In search for diffeomorphism symmetry

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Fate of diffeomorphism symmetry in qg?

- broken? (Horava, Visser, Weinfurtner...) How is it restored at large scales?
- replaced by something else?
- deformed?
- survives in original form?

- for large scales: regain diffeomorphism symmetry to obtain correct semi-classical description

Why might we want to keep diffeomorphism symmetry?

- restricting ambiguities, uniqueness results, avoiding divergencies (Poincare in QFT)

- correct low energy limit

- sum over triangulations:
  diffeo invariant (effective) action could make that precise (triangulation independence), otherwise problems with measure

- control over lattice effects → Lorentz symmetry breaking

Set-up

- lattice models of gravity, i.e. triangulations
- covariant: Regge calculus, spin foam models
- canonical lattice models, "LQG on a fixed lattice" (vs LQG as continuum theory) ...
- problem so far: constraint algebra not closed, inconsistent dynamics, models ruled out
- Are covariant lattice models better?

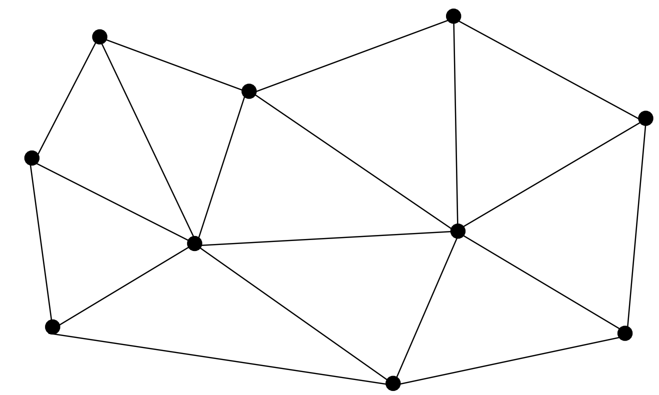
Exact or broken diffeomorphism symmetry in discretized actions?

In Regge calculus

- Rocek and Williams 81: diffeomorphism symmetry for linearization around flat background
- Hamber and Williams 97: should exist also for full theory
- Hartle, Miller, Morse, ...: approximate diffeomorphism symmetry: How is it broken?

Regge calculus

- approximate manifold by triangulation
- lengths of edges fix geometry



$$S_{EH} = \frac{1}{8\pi}\int_{\mathcal{M}} d^D x\,\sqrt{|g|}\,\left(\Lambda - \frac{1}{2}R\right)$$

$$\Downarrow$$

$$S_{\mathcal{T}} = \sum_{h\in\mathcal{T}^\circ} F_h \epsilon_h - \Lambda \sum_{\sigma\subset\mathcal{T}^\circ} V_\sigma$$

4d:triangles
3d: edges

volume of triangle/edge

deficit angle

volume of 4-simplex/tetrahedron

3d Regge calculus

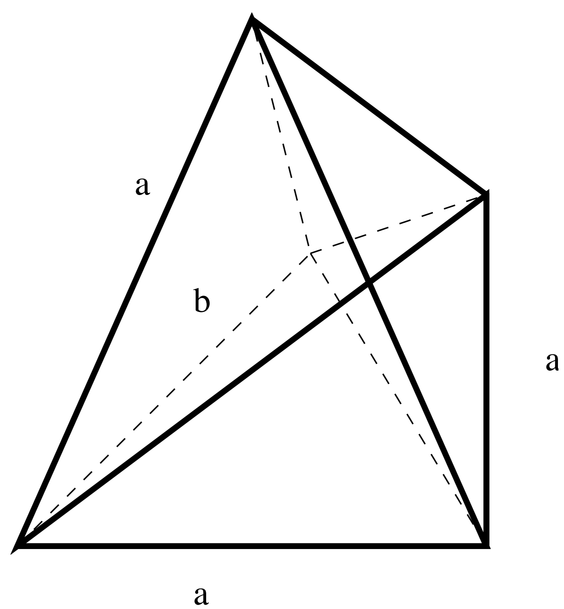
without cosm. constant

- any triangulation of flat space is a solution
- at every vertex 3dim translation symmetry
- Hessian of action has null eigenvalues

with cosm. constant

- unique solutions approximating curved space
- there is no translation symmetry acting
- Hessian of action has small eigenvalues

exact diffeo symmetry



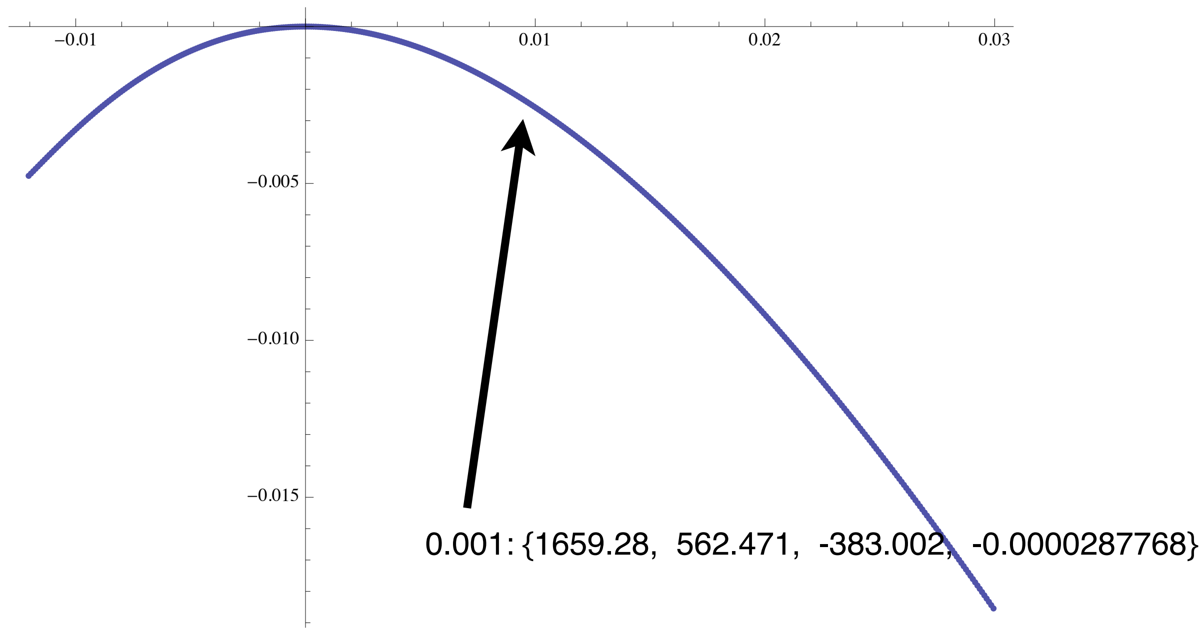
approx. diffeo symmetry

4d Regge calculus?

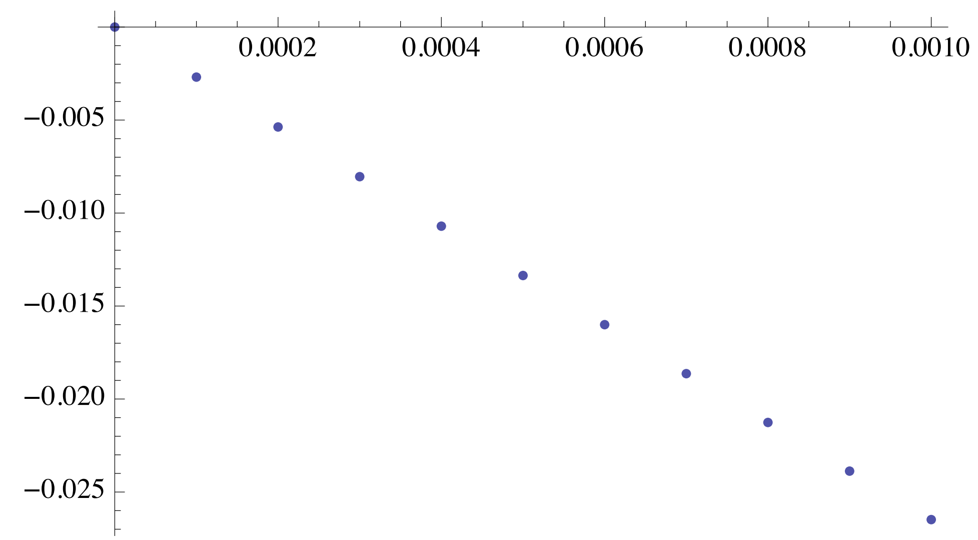
- flat vertices (curvature vanishes at adjacent triangles):display diffeo/translation symmetry.
  There are many curved solutions with flat vertices!

4d solution with curvature:

B.Bahr, BD, 2009



lowest eigenvalues of Hessian as function of deviation parameter from flat configuration



curvature at one triangle as function of deviation parameter from flat configuration

Symmetry is broken, effect quadratic in curvature.

Repercussions for canonical descriptions?

a) Canonical formalism reproducing solutions and (non-)symmetries of discretized action?

b) Constraints? Constraint algebra?

Repercussions for canonical descriptions?

a) Canonical formalism reproducing solutions and symmetries of discretized action?

b) Constraints? Constraint algebra?

a) Yes, we can! B. Bahr, BD 09

b1) Broken Symmetries: Constraints are replaced by pseudo constraints, that depend on lapse and shift. Gauge degrees of freedom seem to be physical. R.Gambini, J.Pullin 05,...

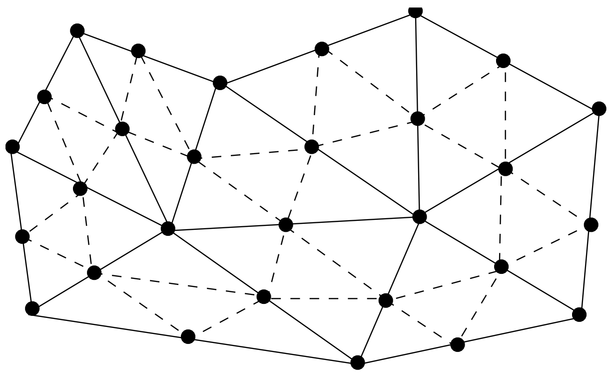
b2) If action has exact symmetries: We obtain exact constraints with first class algebra.

Obtaining a consistent constraint algebra is equivalent to obtaining an action with exact symmetries.

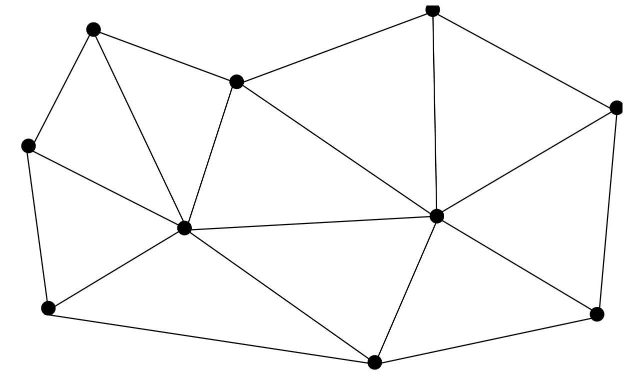
Are there discrete actions with exact symmetries?

Perfect actions

- Lattice QCD Hasenfratz et al 90s
- Actions on lattice, that mirror exactly continuum theory
- reobtain (global) Lorentz symmetry
- here: can we reobtain local gauge symmetries?
- coincides with fixed point action / coarse grain from the continuum
- should be triangulation independent!



integrate out small edge lengths



3d Regge with cc

$$S_{\mathcal{T}} = \sum_e l_e \epsilon_e - 2\Lambda \sum_\sigma V_\sigma$$

action for flat simplices

approximate symmetries

Regge with curved simplices

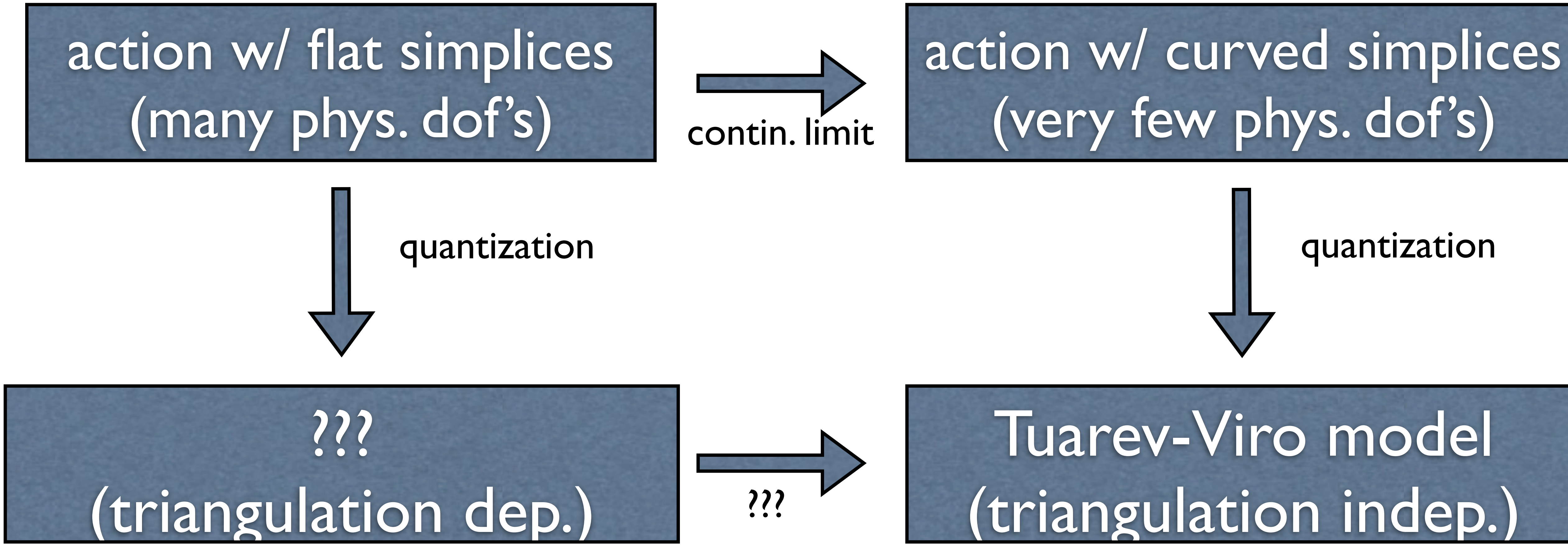
B.Bahr, BD, to appear

$$S^\kappa_{\mathcal{T}} = \sum_E L_E \epsilon^\kappa_E + 2\kappa \sum_\sigma V^\kappa_\Sigma$$

action for simplices with curvature

$$\kappa = \Lambda$$

exact symmetries


action w/ flat simplices
(many phys. dof’s)
contin. limit
action w/ curved simplices
(very few phys. dof’s)
quantization
quantization
???
(triangulation dep.)
???
Tuarev-Viro model
(triangulation indep.)

What can we expect in 4d?

- nonlocal (very complicated) action, but triangulation independent
- canonical: closed constraint algebra
- sum over triangulation/continuum limit either before (perfect action) or after quantization (group field theory)
- Do microscopic details matter? Universality?
- Use (approx) diffeos to derive conditions on quantum models/ amplitudes / measure.
- Explore general mechanisms for regaining gauge symmetries.

Conclusions

- discrete actions generally break diffeomorphism symmetries
- canonical framework exactly mimics covariant symmetries
- can regain symmetries by fine graining (classically, 3d)
- derive conditions for quantum models

Thank you for this great workshop!