Higgs mechanism and Goldstone's bosons

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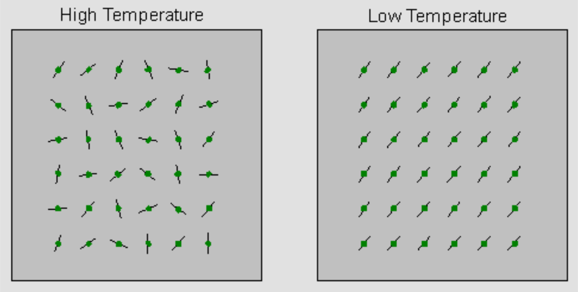
March 15, 2008

Spontaneous symmetry breaking

In physics *spontaneous symmetry breaking* takes place when a system, that is symmetric with respect to some symmetry group, goes into a vacuum state that is not symmetric. At this point the system no longer appears to behave in a symmetric manner.

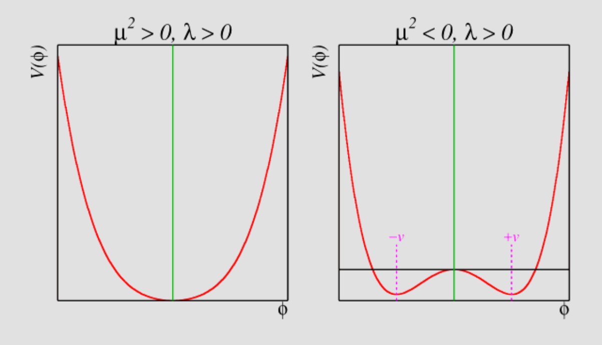


Real scalar field

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \left(\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4\right)$$

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

symmetry: $\phi(x) \rightarrow -\phi(x)$



Minimum:

$$-\nu = -\sqrt{-\frac{\mu^2}{\lambda}}$$

$$+\nu = +\sqrt{-\frac{\mu^2}{\lambda}}$$

Interesting case arising if we take $\mu^2 < 0$. The potential term with a negative value of μ^2 is odd. It appears to represent a particle with **imaginary mass**.

It is easy to see why it is so: it gives a "**negative resistance**" to any attempts of moving it from the origin, where the field is zero.

The potential decreases in both directions, so it is energetically favorable for our field to **roll down** to one of the **two saddle points**.

These lie at the value:

$$\phi = \pm\nu = \pm\sqrt{-\mu^2/\lambda}$$

We like to call the minimum of the potential our "**vacuum**":
$\longrightarrow$ you cannot have less energy than that.

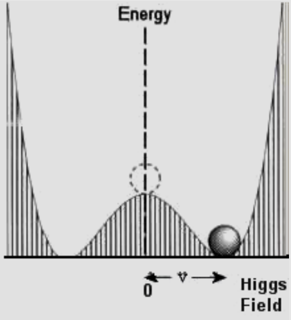
In the case of our potential with negative μ^2, the **vacuum** does **not correspond** to **zero value** of the field! Rather, the field takes the value ν.

There is something wrong here: imaginary mass, vacuum containing non-zero fields We can try to fix it by **redefinition** of our scalar field.

We **shift** it to the minimum at $\pm\nu$ by introducing:

$$\phi(x) = \nu + \eta(x)$$

The physics cannot depend on the shift of the scalar field by a constant value ν, and now the lagrangian takes a different form.



$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)^2 - \lambda(\nu^2\eta^2 - \nu\eta^3 - \frac{1}{4}\eta^4)$$

$$\mathcal{L} = (\tfrac{1}{2}(\partial_\mu \eta)^2 - \lambda \nu^2 \eta^2) - \lambda \nu \eta^3 - \tfrac{\lambda}{4}\eta^4)$$

In terms of the shifted field $\eta(x)$, there is nothing wrong with the lagrangian any more: the vacuum has zero value for the field (it is a real vacuum!), and the field has a mass term of the right sign: $-\frac{1}{2}m^2\eta^2 = -\lambda v^2 \eta^2$.

So the mass of the scalar is now $m = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$, a positive value (forget the rest of terms, they describe self-interactions, not the mass).

However $\mathcal{L}$ is not symmetric for the operation $\eta \to -\eta$ any more!

Complex scalar field

$$\mathcal{L} = (\partial_\mu \Phi)^*(\partial^\mu \Phi) - \mu^2 \Phi^* \Phi - \lambda(\Phi^* \Phi)^2$$

Now we wanted to write a lagrangian which is symmetric under a **continuous transformation law** of the field, not just the simple mirroring as before. (That will **allow** us to state **Goldstones theorem**).

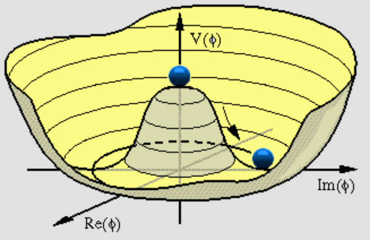
Field $\Phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$ is a complex scalar field.

Lagrangian has global $U(1)$ symmetry introduced by modification of the field by a phase transformation $\Phi \rightarrow \Phi' = exp(i\alpha)\Phi$ (with α the constant phase shift).

Again we have two parameters in the potential energy terms, $\lambda > 0$ and $\mu^2 < 0$ and we're getting a field with an imaginary mass term.

Worse! We now have not just two, but a full circle of minima for the potential, lying at the values of the field satisfying:

$$\phi_1^2 + \phi_2^2 = -\mu^2/\lambda.$$



An infinity of choices for the vacuum!

Having previously worked out the simpler example of one single real scalar field, we are not impressed by the compication, since we know how to get things straight.

We choose one of the vacua for a translated field by writing:

$$\Phi(x) = \frac{1}{\sqrt{2}}\Big(\nu + \xi(x) + i\eta(x)\Big)$$

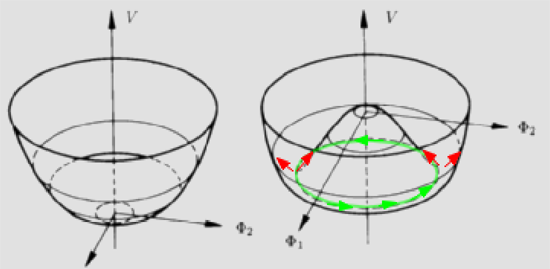
In terms of the shifted fields $\mathcal{L}$ becomes:

$$\mathcal{L} = \frac{1}{2}\Big((\partial_\mu \xi)^2 + (\partial_\mu \eta)^2\Big) + \mu^2 \eta^2$$

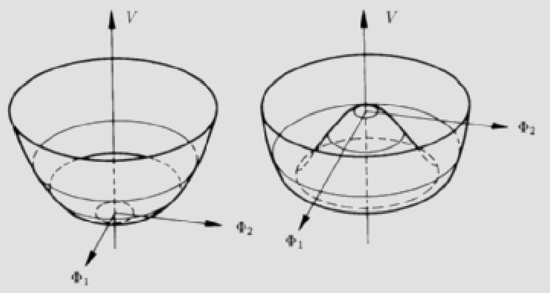
If we examine $\mathcal{L}$ we recognize kinetic terms (the ones with two derivatives of the field) for the scalar fields ξ and η.

But while we also have a **mass term** (the one quadratic in the field) for η.

Field ξ gets no mass term! That means that the field is **massless**.



The two fields correspond to orthogonal oscillations about the vacuum we have chosen.



Massless field corresponds to oscillations along the direction where the potential remains at a minimum - along the circle of minima. Because of that, it encounters no resistance - no inertia, no mass.

The **spontaneous breaking of the symmetry** of the original lagrangian for ϕ has **generated a mass** for one of the two scalars, **and** a further **massless scalar** has appeared in the theory.

This is the Goldstone theorem in a nutshell:

The spontaneous breaking of a continuous symmetry of the lagrangian generates massless scalars. They correspond to fluctuations around the chosen vacuum in the direction described by the neighboring vacua.

Massless scalar particles do not belong to any reasonable theory of nature. Our world would be a quite different place if there were massless scalars around.

We do not observe such particles. Indeed, there is a trick, called the Higgs mechanism, which gets rid of the massless Goldstone bosons.

The **degrees of freedom** of the theory associated to the Goldstone bosons **reappear as the mass terms** for the **weak vector bosons** as will be discussed later.

- We saw what is Goldstones theorem and why should we bother knowing it.

- The theorem is a crucial preliminary to understand the need for a Higgs boson in the Standard Model.

Once agian **Goldstone's theorem** can be stated as follows:

- *If a continous symmetry of the Lagrangian is spontaneously broken, and if there are no long-range forces, then exists a zero-frequency excitation at zero momentum.*

Ferromagnets:

- The absence of long-range forces, which may tend to couple spins at large distances, is necessary for the existance of a mode with $\omega \rightarrow 0$ as $k \rightarrow 0$.

Superconductors:

- In Bardeen-Cooper-Schrieffer (BCS) theory there is a spontaneous breaking of the phase invariance associated with the conservation of the electron number.
- However there is an energy gap (equal to the mass of the Cooper pairs), so there is no Goldstone's boson.
- The reason is that there are long-range electromagnetic forces.

Superfluids (for example low-temperature Bose system):

- The condensate field at $T = 0$ is $< \Phi >= \xi$, which is related to the particle number density by $n = |\xi|^2$.
- The phonon spectrum is:

$$\omega^2 = \frac{k^2}{2m}(\frac{k^2}{2m} + 2nV(\mathbf{k}))$$

- A short-range potential has the property that $V(\mathbf{k})$ (the Fourier transform of the two-body potential) is finite and positive. In that case, $\omega \to \sqrt{nV(\mathbf{k}=0)/m}$ as $k \to 0$. This not so for a long-ange potential.
- For the Coulomb force, $V(k) = \frac{e^2}{k^2}$ and, as $k \to 0$, $\omega \to e\sqrt{n/m} = \omega_p$, the plasma frequency.

Proof of Goldstone's theorem in context of the U(1) scalar field theory

The $U(1)$ symmetry is $\Phi \to \Phi e^{-i\alpha}$, or $\delta\Phi = -i\alpha\Phi$ if $|\alpha|^2 \ll 1$.

The conserved current density in terms of the shifted field:

$$j_\mu = \chi_2 \partial_\mu \chi_1 - \chi_1 \partial_\mu \chi_2 - \sqrt{2}\xi \partial_\mu \chi_2$$

The total charge, $Q = \int d^3x\, j_0(\mathbf{x})$ is conserved.

The change in Φ due to an infinitesimal change in phase can also be expressed in operator form as:

$$\delta\Phi = i\alpha[Q, \Phi]$$

Taking the thermal, or ensemble, average of $\delta\Phi$, we find $< \delta\Phi > = -i\alpha < \Phi > = -i\alpha\xi$.

Taking the thermal average of previous equation (with total charge as the generator of the phase transition) we find an expression for the condensate field:

$$\xi = -\int d^3x \; < [j_0(\mathbf{x}, t), \Phi(\mathbf{0}, 0)] >$$

Now we define the function:

$$F^{\mu}(k_0, \mathbf{k}) = \int d^4x \, e^{ik\cdot x} < T[j^{\mu}(x), \Phi(0)] >$$

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Since $T[j^{\mu}(x), \Phi(0)] = j^{\mu}(x)\Phi(0)\theta(x^0) + \Phi(0)j^{\mu}(x)\theta(-x^0)$ and $\partial_\mu j^{\mu} = 0$ we can write that:

$$k_\mu F^{\mu} = -i \int d^4x\, \partial_\mu \Big(e^{ik\cdot x} < T[j^{\mu}(x), \Phi(0)] > \Big) + \\ + i \int d^3x\, e^{-i\mathbf{k}\cdot\mathbf{x}} < [j^{\mu}(x), \Phi(0)] >$$

Surface term vanishes and second term can be obtain from comparison with $\xi = -\int d^3x\, < [j_0(\mathbf{x}, t), \Phi(\mathbf{0}, 0)] >$

We end up with expression:

$$\lim_{\mathbf{k}\to 0} k_\mu F^\mu = -i\xi$$

- If $\xi \neq 0$, which means that the $U(1)$ symmetry is spontaneously broken, then F has a pole at $k = 0$.
- This pole corresponds to a zero-frequency excitation at zero momentum.

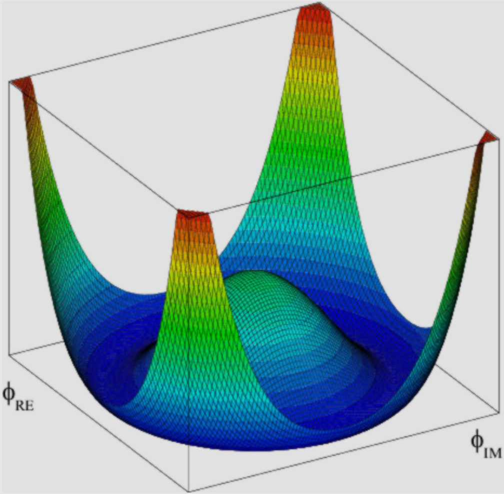
It's not diffcult to determine F^μ. Substituting j_μ in terms of χ_1 and χ_2 into F^μ leads to:

$$F^\mu = -\xi k^\mu \int d^4x\, e^{ikx} < T[\chi_2(x)\chi_2(0)] >= -i\xi k^\mu D_2(k)$$

where D_2 is real time Green's function.

Conbining $F^\mu = -i\xi k^\mu D_2(k)$ with $\lim_{\mathbf{k}\to 0} k_\mu F^\mu = -i\xi$ tells us that the imaginay part of th shifted field has a dispersion relation with the property that $\omega(\mathbf{k} = 0) = 0$. This is the Goldstone boson.

Higgs Mechanism



Glashow-Weinberg-Salam model

$$\mathcal{L} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 - \frac{1}{4} b^{\mu\nu} b_{\mu\nu} - \frac{1}{4} f_a^{\mu\nu} f_{\mu\nu}^a$$

Lagrangian has $SU(2)xU(1)$ symmetry. There is $SU(2)$ gauge field A_μ^a and a $U(1)$ gauge field B_μ.

Strengths are:

- $f_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g\epsilon^{abc} A_\mu^b A_\nu^a$
- $g_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

Covariant derivative: $D_\mu = \partial_\mu + \frac{1}{2} igA_\mu^a \tau_a + \frac{1}{2} ig' B_\mu$

which acts on a complex $SU(2)$ field:

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + i\phi \\ \phi_3 + i\phi_4 \end{pmatrix}$$

Let's introduce the vacuum expectation value:

$$< \Phi > = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \nu \end{pmatrix}$$

$$\Phi = \frac{1}{\sqrt{2}} U^{-1}(\zeta)\begin{pmatrix} 0 \\ \nu + \eta \end{pmatrix}$$

$$U(\zeta) = exp(\frac{-i\zeta\tau}{2\nu})$$

$$\Phi \rightarrow \Phi' = U(\zeta)\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \nu + \eta \end{pmatrix}$$

After some caculations:

$$\mathcal{L} = (\partial_\mu \eta)\partial^\mu \eta) - \frac{1}{2}\mu^2(\nu+\eta)^2 - \frac{1}{4}\lambda(\nu+\eta)^4 - \frac{1}{4}b^{\mu\nu}b_{\mu\nu} - \frac{1}{4}f_a^{\mu\nu}f^a_{\mu\nu} + + \frac{1}{4}\Phi^\dagger(g'B_\mu + g\tau\mathbf{A}_\mu)(g'B^\mu + g\tau\mathbf{A}^\mu)\Phi'$$

This can be written as the sum of classical part, a part quadratic in the fields and a part giving rise to interactions that is cubic and quadratic in the fields

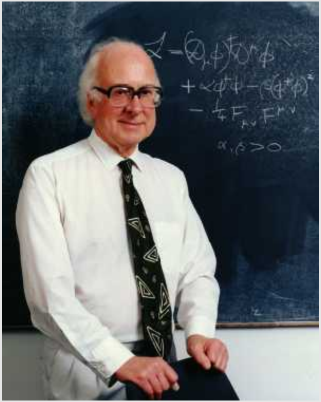
We define:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(A_\mu^1 \pm iA_\mu^2\right)$$

$$Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}}\left(g' B_\mu - gA_\mu^3\right)$$

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}}\left(gB_\mu + g' A_\mu^3\right)$$

The masses:



$$m_A = 0$$

$$m_W = \frac{1}{2} g \nu$$

$$m_z = \frac{1}{2}\sqrt{g^2 + g'^2}$$