DO, 0903.3475 [gr-qc]

EFFECTIVE NON-COMMUTATIVE MATTER FIELDS FROM GFT

- Insights from analog gravity models in condensed matter (C. Barcelo, S. Liberati, M. Visser, gr-qc/0505065, G. Volovik, gr-qc/0005091):
  - effective metric and its dynamics from hydrodynamics of microscopic system $\Rightarrow$ (modified) GR from GFT hydrodynamics?
  - quasi-particles (perturbations around ground state) see only effective metric $\Rightarrow$ effective QFT for matter in emergent metric from GFT? which effective matter QFT? incorporate QG effects? (DO, 0903.3970 [hep-th])
- General hypothesis
  - effective QG spacetime is non-commutative with Lie algebra structure: $[x_\mu, x_\nu] = \mathcal{C}^\alpha_{\mu\nu} x_\alpha$
  - by duality, corresponding momentum space is a (curved) group manifold (Majid)
  - symmetry group of such NC spacetimes is Hopf algebra
  - suggests QFT in such NC spacetime, in momentum space, is a GFT

These NC spacetimes are arena for much current QG phenomenology (G. Amelino-Camelia, 0806.0339 [gr-qc]), especially in the context of Deformed Special Relativity (DSR) (J. Kowalski-Glikman, hep-th/0405273)

Task: derive such NC field theories from more fundamental GFT models
3d Riemannian: W. Fairbairn, E. Livine, gr-qc/0702125
4d Lorentzian: DSR on κ-Minkowski: F.Girelli, E.Livine, DO, 0903.3475 [gr-qc]

EFFECTIVE NON-COMMUTATIVE MATTER FIELDS FROM GFT

- Insights from analog gravity models in condensed matter
  (C. Barcelo, S. Liberati, M. Visser, gr-qc/0505065, G. Volovik, gr-qc/0005091):
  - effective metric and its dynamics from hydrodynamics of microscopic system $\Rightarrow$ (modified) GR from GFT hydrodynamics?
  - quasi-particles (perturbations around ground state) see only effective metric $\Rightarrow$ effective QFT for matter in emergent metric from GFT? which effective matter QFT? incorporate QG effects? (DO, 0903.3970 [hep-th])
- General hypothesis
  - effective QG spacetime is non-commutative with Lie algebra structure: $[x_\mu, x_\nu] = \mathcal{C}^\alpha_{\mu\nu} x_\alpha$
  - by duality, corresponding momentum space is a (curved) group manifold (Majid)
  - symmetry group of such NC spacetimes is Hopf algebra
  - suggests QFT in such NC spacetime, in momentum space, is a GFT

These NC spacetimes are arena for much current QG phenomenology (G. Amelino-Camelia, 0806.0339 [gr-qc]), especially in the context of Deformed Special Relativity (DSR) (J. Kowalski-Glikman, hep-th/0405273)

Task: derive such NC field theories from more fundamental GFT models
3d Riemannian: W. Fairbairn, E. Livine, gr-qc/0702125
4d Lorentzian: DSR on κ-Minkowski: F.Girelli, E.Livine, DO, 0903.3475 [gr-qc]

THE 3D (RIEMANNIAN) CASE - NC FIELD THEORY FROM GFT

We deal with: $SU(2) \leftrightarrow \mathfrak{su}(2) : [X_i, X_j] = \frac{i}{\kappa}\epsilon_{ijk}X_k$ (Freidel, Majid, hep-th/0601004)

NC Fourier transform: $C(SU(2)) \to C_{*,\kappa}(\mathbb{R}^3)$ = functions on $\mathbb{R}^3$ endowed with star product reflecting group multiplication, based on plane waves $e^{i\,Tr(xg)}$ on $\mathbb{R}^3$ ($\vec{x} = x_i\sigma_i$) satisfying: $e^{i\,Tr(xg_1)} * e^{i\,Tr(xg_2)} = e^{i\,Tr(xg_1g_2)}$, $|k| \leq \kappa$.

$$\hat{\phi}(x) = \int_{SU(2)} dg\, e_g(x)\,\phi(g).$$

Scalar field theory in momentum space is:

$$S[\psi] = \frac{1}{2}\int_{SU(2)} dg\psi(g)\mathcal{K}(g)\psi(g^{-1}) - \frac{\lambda}{3!}\int [dg]^3\,\psi(g_1)\psi(g_2)\psi(g_3)\delta(g_1g_2g_3),$$

Boulatov GFT model: field $\varphi : G^3 \to \mathbb{C}$, with $G = SU(2)$, with $\varphi(g_1, g_2, g_3) = \varphi(g_1g, g_2g, g_3g)$, $\forall g \in G.$, and $\varphi(g_3, g_2, g_1) = \overline{\varphi}(g_1, g_2, g_3)$.

$$S[\varphi] = \frac{1}{2}\int (\varphi(g_1, g_2, g_3))^2 - \frac{\lambda}{4!}\int \varphi(g_1, g_2, g_3)\varphi(g_3, g_4, g_5)\varphi(g_5, g_2, g_6)\varphi(g_6, g_4, g_1).$$

Classical solution: $\varphi_0(g_1, g_2, g_3) \equiv \sqrt{\frac{3!}{\lambda}}\int dg\,\delta(g_1g)F(g_2g)\delta(g_3g)$. with $F(g) = F(hgh^{-1})\ \forall h \in G$, $\int F^2 = 1$, $F(g) \in \mathbb{R}$

Interpretation: 'regularized' quantum flat space on some space topology

THE 3D (RIEMANNIAN) CASE - NC FIELD THEORY FROM GFT

We deal with: $SU(2) \leftrightarrow \mathfrak{su}(2) : [X_i, X_j] = \frac{i}{\kappa}\epsilon_{ijk}X_k$ (Freidel, Majid, hep-th/0601004)
NC Fourier transform: $C(SU(2)) \rightarrow C_{*,\kappa}(\mathbb{R}^3)$ = functions on $\mathbb{R}^3$ endowed with star product reflecting group multiplication, based on plane waves $e^{i\,Tr(xg)}$ on $\mathbb{R}^3$ ($\vec{x} = x_i\sigma_i$) satisfying: $e^{i\,Tr(xg_1)} * e^{i\,Tr(xg_2)} = e^{i\,Tr(xg_1g_2)}$, $| k | \leq \kappa$.

$$\hat{\phi}(x) = \int_{SU(2)} dg\, e_g(x)\, \phi(g).$$

Scalar field theory in momentum space is:

$$S[\psi] = \frac{1}{2}\int_{SU(2)} dg\psi(g)\mathcal{K}(g)\psi(g^{-1}) - \frac{\lambda}{3!}\int [dg]^3\, \psi(g_1)\psi(g_2)\psi(g_3)\delta(g_1g_2g_3),$$

Boulatov GFT model: field $\varphi : G^3 \rightarrow \mathbb{C}$, with $G = SU(2)$, with $\varphi(g_1, g_2, g_3) = \varphi(g_1g, g_2g, g_3g)$, $\forall g \in G$., and $\varphi(g_3, g_2, g_1) = \overline{\varphi}(g_1, g_2, g_3)$.

$$S[\varphi] = \frac{1}{2}\int (\varphi(g_1, g_2, g_3))^2 - \frac{\lambda}{4!}\int \varphi(g_1, g_2, g_3)\varphi(g_3, g_4, g_5)\varphi(g_5, g_2, g_6)\varphi(g_6, g_4, g_1).$$

Classical solution: $\varphi_0(g_1, g_2, g_3) \equiv \sqrt{\frac{3!}{\lambda}}\int dg\, \delta(g_1g)F(g_2g)\delta(g_3g)$. with $F(g) = F(hgh^{-1})\ \forall h \in G$, $\int F^2 = 1$, $F(g) \in \mathbb{R}$
Interpretation: 'regularized'quantum flat space on some space topology

THE 3D (RIEMANNIAN) CASE - NC FIELD THEORY FROM GFT

We deal with: $SU(2) \leftrightarrow \mathfrak{su}(2) : [X_i, X_j] = \frac{i}{\kappa}\epsilon_{ijk}X_k$ (Freidel, Majid, hep-th/0601004)
NC Fourier transform: $C(SU(2)) \rightarrow C_{*,\kappa}(\mathbb{R}^3)$ = functions on $\mathbb{R}^3$ endowed with star product reflecting group multiplication, based on plane waves $e^{i\,Tr(xg)}$ on $\mathbb{R}^3$ ($\vec{x} = x_i\sigma_i$) satisfying: $e^{i\,Tr(xg_1)} * e^{i\,Tr(xg_2)} = e^{i\,Tr(xg_1g_2)}$, $|\, k \,| \leq \kappa$.

$$\hat{\phi}(x) = \int_{SU(2)} dg\, e_g(x)\, \phi(g).$$

Scalar field theory in momentum space is:

$$S[\psi] = \frac{1}{2}\int_{SU(2)} dg\psi(g)\mathcal{K}(g)\psi(g^{-1}) - \frac{\lambda}{3!}\int [dg]^3\, \psi(g_1)\psi(g_2)\psi(g_3)\delta(g_1g_2g_3),$$

Boulatov GFT model: field $\varphi : G^3 \rightarrow \mathbb{C}$, with $G = SU(2)$, with $\varphi(g_1, g_2, g_3) = \varphi(g_1g, g_2g, g_3g)$, $\forall g \in G$., and $\varphi(g_3, g_2, g_1) = \overline{\varphi}(g_1, g_2, g_3)$.

$$S[\varphi] = \frac{1}{2}\int (\varphi(g_1, g_2, g_3))^2 - \frac{\lambda}{4!}\int \varphi(g_1, g_2, g_3)\varphi(g_3, g_4, g_5)\varphi(g_5, g_2, g_6)\varphi(g_6, g_4, g_1).$$

Classical solution: $\varphi_0(g_1, g_2, g_3) \equiv \sqrt{\frac{3!}{\lambda}}\int dg\, \delta(g_1g)F(g_2g)\delta(g_3g)$. with $F(g) = F(hgh^{-1})\ \forall h \in G$, $\int F^2 = 1$, $F(g) \in \mathbb{R}$

Interpretation: 'regularized'quantum flat space on some space topology

THE 3D (RIEMANNIAN) CASE - NC FIELD THEORY FROM GFT

We deal with: $SU(2) \leftrightarrow \mathfrak{su}(2) : [X_i, X_j] = \frac{i}{\kappa}\epsilon_{ijk}X_k$ (Freidel, Majid, hep-th/0601004)
NC Fourier transform: $C(SU(2)) \to C_{*,\kappa}(\mathbb{R}^3)$ = functions on $\mathbb{R}^3$ endowed with star product reflecting group multiplication, based on plane waves $e^{i\,Tr(xg)}$ on $\mathbb{R}^3$ $(\vec{x} = x_i\sigma_i)$ satisfying: $e^{i\,Tr(xg_1)} * e^{i\,Tr(xg_2)} = e^{i\,Tr(xg_1g_2)}$, $|\,k\,| \leq \kappa$.

$$\hat{\phi}(x) = \int_{SU(2)} dg\, e_g(x)\, \phi(g).$$

Scalar field theory in momentum space is:

$$S[\psi] = \frac{1}{2}\int_{SU(2)} dg\psi(g)\mathcal{K}(g)\psi(g^{-1}) - \frac{\lambda}{3!}\int [dg]^3\, \psi(g_1)\psi(g_2)\psi(g_3)\delta(g_1g_2g_3),$$

Boulatov GFT model: field $\varphi : G^3 \to \mathbb{C}$, with $G = SU(2)$, with
$\varphi(g_1, g_2, g_3) = \varphi(g_1g, g_2g, g_3g)$, $\forall g \in G$., and $\varphi(g_3, g_2, g_1) = \overline{\varphi}(g_1, g_2, g_3)$.

$$S[\varphi] = \frac{1}{2}\int (\varphi(g_1, g_2, g_3))^2 - \frac{\lambda}{4!}\int \varphi(g_1, g_2, g_3)\varphi(g_3, g_4, g_5)\varphi(g_5, g_2, g_6)\varphi(g_6, g_4, g_1).$$

Classical solution: $\varphi_0(g_1, g_2, g_3) \equiv \sqrt{\frac{3!}{\lambda}}\int dg\, \delta(g_1g)F(g_2g)\delta(g_3g)$. with
$F(g) = F(hgh^{-1})\ \forall h \in G$, $\int F^2 = 1$, $F(g) \in \mathbb{R}$
Interpretation: 'regularized' quantum flat space on some space topology

THE 3D (RIEMANNIAN) CASE - NC FIELD THEORY FROM GFT

We deal with: $SU(2) \leftrightarrow \mathfrak{su}(2) : [X_i, X_j] = \frac{i}{\kappa}\epsilon_{ijk}X_k$ (Freidel, Majid, hep-th/0601004)
NC Fourier transform: $C(SU(2)) \rightarrow C_{*,\kappa}(\mathbb{R}^3)$ = functions on $\mathbb{R}^3$ endowed with star product reflecting group multiplication, based on plane waves $e^{i\,Tr(xg)}$ on $\mathbb{R}^3$
($\vec{x} = x_i\sigma_i$) satisfying: $e^{i\,Tr(xg_1)} * e^{i\,Tr(xg_2)} = e^{i\,Tr(xg_1g_2)}$, $\mid k \mid \leq \kappa$.

$$\hat{\phi}(x) = \int_{SU(2)} dg\, e_g(x)\, \phi(g).$$

Scalar field theory in momentum space is:

$$S[\psi] \;=\; \frac{1}{2}\int_{SU(2)} dg\psi(g)\mathcal{K}(g)\psi(g^{-1}) - \frac{\lambda}{3!}\int [dg]^3\, \psi(g_1)\psi(g_2)\psi(g_3)\delta(g_1g_2g_3),$$

Boulatov GFT model: field $\varphi : G^3 \rightarrow \mathbb{C}$, with $G = SU(2)$, with
$\varphi(g_1, g_2, g_3) \;=\; \varphi(g_1g, g_2g, g_3g),\;\; \forall g \in G.$, and $\varphi(g_3, g_2, g_1) = \overline{\varphi}(g_1, g_2, g_3)$.

$$S[\varphi] = \frac{1}{2}\int (\varphi(g_1, g_2, g_3))^2 - \frac{\lambda}{4!}\int \varphi(g_1, g_2, g_3)\varphi(g_3, g_4, g_5)\varphi(g_5, g_2, g_6)\varphi(g_6, g_4, g_1).$$

Classical solution: $\varphi_0(g_1, g_2, g_3) \;\equiv\; \sqrt{\frac{3!}{\lambda}}\int dg\, \delta(g_1g)F(g_2g)\delta(g_3g)$. with
$F(g) = F(hgh^{-1})\; \forall h \in G$, $\int F^2 = 1$, $F(g) \in \mathbb{R}$
Interpretation: 'regularized' quantum flat space on some space topology

NC FIELD THEORY FROM GFT: 2D PERTURBATIONS AND EFFECTIVE FT

Look at "two-dimensional" perturbation $\psi(g_1, g_3) = \psi(g_1 g_3^{-1})$ of field φ around φ_0:
$\varphi(g_1, g_2, g_3) = \varphi_0(g_1, g_2, g_3) + \psi(g_1, g_3)$ Then effective action for ψ is

$$S_{eff}[\psi] \equiv S[\varphi_0 + \psi] - S[\varphi_0] = \frac{1}{2}\int \psi(g)\mathcal{K}(g)\psi(g^{-1}) -$$

$$-\frac{\mu}{3!}\int [dg]^3\, \psi(g_1)\psi(g_2)\psi(g_3)\delta(g_1 g_2 g_3) - \frac{\lambda}{4!}\int [dg]^4\, \psi(g_1)..\psi(g_4)\delta(g_1..g_4),$$

with

$$\mathcal{K}(g) = 1 - 2\left(\int F\right)^2 - \int dh F(h)F(hg), \qquad \frac{\mu}{3!} = \sqrt{\frac{\lambda}{3!}}\int F$$

The effective action S_{eff} depends on the solution φ_0 through F, and is invariant under $DSU(2)$ (quantum double of SU(2)), deformation of Poincaré group
F can be expanded in group characters: $F(g) = \sum_{j\in\mathbb{N}/2} F_j \chi_j(g)$

$$\mathcal{K}(g) = \sum_{j>0} F_j^2 \left(1 - \frac{\chi_j(g)}{d_j}\right) - 2F_0^2 \equiv Q^2(g) - M^2.$$

$Q^2(g) \geq 0$, $Q(I) = 0$, generalized "Laplacian"
For $F(g) = a + \sqrt{1-a^2}\chi_1(g)$ we get $\mathcal{K}(g) = \frac{4}{3}(1-a^2)\vec{k}^2 - 2a^2$

NC FIELD THEORY FROM GFT: 2D PERTURBATIONS AND EFFECTIVE FT

Look at "two-dimensional" perturbation $\psi(g_1, g_3) = \psi(g_1 g_3^{-1})$ of field φ around φ_0:
$\varphi(g_1, g_2, g_3) = \varphi_0(g_1, g_2, g_3) + \psi(g_1, g_3)$ Then effective action for ψ is

$$S_{eff}[\psi] \equiv S[\varphi_0 + \psi] - S[\varphi_0] = \frac{1}{2} \int \psi(g) \mathcal{K}(g) \psi(g^{-1}) -$$

$$- \frac{\mu}{3!} \int [dg]^3 \, \psi(g_1)\psi(g_2)\psi(g_3)\delta(g_1 g_2 g_3) - \frac{\lambda}{4!} \int [dg]^4 \, \psi(g_1)..\psi(g_4)\delta(g_1..g_4),$$

with

$$\mathcal{K}(g) = 1 - 2\left(\int F\right)^2 - \int dh F(h) F(hg), \qquad \frac{\mu}{3!} = \sqrt{\frac{\lambda}{3!}} \int F$$

The effective action S_{eff} depends on the solution φ_0 through F, and is invariant under $\mathcal{D}SU(2)$ (quantum double of SU(2)), deformation of Poincaré group
F can be expanded in group characters: $F(g) = \sum_{j \in \mathbb{N}/2} F_j \chi_j(g)$

$$\mathcal{K}(g) = \sum_{j>0} F_j^2 \left(1 - \frac{\chi_j(g)}{d_j}\right) - 2F_0^2 \equiv Q^2(g) - M^2.$$

$Q^2(g) \geq 0$, $Q(I) = 0$, generalized "Laplacian"
For $F(g) = a + \sqrt{1-a^2}\chi_1(g)$ we get $\mathcal{K}(g) = \frac{4}{3}(1-a^2)\vec{k}^2 - 2a^2$

NC field theory from GFT: 2D perturbations and effective FT

Look at "two-dimensional" perturbation $\psi(g_1, g_3) = \psi(g_1 g_3^{-1})$ of field φ around φ_0:
$\varphi(g_1, g_2, g_3) = \varphi_0(g_1, g_2, g_3) + \psi(g_1, g_3)$ Then effective action for ψ is

$$S_{eff}[\psi] \equiv S[\varphi_0 + \psi] - S[\varphi_0] = \frac{1}{2} \int \psi(g)\mathcal{K}(g)\psi(g^{-1}) -$$

$$- \frac{\mu}{3!} \int [dg]^3 \psi(g_1)\psi(g_2)\psi(g_3)\delta(g_1 g_2 g_3) - \frac{\lambda}{4!} \int [dg]^4 \psi(g_1)..\psi(g_4)\delta(g_1..g_4),$$

with

$$\mathcal{K}(g) = 1 - 2\left(\int F\right)^2 - \int dh F(h)F(hg), \qquad \frac{\mu}{3!} = \sqrt{\frac{\lambda}{3!}} \int F$$

The effective action S_{eff} depends on the solution φ_0 through F, and is invariant under DSU(2) (quantum double of SU(2)), deformation of Poincaré group

F can be expanded in group characters: $F(g) = \sum_{j \in \mathbb{N}/2} F_j \chi_j(g)$

$$\mathcal{K}(g) = \sum_{j>0} F_j^2 \left(1 - \frac{\chi_j(g)}{d_j}\right) - 2F_0^2 \equiv Q^2(g) - M^2.$$

$Q^2(g) \geq 0$, $Q(I) = 0$, generalized "Laplacian"

For $F(g) = a + \sqrt{1 - a^2}\chi_1(g)$ we get $\mathcal{K}(g) = \frac{4}{3}(1 - a^2)\vec{k}^2 - 2a^2$

NC FIELD THEORY FROM GFT: 2D PERTURBATIONS AND EFFECTIVE FT

Look at "two-dimensional" perturbation $\psi(g_1, g_3) = \psi(g_1 g_3^{-1})$ of field φ around φ_0:
$\varphi(g_1, g_2, g_3) = \varphi_0(g_1, g_2, g_3) + \psi(g_1, g_3)$ Then effective action for ψ is

$$S_{eff}[\psi] \equiv S[\varphi_0 + \psi] - S[\varphi_0] = \frac{1}{2}\int \psi(g)\mathcal{K}(g)\psi(g^{-1}) -$$

$$-\frac{\mu}{3!}\int [dg]^3\,\psi(g_1)\psi(g_2)\psi(g_3)\delta(g_1 g_2 g_3) - \frac{\lambda}{4!}\int [dg]^4\,\psi(g_1)..\psi(g_4)\delta(g_1..g_4),$$

with

$$\mathcal{K}(g) = 1 - 2\left(\int F\right)^2 - \int dhF(h)F(hg), \qquad \frac{\mu}{3!} = \sqrt{\frac{\lambda}{3!}}\int F$$

The effective action S_{eff} depends on the solution φ_0 through F, and is invariant under DSU(2) (quantum double of SU(2)), deformation of Poincaré group
F can be expanded in group characters: $F(g) = \sum_{j\in\mathbb{N}/2} F_j \chi_j(g)$

$$\mathcal{K}(g) = \sum_{j>0} F_j^2\left(1 - \frac{\chi_j(g)}{d_j}\right) - 2F_0^2 \equiv Q^2(g) - M^2.$$

$Q^2(g) \geq 0$, $Q(I) = 0$, generalized "Laplacian"
For $F(g) = a + \sqrt{1-a^2}\chi_1(g)$ we get $\mathcal{K}(g) = \frac{4}{3}(1-a^2)\,\vec{k}^{\,2} - 2a^2$

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- main theme: gravity as constrained (BF) topological field theory

$$S(B,\omega,\lambda) = \int B \wedge F(\omega) + \int \lambda\, C(B) \approx_{C(B)=0} S_{GR}(e,\omega)$$

$\Rightarrow$ strategy: construct SF/GFT models as properly constrained SF/GFT models for 4D discrete BF theory ($B \to \mathfrak{so}(4)$ elements, $\omega \to \mathrm{SO}(4)$ elements)

$$S[\phi] = \int dg_i\, \phi(g_1,g_2,g_3,g_4)^2 + \lambda \int dg_j\, \phi(g_1,g_2,g_3,g_4)\phi(g_4,g_5,g_6,g_7)....\phi(g_{10},g_8,g_6,g_1)$$

- ANY spin foam model gives rise to/is defined by a GFT $\Rightarrow$ new SF models (EPR, Freidel-Krasnov) (cf. Barrett's talk) $\approx$ new GFT models for 4D QG
- work directly at GFT level: encode simplicial geometry in GFT action $\Rightarrow$ GFT amplitudes are simplicial gravity path integrals
- immediate difficulty: constraints have to be imposed on B variables............but no B variables in GFT formulation
- a possible strategy (usual SF/GFT construction)(from geometric quantization):
  - expand functions of G in representations j
  - identify $B^{IJ} \simeq T_j^{IJ}$ generators of $SO(4)$ (use conjugacy relation between B and ω)
  - identify (some) quantum parameters in representation space j as discrete B variables and impose constraints $C(B)=0$ on them
  - issues......

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- main theme: gravity as constrained (BF) topological field theory

$$S(B,\omega,\lambda) = \int B \wedge F(\omega) + \int \lambda\, C(B) \approx_{C(B)=0} S_{GR}(e,\omega)$$

$\Rightarrow$ strategy: construct SF/GFT models as properly constrained SF/GFT models for 4D discrete BF theory ($B \to \mathfrak{so}(4)$ elements, $\omega \to \mathrm{SO}(4)$ elements)

$$S[\phi] = \int dg_i\, \phi(g_1,g_2,g_3,g_4)^2 + \lambda \int dg_j\, \phi(g_1,g_2,g_3,g_4)\phi(g_4,g_5,g_6,g_7)....\phi(g_{10},g_8,g_6,g_1)$$

- ANY spin foam model gives rise to/is defined by a GFT $\Rightarrow$ new SF models (EPR, Freidel-Krasnov) (cf. Barrett's talk) $\approx$ new GFT models for 4D QG
- work directly at GFT level: encode simplicial geometry in GFT action $\Rightarrow$ GFT amplitudes are simplicial gravity path integrals
- immediate difficulty: constraints have to be imposed on B variables............but no B variables in GFT formulation
- a possible strategy (usual SF/GFT construction)(from geometric quantization):
  - expand functions of G in representations j
  - identify $B^{IJ} \simeq T_j^{IJ}$ generators of $SO(4)$ (use conjugacy relation between B and ω)
  - identify (some) quantum parameters in representation space j as discrete B variables and impose constraints $C(B) = 0$ on them
  - issues......

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- main theme: gravity as constrained (BF) topological field theory

$$S(B,\omega,\lambda) \;=\; \int B \wedge F(\omega) \;+\; \int \lambda\, C(B) \;\approx_{C(B)=0}\; S_{GR}(e,\omega)$$

$\Rightarrow$ strategy: construct SF/GFT models as properly constrained SF/GFT models for 4D discrete BF theory ($B \rightarrow \mathfrak{so}(4)$ elements, $\omega \rightarrow \mathrm{SO}(4)$ elements)

$$S[\phi] = \int dg_i\, \phi(g_1,g_2,g_3,g_4)^2 + \lambda \int dg_j\, \phi(g_1,g_2,g_3,g_4)\phi(g_4,g_5,g_6,g_7)....\phi(g_{10},g_8,g_6,g_1)$$

- ANY spin foam model gives rise to/is defined by a GFT $\Rightarrow$ new SF models (EPR, Freidel-Krasnov) (cf. Barrett's talk) $\approx$ new GFT models for 4D QG
- work directly at GFT level: encode simplicial geometry in GFT action $\Rightarrow$ GFT amplitudes are simplicial gravity path integrals
- immediate difficulty: constraints have to be imposed on B variables............but no B variables in GFT formulation
- a possible strategy (usual SF/GFT construction)(from geometric quantization):
  - expand functions of G in representations j
  - identify $B^{IJ} \simeq T_j^{IJ}$ generators of $SO(4)$ (use conjugacy relation between B and ω)
  - identify (some) quantum parameters in representation space j as discrete B variables and impose constraints $C(B)=0$ on them
  - issues......

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- main theme: gravity as constrained (BF) topological field theory

$$S(B,\omega,\lambda) = \int B \wedge F(\omega) + \int \lambda\, C(B) \approx_{C(B)=0} S_{GR}(e,\omega)$$

$\Rightarrow$ strategy: construct SF/GFT models as properly constrained SF/GFT models for 4D discrete BF theory ($B \to \mathfrak{so}(4)$ elements, $\omega \to \mathrm{SO}(4)$ elements)

$$S[\phi] = \int dg_i\, \phi(g_1,g_2,g_3,g_4)^2 + \lambda \int dg_j\, \phi(g_1,g_2,g_3,g_4)\phi(g_4,g_5,g_6,g_7)....\phi(g_{10},g_8,g_6,g_1)$$

- ANY spin foam model gives rise to/is defined by a GFT $\Rightarrow$ new SF models (EPR, Freidel-Krasnov) (cf. Barrett's talk) $\approx$ new GFT models for 4D QG
- work directly at GFT level: encode simplicial geometry in GFT action $\Rightarrow$ GFT amplitudes are simplicial gravity path integrals
- immediate difficulty: constraints have to be imposed on B variables............but no B variables in GFT formulation
- a possible strategy (usual SF/GFT construction)(from geometric quantization):
  - expand functions of G in representations j
  - identify $B^{IJ} \simeq T_j^{IJ}$ generators of $SO(4)$ (use conjugacy relation between B and ω)
  - identify (some) quantum parameters in representation space j as discrete B variables and impose constraints $C(B) = 0$ on them
  - issues......

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- main theme: gravity as constrained (BF) topological field theory

$$S(B,\omega,\lambda) = \int B \wedge F(\omega) + \int \lambda\, C(B) \approx_{C(B)=0} S_{GR}(e,\omega)$$

  $\Rightarrow$ strategy: construct SF/GFT models as properly constrained SF/GFT models for 4D discrete BF theory ($B \rightarrow \mathfrak{so}(4)$ elements, $\omega \rightarrow \mathrm{SO}(4)$ elements)

$$S[\phi] = \int dg_i\, \phi(g_1,g_2,g_3,g_4)^2 + \lambda \int dg_j\, \phi(g_1,g_2,g_3,g_4)\phi(g_4,g_5,g_6,g_7)....\phi(g_{10},g_8,g_6,g_1)$$

- ANY spin foam model gives rise to/is defined by a GFT $\Rightarrow$ new SF models (EPR, Freidel-Krasnov) (cf. Barrett's talk) $\approx$ new GFT models for 4D QG
- work directly at GFT level: encode simplicial geometry in GFT action $\Rightarrow$ GFT amplitudes are simplicial gravity path integrals
- immediate difficulty: constraints have to be imposed on B variables............but no B variables in GFT formulation
- a possible strategy (usual SF/GFT construction)(from geometric quantization):
  - expand functions of G in representations j
  - identify $B^{IJ} \simeq T_j^{IJ}$ generators of $SO(4)$ (use conjugacy relation between B and ω)
  - identify (some) quantum parameters in representation space j as discrete B variables and impose constraints $C(B) = 0$ on them
  - issues......

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- main theme: gravity as constrained (BF) topological field theory

$$S(B,\omega,\lambda) = \int B \wedge F(\omega) + \int \lambda\, C(B) \approx_{C(B)=0} S_{GR}(e,\omega)$$

$\Rightarrow$ strategy: construct SF/GFT models as properly constrained SF/GFT models for 4D discrete BF theory ($B \rightarrow \mathfrak{so}(4)$ elements, $\omega \rightarrow \mathrm{SO}(4)$ elements)

$$S[\phi] = \int dg_i\, \phi(g_1,g_2,g_3,g_4)^2 + \lambda \int dg_j\, \phi(g_1,g_2,g_3,g_4)\phi(g_4,g_5,g_6,g_7)....\phi(g_{10},g_8,g_6,g_1)$$

- ANY spin foam model gives rise to/is defined by a GFT $\Rightarrow$ new SF models (EPR, Freidel-Krasnov) (cf. Barrett's talk) $\approx$ new GFT models for 4D QG
- work directly at GFT level: encode simplicial geometry in GFT action $\Rightarrow$ GFT amplitudes are simplicial gravity path integrals
- immediate difficulty: constraints have to be imposed on B variables............but no B variables in GFT formulation
- a possible strategy (usual SF/GFT construction)(from geometric quantization):
  - expand functions of G in representations j
  - identify $B^{IJ} \simeq T_j^{IJ}$ generators of $SO(4)$ (use conjugacy relation between B and ω)
  - identify (some) quantum parameters in representation space j as discrete B variables and impose constraints $C(B) = 0$ on them
  - issues......

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- main theme: gravity as constrained (BF) topological field theory

$$S(B,\omega,\lambda) = \int B \wedge F(\omega) + \int \lambda\, C(B) \approx_{C(B)=0} S_{GR}(e,\omega)$$

$\Rightarrow$ strategy: construct SF/GFT models as properly constrained SF/GFT models for 4D discrete BF theory ($B \to \mathfrak{so}(4)$ elements, $\omega \to \mathrm{SO}(4)$ elements)

$$S[\phi] = \int dg_i\, \phi(g_1,g_2,g_3,g_4)^2 + \lambda \int dg_j\, \phi(g_1,g_2,g_3,g_4)\phi(g_4,g_5,g_6,g_7)....\phi(g_{10},g_8,g_6,g_1)$$

- ANY spin foam model gives rise to/is defined by a GFT $\Rightarrow$ new SF models (EPR, Freidel-Krasnov) (cf. Barrett's talk) $\approx$ new GFT models for 4D QG
- work directly at GFT level: encode simplicial geometry in GFT action $\Rightarrow$ GFT amplitudes are simplicial gravity path integrals
- immediate difficulty: constraints have to be imposed on B variables............but no B variables in GFT formulation
- a possible strategy (usual SF/GFT construction)(from geometric quantization):
  - expand functions of G in representations j
  - identify $B^{IJ} \simeq T^{IJ}_j$ generators of $SO(4)$ (use conjugacy relation between B and ω)
  - identify (some) quantum parameters in representation space j as discrete B variables and impose constraints $C(B)=0$ on them
  - issues......

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- main theme: gravity as constrained (BF) topological field theory

$$S(B,\omega,\lambda) = \int B \wedge F(\omega) + \int \lambda C(B) \approx_{C(B)=0} S_{GR}(e,\omega)$$

$\Rightarrow$ strategy: construct SF/GFT models as properly constrained SF/GFT models for 4D discrete BF theory ($B \to \mathfrak{so}(4)$ elements, $\omega \to \mathrm{SO}(4)$ elements)

$$S[\phi] = \int dg_i\, \phi(g_1,g_2,g_3,g_4)^2 + \lambda \int dg_j\, \phi(g_1,g_2,g_3,g_4)\phi(g_4,g_5,g_6,g_7)....\phi(g_{10},g_8,g_6,g_1)$$

- ANY spin foam model gives rise to/is defined by a GFT $\Rightarrow$ new SF models (EPR, Freidel-Krasnov) (cf. Barrett's talk) $\approx$ new GFT models for 4D QG
- work directly at GFT level: encode simplicial geometry in GFT action $\Rightarrow$ GFT amplitudes are simplicial gravity path integrals
- immediate difficulty: constraints have to be imposed on B variables............but no B variables in GFT formulation
- a possible strategy (usual SF/GFT construction)(from geometric quantization):
  - expand functions of G in representations j
  - identify $B^{IJ} \simeq T_j^{IJ}$ generators of $SO(4)$ (use conjugacy relation between B and ω)
  - identify (some) quantum parameters in representation space j as discrete B variables and impose constraints $C(B) = 0$ on them
  - issues......

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- main theme: gravity as constrained (BF) topological field theory

$$S(B,\omega,\lambda) = \int B \wedge F(\omega) + \int \lambda\, C(B) \approx_{C(B)=0} S_{GR}(e,\omega)$$

$\Rightarrow$ strategy: construct SF/GFT models as properly constrained SF/GFT models for 4D discrete BF theory ($B \rightarrow \mathfrak{so}(4)$ elements, $\omega \rightarrow \mathrm{SO}(4)$ elements)

$$S[\phi] = \int dg_i\, \phi(g_1,g_2,g_3,g_4)^2 + \lambda \int dg_j\, \phi(g_1,g_2,g_3,g_4)\phi(g_4,g_5,g_6,g_7)....\phi(g_{10},g_8,g_6,g_1)$$

- ANY spin foam model gives rise to/is defined by a GFT $\Rightarrow$ new SF models (EPR, Freidel-Krasnov) (cf. Barrett's talk) $\approx$ new GFT models for 4D QG
- work directly at GFT level: encode simplicial geometry in GFT action $\Rightarrow$ GFT amplitudes are simplicial gravity path integrals
- immediate difficulty: constraints have to be imposed on B variables............but no B variables in GFT formulation
- a possible strategy (usual SF/GFT construction)(from geometric quantization):
  - expand functions of G in representations j
  - identify $B^{IJ} \simeq T_j^{IJ}$ generators of $SO(4)$ (use conjugacy relation between B and ω)
  - identify (some) quantum parameters in representation space j as discrete B variables and impose constraints $C(B) = 0$ on them
  - issues......

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- alternative 1(DO, 0902.3903 [gr-qc]) :
  - extend the GFT formalism $\phi(g_i) \longrightarrow \phi(g_i, B_i) : (G \times \mathfrak{g})^{\times 4}$
  - relax relation $B^{IJ} \simeq T_j^{IJ} \longrightarrow$ kinetic operator: $\mathcal{K} = \prod_i (B_i^2 + \Box_i)$
  - simple BF vertex term
  - impose constraints $C(B_i)$ on B variables in GFT action

$$S = \int C(B_i)\phi(g_i, B_i) \left(\prod_{i=1} (\Box_i + B_i^2) \right) \phi(g_i, B_i) + \lambda \int [\phi(g, B)]^5$$

result: simplicial path integral for BF-like simplicial gravity

$$Z(\Gamma) = \int dB \int dg \, \mu(g, B) \, e^{iS_R[B,H(g)]} \, e^{iS_c[B,H(g)]}$$

measure contains contains primary constraints on B and secondary constraints on g, S_R is Regge action, S_c is quantum correction to bare action

- issues: measure, S_c corrections, boundary configurations
  $\Rightarrow$ conjugate nature of (B_i, g_i), non-commutative nature of B_i

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- alternative 1(DO, 0902.3903 [gr-qc]) :
  - extend the GFT formalism $\phi(g_i) \longrightarrow \phi(g_i, B_i) : (G \times \mathfrak{g})^{\times 4}$
  - relax relation $B^{IJ} \simeq T_j^{IJ} \longrightarrow$ kinetic operator: $\mathcal{K} = \prod_i (B_i^2 + \Box_i)$
  - simple BF vertex term
  - impose constraints $C(B_i)$ on B variables in GFT action

$$S = \int C(B_i)\phi(g_i, B_i) \left(\prod_{i=1} (\Box_i + B_i^2) \right) \phi(g_i, B_i) + \lambda \int [\phi(g, B)]^5$$

result: simplicial path integral for BF-like simplicial gravity

$$Z(\Gamma) = \int dB \int dg\, \mu(g, B)\, e^{iS_R[B,H(g)]}\, e^{iS_c[B,H(g)]}$$

measure contains contains primary constraints on B and secondary constraints on g, S_R is Regge action, S_c is quantum correction to bare action

- issues: measure, S_c corrections, boundary configurations
  $\Rightarrow$ conjugate nature of (B_i, g_i), non-commutative nature of B_i

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- alternative 1(DO, 0902.3903 [gr-qc]) :
  - extend the GFT formalism $\phi(g_i) \longrightarrow \phi(g_i, B_i) : (G \times \mathfrak{g})^{\times 4}$
  - relax relation $B^{IJ} \simeq T_j^{IJ} \longrightarrow$ kinetic operator: $\mathcal{K} = \prod_i (B_i^2 + \Box_i)$
  - simple BF vertex term
  - impose constraints $C(B_i)$ on B variables in GFT action

$$S = \int C(B_i)\phi(g_i, B_i) \left(\prod_{i=1} (\Box_i + B_i^2) \right) \phi(g_i, B_i) + \lambda \int [\phi(g, B)]^5$$

  result: simplicial path integral for BF-like simplicial gravity

$$Z(\Gamma) = \int dB \int dg \, \mu(g, B) \, e^{iS_R[B,H(g)]} \, e^{iS_c[B,H(g)]}$$

  measure contains contains primary constraints on B and secondary constraints on g, S_R is Regge action, S_c is quantum correction to bare action

- issues: measure, S_c corrections, boundary configurations
  $\Rightarrow$ conjugate nature of (B_i, g_i), non-commutative nature of B_i

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- alternative 1(DO, 0902.3903 [gr-qc]) :
  - extend the GFT formalism $\phi(g_i) \longrightarrow \phi(g_i, B_i) : (G \times \mathfrak{g})^{\times 4}$
  - relax relation $B^{IJ} \simeq T_j^{IJ} \longrightarrow$ kinetic operator: $\mathcal{K} = \prod_i (B_i^2 + \Box_i)$
  - simple BF vertex term
  - impose constraints $C(B_i)$ on B variables in GFT action

$$S = \int C(B_i)\phi(g_i, B_i) \left(\prod_{i=1} (\Box_i + B_i^2) \right) \phi(g_i, B_i) + \lambda \int [\phi(g, B)]^5$$

result: simplicial path integral for BF-like simplicial gravity

$$Z(\Gamma) = \int dB \int dg \, \mu(g, B) \, e^{iS_R[B,H(g)]} \, e^{iS_c[B,H(g)]}$$

measure contains contains primary constraints on B and secondary constraints on g, S_R is Regge action, S_c is quantum correction to bare action

- issues: measure, S_c corrections, boundary configurations
  $\Rightarrow$ conjugate nature of (B_i, g_i), non-commutative nature of B_i

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- alternative 1(DO, 0902.3903 [gr-qc]) :
  - extend the GFT formalism $\phi(g_i) \rightarrow \phi(g_i, B_i)$: $(G \times \mathfrak{g})^{\times 4}$
  - relax relation $B^{IJ} \simeq T_j^{IJ} \rightarrow$ kinetic operator: $\mathcal{K} = \prod_i (B_i^2 + \Box_i)$
  - simple BF vertex term
  - impose constraints $C(B_i)$ on B variables in GFT action

$$S = \int C(B_i)\phi(g_i, B_i) \left(\prod_{i=1} (\Box_i + B_i^2) \right) \phi(g_i, B_i) + \lambda \int [\phi(g, B)]^5$$

result: simplicial path integral for BF-like simplicial gravity

$$Z(\Gamma) = \int dB \int dg \, \mu(g, B) \, e^{iS_R[B, H(g)]} \, e^{iS_c[B, H(g)]}$$

measure contains contains primary constraints on B and secondary constraints on g, S_R is Regge action, S_c is quantum correction to bare action

- issues: measure, S_c corrections, boundary configurations
  $\Rightarrow$ conjugate nature of (B_i, g_i), non-commutative nature of B_i

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- alternative 1(DO, 0902.3903 [gr-qc]) :
  - extend the GFT formalism $\phi(g_i) \rightarrow \phi(g_i, B_i) : (G \times \mathfrak{g})^{\times 4}$
  - relax relation $B^{IJ} \simeq T_j^{IJ} \rightarrow$ kinetic operator: $\mathcal{K} = \prod_i (B_i^2 + \Box_i)$
  - simple BF vertex term
  - impose constraints $C(B_i)$ on B variables in GFT action

$$S = \int C(B_i)\phi(g_i, B_i) \left(\prod_{i=1} (\Box_i + B_i^2) \right) \phi(g_i, B_i) + \lambda \int [\phi(g, B)]^5$$

result: simplicial path integral for BF-like simplicial gravity

$$Z(\Gamma) = \int dB \int dg\, \mu(g, B)\, e^{iS_R[B,H(g)]}\, e^{iS_c[B,H(g)]}$$

measure contains contains primary constraints on B and secondary constraints on g, S_R is Regge action, S_c is quantum correction to bare action

- issues: measure, S_c corrections, boundary configurations
  $\Rightarrow$ conjugate nature of (B_i, g_i), non-commutative nature of B_i

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- alternative 2 (A. Baratin, DO, in preparation) :
  - introduce B variables by no-commutatative Fourier transform $\phi(g_i) \longrightarrow \phi(B_i)$ $C(SO(4)) \longrightarrow C_{*,\kappa}(\wedge^2\mathbb{R}^3)$ = functions on $\mathfrak{g} \simeq \wedge^2\mathbb{R}^3$ with star product, based on plane waves satisfying: $e^{i\,Tr(Bg_1)} * e^{i\,Tr(Bg_2)} = e^{i\,Tr(Bg_1 g_2)}$
  - turn GFT into non-commutative, combinatorially non-local QFT on algebra $\mathfrak{g}^{\times 4}$
  - impose constraints $C(B_i)$ on B variables in GFT action

results:

- pure BF:

$$Z(\Gamma) = \int dB \int dg\, e^{i\sum_e Tr(B_e H_e(g))}$$

  correct simplicial geometry, correct symmetries, correct boundary terms, ...
- 4D gravity: $C(B_i)$ correctly implemented:

$$Z(\Gamma) = \int \mathcal{D}B \int \mathcal{D}g\, C(\{B_e\}) e^{i\sum_e Tr(B_e H_e(g))}$$

  correct secondary constraints?, relation to other SF models?, details of simplicial geometry?,.....
- in progress.....

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- alternative 2 (A. Baratin, DO, in preparation) :
  - introduce B variables by no-commutatative Fourier transform $\phi(g_i) \longrightarrow \phi(B_i)$
    $C(SO(4)) \longrightarrow C_{*,\kappa}(\wedge^2\mathbb{R}^3)$ = functions on $\mathfrak{g} \simeq \wedge^2\mathbb{R}^3$ with star product, based on plane waves satisfying: $e^{i\,Tr(Bg_1)} * e^{i\,Tr(Bg_2)} = e^{i\,Tr(Bg_1g_2)}$
  - turn GFT into non-commutative, combinatorially non-local QFT on algebra $\mathfrak{g}^{\times 4}$
  - impose constraints $C(B_i)$ on B variables in GFT action

results:

  - pure BF:

$$Z(\Gamma) = \int dB \int dg\, e^{i\sum_e Tr(B_e H_e(g))}$$

    correct simplicial geometry, correct symmetries, correct boundary terms, ...
  - 4D gravity: $C(B_i)$ correctly implemented:

$$Z(\Gamma) = \int \mathcal{D}B \int \mathcal{D}g\, C(\{B_e\}) e^{i\sum_e Tr(B_e H_e(g))}$$

    correct secondary constraints?, relation to other SF models?, details of simplicial geometry?,.....
  - in progress.....

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- alternative 2 (A. Baratin, DO, in preparation) :
  - introduce B variables by no-commutatative Fourier transform $\phi(g_i) \longrightarrow \phi(B_i)$
    $C(SO(4)) \longrightarrow C_{*,\kappa}(\wedge^2\mathbb{R}^3)$ = functions on $\mathfrak{g} \simeq \wedge^2\mathbb{R}^3$ with star product, based on plane waves satisfying: $e^{i\,Tr(Bg_1)} * e^{i\,Tr(Bg_2)} = e^{i\,Tr(Bg_1g_2)}$
  - turn GFT into non-commutative, combinatorially non-local QFT on algebra $\mathfrak{g}^{\times 4}$
  - impose constraints $C(B_i)$ on B variables in GFT action

  results:
  - pure BF:

$$Z(\Gamma) = \int dB \int dg\, e^{i\sum_e Tr(B_e H_e(g))}$$

    correct simplicial geometry, correct symmetries, correct boundary terms, ...
  - 4D gravity: $C(B_i)$ correctly implemented:

$$Z(\Gamma) = \int \mathcal{D}B \int \mathcal{D}g\, C(\{B_e\}) e^{i\sum_e Tr(B_e H_e(g))}$$

    correct secondary constraints?, relation to other SF models?, details of simplicial geometry?,.....
  - in progress.....

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- alternative 2 (A. Baratin, DO, in preparation) :
  - introduce B variables by no-commutatative Fourier transform $\phi(g_i) \longrightarrow \phi(B_i)$ $C(\mathrm{SO}(4)) \longrightarrow C_{*,\kappa}(\wedge^2\mathbb{R}^3) =$ functions on $\mathfrak{g} \simeq \wedge^2\mathbb{R}^3$ with star product, based on plane waves satisfying: $e^{i\,Tr(Bg_1)} * e^{i\,Tr(Bg_2)} = e^{i\,Tr(Bg_1g_2)}$
  - turn GFT into non-commutative, combinatorially non-local QFT on algebra $\mathfrak{g}^{\times 4}$
  - impose constraints $C(B_i)$ on B variables in GFT action

results:

- pure BF:

$$Z(\Gamma) = \int dB \int dg\, e^{i\sum_e Tr(B_e H_e(g))}$$

  correct simplicial geometry, correct symmetries, correct boundary terms, ...
- 4D gravity: $C(B_i)$ correctly implemented:

$$Z(\Gamma) = \int \mathcal{D}B \int \mathcal{D}g\, C(\{B_e\}) e^{i\sum_e Tr(B_e H_e(g))}$$

  correct secondary constraints?, relation to other SF models?, details of simplicial geometry?,.....
- in progress.....

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- alternative 2 (A. Baratin, DO, in preparation) :
  - introduce B variables by no-commutatative Fourier transform $\phi(g_i) \longrightarrow \phi(B_i)$
    $C(SO(4)) \longrightarrow C_{*,\kappa}(\wedge^2\mathbb{R}^3)$ = functions on $\mathfrak{g} \simeq \wedge^2\mathbb{R}^3$ with star product, based on plane waves satisfying: $e^{i\,Tr(Bg_1)} * e^{i\,Tr(Bg_2)} = e^{i\,Tr(Bg_1g_2)}$
  - turn GFT into non-commutative, combinatorially non-local QFT on algebra $\mathfrak{g}^{\times 4}$
  - impose constraints $C(B_i)$ on B variables in GFT action

results:

- pure BF:

$$Z(\Gamma) = \int dB \int dg\, e^{i\sum_e Tr(B_e H_e(g))}$$

  correct simplicial geometry, correct symmetries, correct boundary terms, ...
- 4D gravity: $C(B_i)$ correctly implemented:

$$Z(\Gamma) = \int \mathcal{D}B \int \mathcal{D}g\, C(\{B_e\}) e^{i\sum_e Tr(B_e H_e(g))}$$

  correct secondary constraints?, relation to other SF models?, details of simplicial geometry?,.....
- in progress.....

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- alternative 2 (A. Baratin, DO, in preparation) :
  - introduce B variables by no-commutatative Fourier transform $\phi(g_i) \longrightarrow \phi(B_i)$ $C(SO(4)) \longrightarrow C_{*,\kappa}(\wedge^2\mathbb{R}^3)$ = functions on $\mathfrak{g} \simeq \wedge^2\mathbb{R}^3$ with star product, based on plane waves satisfying: $e^{i\,Tr(Bg_1)} * e^{i\,Tr(Bg_2)} = e^{i\,Tr(Bg_1 g_2)}$
  - turn GFT into non-commutative, combinatorially non-local QFT on algebra $\mathfrak{g}^{\times 4}$
  - impose constraints $C(B_i)$ on B variables in GFT action

results:

- pure BF:

$$Z(\Gamma) = \int dB \int dg\, e^{i \sum_e Tr(B_e H_e(g))}$$

correct simplicial geometry, correct symmetries, correct boundary terms, ...

- 4D gravity: $C(B_i)$ correctly implemented:

$$Z(\Gamma) = \int \mathcal{D}B \int \mathcal{D}g\, C(\{B_e\}) e^{i \sum_e Tr(B_e H_e(g))}$$

correct secondary constraints?, relation to other SF models?, details of simplicial geometry?,.....

- in progress.....

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- alternative 2 (A. Baratin, DO, in preparation) :
  - introduce B variables by no-commutatative Fourier transform $\phi(g_i) \longrightarrow \phi(B_i)$
    $C(SO(4)) \longrightarrow C_{*,\kappa}(\wedge^2\mathbb{R}^3)$ = functions on $\mathfrak{g} \simeq \wedge^2\mathbb{R}^3$ with star product, based on plane waves satisfying: $e^{i\,Tr(Bg_1)} * e^{i\,Tr(Bg_2)} = e^{i\,Tr(Bg_1g_2)}$
  - turn GFT into non-commutative, combinatorially non-local QFT on algebra $\mathfrak{g}^{\times 4}$
  - impose constraints $C(B_i)$ on B variables in GFT action

results:

- pure BF:

$$Z(\Gamma) = \int dB \int dg\, e^{i\sum_e Tr(B_e H_e(g))}$$

  correct simplicial geometry, correct symmetries, correct boundary terms, ...
- 4D gravity: $C(B_i)$ correctly implemented:

$$Z(\Gamma) = \int \mathcal{D}B \int \mathcal{D}g\, C(\{B_e\}) e^{i\sum_e Tr(B_e H_e(g))}$$

  correct secondary constraints?, relation to other SF models?, details of simplicial geometry?,.....
- in progress.....

NEW GFT MODELS FOR 4D QUANTUM GRAVITY

- alternative 2 (A. Baratin, DO, in preparation) :
  - introduce B variables by no-commutatative Fourier transform $\phi(g_i) \longrightarrow \phi(B_i)$ $C(SO(4)) \longrightarrow C_{*,\kappa}(\wedge^2\mathbb{R}^3)$ = functions on $\mathfrak{g} \simeq \wedge^2\mathbb{R}^3$ with star product, based on plane waves satisfying: $e^{i\,Tr(Bg_1)} * e^{i\,Tr(Bg_2)} = e^{i\,Tr(Bg_1 g_2)}$
  - turn GFT into non-commutative, combinatorially non-local QFT on algebra $\mathfrak{g}^{\times 4}$
  - impose constraints $C(B_i)$ on B variables in GFT action

results:

- pure BF:

$$Z(\Gamma) = \int dB \int dg\, e^{i\sum_e Tr(B_e H_e(g))}$$

  correct simplicial geometry, correct symmetries, correct boundary terms, ...
- 4D gravity: $C(B_i)$ correctly implemented:

$$Z(\Gamma) = \int \mathcal{D}B \int \mathcal{D}g\, C(\{B_e\}) e^{i\sum_e Tr(B_e H_e(g))}$$

  correct secondary constraints?, relation to other SF models?, details of simplicial geometry?,.....
- in progress.....

CONCLUSIONS

- Group Field Theories are promising framework for quantum gravity
- they are a generalization of matrix models to dimensions higher than 2
- they arise naturally from and share structures with several other approaches to quantum gravity
- allow for the use of quantum field theory tools and ideas in a background independent context, for studying both the kinematics and, most important, the dynamics of quantum spacetime, and continuum/semi-classical approximation
- many results have already been obtained, but there is plenty more to explore and understand!

THANK YOU FOR YOUR ATTENTION

CONCLUSIONS

- Group Field Theories are promising framework for quantum gravity
- they are a generalization of matrix models to dimensions higher than 2
- they arise naturally from and share structures with several other approaches to quantum gravity
- allow for the use of quantum field theory tools and ideas in a background independent context, for studying both the kinematics and, most important, the dynamics of quantum spacetime, and continuum/semi-classical approximation
- many results have already been obtained, but there is plenty more to explore and understand!

THANK YOU FOR YOUR ATTENTION

CONCLUSIONS

- Group Field Theories are promising framework for quantum gravity
- they are a generalization of matrix models to dimensions higher than 2
- they arise naturally from and share structures with several other approaches to quantum gravity
- allow for the use of quantum field theory tools and ideas in a background independent context, for studying both the kinematics and, most important, the dynamics of quantum spacetime, and continuum/semi-classical approximation
- many results have already been obtained, but there is plenty more to explore and understand!

THANK YOU FOR YOUR ATTENTION

CONCLUSIONS

- Group Field Theories are promising framework for quantum gravity
- they are a generalization of matrix models to dimensions higher than 2
- they arise naturally from and share structures with several other approaches to quantum gravity
- allow for the use of quantum field theory tools and ideas in a background independent context, for studying both the kinematics and, most important, the dynamics of quantum spacetime, and continuum/semi-classical approximation
- many results have already been obtained, but there is plenty more to explore and understand!

THANK YOU FOR YOUR ATTENTION

CONCLUSIONS

- Group Field Theories are promising framework for quantum gravity
- they are a generalization of matrix models to dimensions higher than 2
- they arise naturally from and share structures with several other approaches to quantum gravity
- allow for the use of quantum field theory tools and ideas in a background independent context, for studying both the kinematics and, most important, the dynamics of quantum spacetime, and continuum/semi-classical approximation
- many results have already been obtained, but there is plenty more to explore and understand!

THANK YOU FOR YOUR ATTENTION

CONCLUSIONS

- Group Field Theories are promising framework for quantum gravity
- they are a generalization of matrix models to dimensions higher than 2
- they arise naturally from and share structures with several other approaches to quantum gravity
- allow for the use of quantum field theory tools and ideas in a background independent context, for studying both the kinematics and, most important, the dynamics of quantum spacetime, and continuum/semi-classical approximation
- many results have already been obtained, but there is plenty more to explore and understand!

THANK YOU FOR YOUR ATTENTION