Quantum Gravity is Coming to Town

Instytut Fizyki Teoretycznej
Uniwersytet Wrocławski
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Quantum Gravity is Coming to Town

The Fight Between Classical and Quantum
Quantum Gravity is Coming to Town
## 1. Cover pages

- Cover page: conference 1
- Cover page: title 2
- Cover page: picture 3

## 2. Observations before Ligo-Virgo

- A history of the GW theoretical research 5
- First (indirect) GW detection by astrophysicists 6
- Quadrupole, binary pulsar 7

## 3. Ligo-Virgo breakthrough

- Inspiral, merger, ringdown 8
- Frequencies (1) 9
- Frequencies (2) 10
- GW power $c^5/G$ 11
- Afterglows 12
- Fits well masses 13

## 4. That is the Question

- The question 14
- Advocatus diaboli 15
- The single graviton issue 16
- Separating gravity from quantum is OK 17
- Quantum Gravity meaningless 18
- The Hawking radiation 19
- Primordial black holes 20
- Dyson says ... but... 21

## 5. Strong hopes, vague ideas

- Vitor Cardoso dixit: there are observable differences... 22
  ... in waves from horizon-less objects 23

## 6. The Schwarzschild star

- Schwarzschild at his desk 24
- The exterior Schwarzschild 25
- The interior Schwarzschild 26

## 7. The waves

- Wave equation in Minkowski 27
- The effective potential 28
- General mathematical formulation 29
- Waves trapped in the potential well 30

## 8. OUR SOLUTION

- The solution method 31
- Analytic formulae 32
- Analytic versus numerical 33

## 9. Optical geometry

- Calculating Re($\omega$) 34
- Calculating Im($\omega$) 35
- Black body does not fit 36

## 10. NEW RESULTS AND CONCLUSIONS

The last page 37
A history of the theoretical research on gravitational waves.

Näherungsweise Integration der Feldgleichungen der Gravitation
Albert Einstein
1916
Der Königlich Preußischen Akademie der Wissenschaften
(Berlin), pp. 688-696.

First hint

40 years of efforts and confusions

Spherical Gravitational Waves
Ivor Robinson, Andrzej Trautman
Phys.Rev.Lett. 4, 431-432
Final understanding
ABSTRACT: The rate of period changes and the time scale of collapse caused by the gravitational radiation is tabulated as a function of orbital period of a binary. It is shown that the time scale of collapse is of the same order of magnitude as the time scale of the nuclear evolution for W UMa type binaries. The influence of gravitational radiation on the evolution of novae, U Geminorum type stars, and pairs of white dwarfs is discussed. It is shown that the evolution of WZ Sge and HZ 29 might be considerably affected by this phenomenon.
The Einstein (Landau-Lifshitz) quadrupole formula

\[ A = \frac{\kappa}{24\pi} \sum_{\alpha\beta} \left( \frac{\partial^3 J_{\alpha\beta}}{\partial t^3} \right)^2 \]

**PSR B1913+16 Hulse-Taylor Binary Pulsar**

*Nobel in 1993*

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*Marek.Abramowicz@physics.gu.se–Horak@astro.cas.cz–QUANTUM–GRAVITY–IS–COMING–TO–TOWN–12.III.2021*
LIGO-Virgo observations: inspiral, merger, ringdown
GWTC-1: A Gravitational-Wave Transient Catalog of Compact Binary Mergers Observed by LIGO and Virgo during the First and Second Observing Runs

B. P. Abbott et al.*

(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 14 December 2018; revised manuscript received 27 March 2019; published 4 September 2019)
LIGO-Virgo observations: frequencies (2)
The maximal energy available from an object with the mass $M$ is $E_{\text{max}} = Mc^2$. The minimal time in which this energy may be liberated is $t_{\text{min}} = R_G/c$, where the gravitational radius $R_G = GM/c^2$ corresponds to a minimal size the object may have. Thus, the maximal power

$$L_{\text{max}} = E_{\text{max}}/t_{\text{min}} = c^5/G = L_{\text{Planck}},$$

where $L_{\text{Planck}}$ is the Planck power, i.e. the power expressed in Planck’s units,

$$L_{\text{Planck}} = c^5/G = 10^{58} \text{[erg/sec]} = 10^{52} \text{[Watts]}.$$

Rather surprisingly, it does not depend on the Planck constant $h$.

This is the absolute upper limit for power of anything in the Universe: all objects, phenomena, explosions, and evil empires. A power needed for the Creation was the rate at which the Big Bang transferred energy from a pre-Planck to the post-Planck state. For the reason outlined here, no more than $10^{52}$ Watts was needed to create the Universe.

The peak power in the LIGO-Virgo GW events is close:

$$\ell_P \approx 3 \times 10^{-2} L_{\text{Planck}}.$$

These are the most powerful events ever observed by humans.

Marek.Abramowicz@physics.gu.se–Horak@astro.cas.cz–QUANTUM–GRAVITY–IS–COMING–TO–TOWN–12.III.2021
LIGO-Virgo observations: afterglows
LIGO-Virgo observations fit the black hole collision hypothesis

<table>
<thead>
<tr>
<th>Event</th>
<th>$m_1/M_\odot$</th>
<th>$m_2/M_\odot$</th>
<th>$M_f/M_\odot$</th>
<th>$a_f$</th>
<th>$E_{\text{rad}}/(M_\odot c^2)$</th>
<th>$\epsilon_{\text{peak}}/(\text{erg s}^{-1})$</th>
<th>$d_L/\text{Mpc}$</th>
<th>$\Delta\Omega/\text{deg}^2$</th>
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<tbody>
<tr>
<td>GW150914</td>
<td>35.6$^{+4.7}_{-3.1}$</td>
<td>30.6$^{+3.0}_{-4.4}$</td>
<td>63.1$^{+3.4}_{-3.0}$</td>
<td>0.69$^{+0.05}_{-0.04}$</td>
<td>3.1$^{+0.4}_{-0.4}$</td>
<td>3.6$^{+0.4}_{-0.4} \times 10^{56}$</td>
<td>440$^{+150}_{-170}$</td>
<td>182</td>
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<tr>
<td>GW151012</td>
<td>23.2$^{+14.9}_{-5.5}$</td>
<td>13.6$^{+4.1}_{-4.8}$</td>
<td>35.6$^{+10.8}_{-3.8}$</td>
<td>0.67$^{+0.13}_{-0.11}$</td>
<td>1.6$^{+0.6}_{-0.5}$</td>
<td>3.2$^{+0.8}_{-1.7} \times 10^{56}$</td>
<td>1080$^{+550}_{-490}$</td>
<td>1523</td>
</tr>
<tr>
<td>GW151226</td>
<td>13.7$^{+8.8}_{-3.2}$</td>
<td>7.7$^{+2.2}_{-2.3}$</td>
<td>20.5$^{+6.4}_{-1.5}$</td>
<td>0.74$^{+0.07}_{-0.05}$</td>
<td>1.0$^{+0.1}_{-0.2}$</td>
<td>3.4$^{+0.7}_{-1.7} \times 10^{56}$</td>
<td>450$^{+180}_{-190}$</td>
<td>1033</td>
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<tr>
<td>GW170104</td>
<td>30.8$^{+7.3}_{-5.6}$</td>
<td>20.0$^{+4.9}_{-4.6}$</td>
<td>48.9$^{+5.1}_{-4.0}$</td>
<td>0.66$^{+0.08}_{-0.11}$</td>
<td>2.2$^{+0.5}_{-0.5}$</td>
<td>3.3$^{+0.6}_{-1.0} \times 10^{56}$</td>
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<td>GW170608</td>
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<td>7.6$^{+1.4}_{-2.2}$</td>
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<td>0.9$^{+0.0}_{-0.1}$</td>
<td>3.5$^{+0.4}_{-1.3} \times 10^{56}$</td>
<td>320$^{+120}_{-110}$</td>
<td>392</td>
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<tr>
<td>GW170729</td>
<td>50.2$^{+16.2}_{-10.2}$</td>
<td>34.0$^{+9.1}_{-10.1}$</td>
<td>79.5$^{+14.7}_{-10.2}$</td>
<td>0.81$^{+0.07}_{-0.13}$</td>
<td>4.8$^{+1.7}_{-1.7}$</td>
<td>4.2$^{+0.9}_{-1.5} \times 10^{56}$</td>
<td>2840$^{+1400}_{-1500}$</td>
<td>1041</td>
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<tr>
<td>GW170809</td>
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<td>23.8$^{+5.1}_{-5.2}$</td>
<td>56.3$^{+5.2}_{-3.8}$</td>
<td>0.70$^{+0.08}_{-0.09}$</td>
<td>2.7$^{+0.6}_{-0.6}$</td>
<td>3.5$^{+0.6}_{-0.9} \times 10^{56}$</td>
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<td>308</td>
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<td>GW170814</td>
<td>30.6$^{+5.6}_{-3.0}$</td>
<td>25.2$^{+2.8}_{-4.0}$</td>
<td>53.2$^{+3.2}_{-2.4}$</td>
<td>0.72$^{+0.07}_{-0.05}$</td>
<td>2.7$^{+0.4}_{-0.3}$</td>
<td>3.7$^{+0.4}_{-0.5} \times 10^{56}$</td>
<td>600$^{+150}_{-220}$</td>
<td>87</td>
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<td>GW170817</td>
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<td>1.27$^{+0.09}_{-0.09}$</td>
<td>$\leq 2.8$</td>
<td>$\leq 0.89$</td>
<td>$\geq 0.04$</td>
<td>$\geq 0.1 \times 10^{56}$</td>
<td>40$^{+7}_{-15}$</td>
<td>16</td>
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<tr>
<td>GW170818</td>
<td>35.4$^{+7.5}_{-4.7}$</td>
<td>26.7$^{+4.3}_{-5.2}$</td>
<td>59.4$^{+4.9}_{-3.8}$</td>
<td>0.67$^{+0.07}_{-0.08}$</td>
<td>2.7$^{+0.5}_{-0.5}$</td>
<td>3.4$^{+0.5}_{-0.7} \times 10^{56}$</td>
<td>1060$^{+420}_{-380}$</td>
<td>39</td>
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<tr>
<td>GW170823</td>
<td>39.5$^{+11.2}_{-6.7}$</td>
<td>29.0$^{+6.7}_{-7.8}$</td>
<td>65.4$^{+10.1}_{-7.4}$</td>
<td>0.72$^{+0.09}_{-0.12}$</td>
<td>3.3$^{+1.0}_{-0.9}$</td>
<td>3.6$^{+0.7}_{-1.1} \times 10^{56}$</td>
<td>1940$^{+970}_{-900}$</td>
<td>1666</td>
</tr>
</tbody>
</table>
The question:

May the LIGO-Virgo (and EHT) observations put meaningful constraints on Quantum Gravity?
Freeman Dyson:

The essence of any theory of quantum gravity is that there exists a graviton.

If individual gravitons cannot be observed in any conceivable experiment, then they have no physical reality and we might as well consider them non-existent, and quantum gravity as physically meaningless.
Freeman Dyson against Quantum Gravity.

Ontology: no gravitons

Gravitons not detectable by LIGO in practise

[01] Observations: strain $f = 10^{-21}$, frequency $\omega = \text{kHz}$

[02] Energy density $E = (c^2/32\pi G)\omega^2 f^2 = 10^{-10} \text{erg/cm}^3$

[03] Graviton’s minimal size is $(c/\omega)$

[04] Graviton’s energy density $E_S = \omega \hbar/(c/\omega)^3$

[05] Thus, $E_S = 10^{-47} \text{erg/cm}^3$

[06] Number of gravitons in the wave $E/E_S = 10^{37}$.

Gravitons not detectable in principle

[07] From $E = E_S$ follows $f = (32\pi)^{1/2}(L_P \omega/c)$,

[08] where $L_P = (G\hbar/c^3)^{1/2}$ is the Planck length.

[09] Wave detected by variation of distance $\delta = fD$

[10] between two masses $M$ separated by $D$.


[12] Thus, $\delta = (32\pi)^{1/2}L_P \approx L_P$ independently of $\omega$

[13] Heisenberg: $M(\delta/T)\delta > \hbar$,

[14] where $T > (D/c)$ is the measurement time.


Sensitivity of LIGO should be $10^{37}$ times better to detect a single graviton.

The measuring apparatus collapses !!!
Freeman Dyson against Quantum Gravity.

Epistemology: separate theories for large and small are OK.

The great majority of physicists insist that the division of physics into separate theories for large and small objects is unacceptable.

As a conservative, I do not agree with that.
Epistemology: Quantum Gravity may be meaningless

Instead of insisting dogmatically on unification, I prefer to ask the question whether a unified theory would have any real physical meaning.
Example: the Hawking radiation

<table>
<thead>
<tr>
<th>Hawking temperature and radiation</th>
<th>Hawking’s effects not detectable in practice</th>
</tr>
</thead>
<tbody>
<tr>
<td>[01] Strong gravity’s frequency: $\omega_H = c/R_G$</td>
<td></td>
</tr>
<tr>
<td>[02] $E_H = \omega_H \hbar = kT_H \Rightarrow T_H \sim h\omega_H/k$</td>
<td></td>
</tr>
<tr>
<td>[03] $T_H = (\hbar c^3)/(8\pi kGM(\odot)(M/M(\odot))^{-1}$</td>
<td></td>
</tr>
<tr>
<td>[04] $T_H = 6 \times 10^{-8}(M/M(\odot))^{-1} , [\circ K]$</td>
<td></td>
</tr>
<tr>
<td>[05] Radiation flux: $F_H \sim T_H^4 \sim M^{-4}$</td>
<td></td>
</tr>
<tr>
<td>[06] Radiation power: $L_H \sim F_H A \sim F_H M^2 \sim M^{-2}$</td>
<td></td>
</tr>
<tr>
<td>[07] Radiation losses: $\partial_t (Mc^2) \sim L_H \sim M^{-2}$</td>
<td></td>
</tr>
<tr>
<td>[08] Evaporation time: $M/t_H \sim M^{-2} \Rightarrow t_H \sim M^3$</td>
<td></td>
</tr>
<tr>
<td>[09] For $t_H = \text{Hubble time}: M = 10^{15} [g]$</td>
<td></td>
</tr>
</tbody>
</table>

Hawking’s radiation is totally negligible for astrophysical black holes.

Hawking’s radiation has never been observed in Lab.

BTW: primordial black holes have never been detected.
Primordial black holes

Dyson says, QG may be meaningless with no gravitons detected...

... however

“If individual gravitons cannot be observed in any conceivable experiment, then they have no physical reality and we might as well consider them non-existent, and quantum gravity as physically meaningless.”

... an analogy with electromagnetism shows that even when individual photons have not been observed, the quantum nature of light was clearly recognized and demonstrated.

Electromagnetic radiation of energy density $E$ trapped in a container leaks through a small hole, with the emerging flux $F$. The distribution function $f(\nu)$ needs not to be specified. Two assumptions are crucial: (1) individual light quanta have energies $h\nu$. (2) They move with velocity $c$. The relation between $E$ and $F$ does not contain the Planck constant $\hbar$,

$$F = \frac{c}{4} E$$

They determine the damping time $t_0$ of the energy decay due to the leaking,

$$t_0 = \frac{VE}{AF} = \frac{4V}{cA}.$$ 

$V$ is the container volume and $A$ is the hole area.
The GW are partially trapped by the space time curvature of an ultra compact star as the EM waves are partially trapped in a spherical container with a small hole.

**NOT OBVIOUS:** is the GW ringdown decay time determined to some degree by quantum effects, i.e. the existence of gravitons?

Vitor Cardoso: the electromagnetic and gravitational waves in non black hole space times postulated by QG propagate differently than in the black hole space time.
Strong hopes and vague ideas

The GW are partially trapped by the space time curvature of an ultra compact star as the EM waves are partially trapped in a spherical container with a small hole.

NOT OBVIOUS: is the GW ringdown decay time determined to some degree by quantum effects, i.e. the existence of gravitons?

Vitor Cardoso: the electromagnetic and gravitational waves in non black hole space times postulated by QG propagate differently than in the black hole space time.

EHT: wormhole image   LIGO: wormhole ringdown
The exterior and interior Schwarzschild solutions
The exterior Schwarzschild solution

\[ ds^2 = (1 - \frac{r_g}{r}) dt^2 - (1 - \frac{r_g}{r})^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \]

\[ = (1 - \frac{r_g}{r}) \left[ dt^2 - (1 - \frac{r_g}{r})^2 dr^2 - \frac{r^2}{(1 - \frac{r_g}{r})} (d\theta^2 + \sin^2 \theta d\phi^2) \right] \]

\[ = e^{2\Phi} \left[ dt^2 - dr_*^2 - \frac{r^2}{\nu^2} (d\theta^2 + \sin^2 \theta d\phi^2) \right] \]

\[ e^{2\Phi} = g_{tt}, \quad r_* = \int (1 - \frac{r_g}{r})^{-1} dr, \quad \nu^2 = -\frac{g_{\phi\phi}}{g_{tt}} \]
The internal Schwarzschild solution

Because along light trajectories $ds = 0$, and therefore $\int dt = \int dh$, one concludes that $\int dh$ is minimal along a light trajectory in the optical geometry: light trajectories are spatial geodesics in the optical space.

The optical geometry corresponding to the interior of a constant density Schwarzschild star is spherical and isometric with spatial sections of the static Einstein Universe with the curvature scalar $\mathcal{R} = 6/\tilde{a}^2$.

$$-\mathit{d}h^2 = \tilde{a}^2 \left[ \mathit{d}(r_*/\tilde{a})^2 + \sin^2(r_*/\tilde{a}) \left( \mathit{d}\theta^2 + \sin^2 \theta \mathit{d}\phi^2 \right) \right]$$

Its meridional cut $\theta = \pi/2$ has a geometry of a two sphere, $-\mathit{d}h^2 = \tilde{a}^2 \left[ \mathit{d}(r_*/\tilde{a})^2 + \sin^2(r_*/\tilde{a}) \mathit{d}\phi^2 \right]$. Functions and parameters that appear in equations have specific forms, expressed in the Schwarzschild radial coordinate $r$ and the mass $M$ and radius $R$ of the star:

$$e^{\Phi(r)} = \frac{3}{2} \left( 1 - \frac{2M}{R} \right)^{1/2} - \frac{1}{2} \left( 1 - \frac{r^2}{\alpha^2} \right) ; \quad a = (R^3/2M)^{1/2}$$

$$r_*(r) = \int_0^r \left[ e^{-\Phi(r)} \left( 1 - \frac{r^2}{\alpha^2} \right)^{-1/2} \right] \mathit{d}r$$

$$\tilde{a} = \frac{R}{2} \left( \frac{R}{M} \right)^{1/2} \left( 1 - \frac{9M}{4R} \right)^{-1/2} = \text{const}$$

The Figure shows the intrinsic geometry of the full $\theta = \pi/2$ cut, including both the vacuum exterior and the $\rho = \text{const}$ interior (indicated by a shadow) of a Schwarzschild star.
A reminder: the effective potential for photon’s motion

The d’Alambert operator in Minkowski, in \( \{t, r, \theta, \phi\} \) spherical coordinates:

\[
\Box \Psi \equiv \left( \frac{\partial^2 \Psi}{\partial t^2} \right) - \frac{1}{r^2} \left( r^2 \frac{\partial^2 \Psi}{\partial r^2} \right) - \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \Psi}{\partial \theta} \right) - \frac{1}{r^2 \sin^2 \theta} \left( \frac{\partial^2 \Psi}{\partial \phi^2} \right)
\]

The wave function \( \Psi(t, r, \theta, \phi) \):

\[
\Psi(t, r, \theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \{ W_{\ell m}(r) P_{\ell}^m(\cos \theta) e^{-i\omega t} e^{-im\phi} \}
\]

where \( P_{\ell}^m(\cos \theta) \) are Legendre’s polynomials. In the axially symmetric case \( \partial\phi = 0, m = 0 \) the radial part of the wave equation:

\[
\Box_r W = \left[ \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial \tilde{r}^2_*} + \frac{\ell(1 + \ell)}{\tilde{r}^2} + \delta V \right] W = 0.
\]

In the Minkowski space-time it is \( r_* = \tilde{r} = r \) and \( \delta V = 0 \) but in general:

\[
r_* = \int (g_{tt})^{-1} dr, \quad \tilde{r}^2 = -g_{\phi\phi}/g_{tt}, \quad \tilde{V} = V + \delta V
\]

\( V = L^2/\tilde{r}^2 \) is the effective potential for a photon with angular momentum \( L \).
Effective potential for ultra compact objects

QUANTUM GRAVITY IS COMING TO TOWN – 12. III. 2021
The mathematical formulation of the problem

\[ W \sim e^{-i\omega_\ast t}, \quad \text{Re}(\omega_\ast) = \omega, \quad \text{Im}(\omega_\ast) = 1/t_0. \]

\[ \frac{\partial^2 W}{\partial t^2} - \frac{\partial^2 W}{\partial r_*^2} - \left[ V_\ell(r_\ast) + \delta V(r_\ast) \right] W = 0, \]

\[ \tilde{r} \equiv \frac{r}{(\eta^i \eta_i)^{1/2}} = \begin{cases} \tilde{a}^2 \sin^2 (r_\ast / \tilde{a}) & \text{for } r < R, \\ r(1 - 2M/r)^{-1/2} & \text{for } r > R. \end{cases} \]

\[ V_\ell(r_\ast) = \frac{\ell(1 + \ell)}{\tilde{r}^2}, \quad \delta V(r_\ast) \ll V_\ell(r_\ast) \]

\[ \delta V(r_\ast) = \begin{cases} -(1 - 2M/R)^{1/2}(3M/R^3)e^\Phi & \text{for } r < R, \\ -6Mr^{-3}(1 - 2M/r) & \text{for } r > R. \end{cases} \]
Waves trapped in the potential well
Our solution step by step

1. The equation

For weakly damped modes we solve the Schrödinger-like Teukolsky’s equation (the radial part):

\[
\frac{d^2 W}{d\chi^2} + \left[ \sigma^2 - \tilde{a}^2 \left( V_\ell + \delta V \right) \right] W = 0; \quad V_\ell = \frac{\ell(\ell + 1)}{\tilde{r}^2} = \text{(effective potential)}
\]

\[\delta V \ll V_\ell, \quad \chi \equiv r_*/\tilde{a} \quad \text{and} \quad \sigma \equiv \tilde{a} \omega_*\]

2. The boundary conditions

We adopt the regularity boundary condition at the center of the star and outgoing-wave boundary at infinity:

\[W'(0) = 0, \quad [W' - i\sigma W]_{\chi \to \infty} = 0\]

3. Exterior and interior solution:

The equation is solved separately in the exterior and interior spacetimes and matched at the stellar surface. In the exterior, the solution is given by WKBJ approximation, in the interior it is given by the associated Legendre function,

\[W_{\text{ext}} = \left( \frac{\Theta}{\gamma} \right)^{1/4} \left[ \text{Ai}(\Theta) - i\text{Bi}(\Theta) \right] \quad W_{\text{int}}(\chi) = (\sin \chi)^{1/2} Q^\mu_\nu(\cos \chi),\]

\[\Theta \equiv \left[ \frac{3}{2} \int_{r(\chi)}^{r_\star} \gamma(r) \left( \frac{dx}{dr} \right) dr \right]^{2/3}, \quad \gamma(r) \equiv \sqrt{\tilde{a}^2 V_{\text{ext}}(r) - \sigma^2}.
\]

4. The corresponding eigenfrequencies

\[\text{Re}(\sigma) = n; \quad \text{Im}(\sigma) = -A_\ell \frac{n^{2\lambda}(\ell + n)!}{(n - \ell - 1)!} \exp \left( \frac{32}{27} \pi n x^{1/2} \right) x^{\lambda + \ell + 1/2}; \quad A_2 = 1.165 \times 10^{-2}.\]
The solution: analytic formulae

\[
\text{Re} (M \omega_*) = \frac{16}{27} nx^{1/2},
\]

\[
\text{Im} (M \omega_*) = - \frac{16}{27} \frac{n^{2\lambda}(\ell + n)!}{(n - \ell - 1)!} \times A_\ell \exp \left( \frac{32}{27} \pi n x^{1/2} \right) x^{\lambda + \ell + 1}
\]
Results: analytic formulae versus numerical results.

\[ \text{Re}(M\omega_*) \]

Numerical simulation points:
Andersson, Kojima and Kokkotas

\[ x = 1 - 9M/4R \]

\[ \text{Im}(M\omega_*) \]

\[ x = 1 - 9M/4R \]
Optical geometry calculation of $Re(\omega)$

The condition for a standing wave with $n$ nodes
\[ \tilde{a} \omega_n = n + 1 \]

\[ \omega_n R = 3(n + 1) \frac{M}{R} \left( \frac{4}{9} \frac{R}{M} - 1 \right)^{1/2} \]

Optical geometry gives correct result in one line of calculation

\[ R \ll \tilde{a} \]
One may estimate the mode decay time $t_0$ by considering the black body radiation with a uniform energy density $\varepsilon$ enclosed in a container. The radiation flux that leaks from a small hole in the container if $f$. In the case of the black body radiation it is

$$\varepsilon \left( \frac{4\sigma}{3c} \right) T^4, \quad f = \sigma T^4, \quad T = \text{temperature}$$

Therefore, the characteristic decay time is given by:

$$t_0 = \frac{\varepsilon V}{f A} = \left[ \left( \frac{4\pi \tilde{a}^3}{3} \right) \left( \frac{4\sigma}{3c} \right) T^4 \right] \times \left[ (4\pi R^2) \left( \sigma T^4 \right) \right]^{-1}.$$

Here $V(R)$ is the volume of the compact star and $A(R)$ is the area of its surface. Using the explicit form of these two functions that are known in the Schwarzschild internal solution, one arrives at

$$\frac{4}{9} \left( 1 - \frac{9M}{4R} \right)^{-q} \left( \frac{R}{M} \right)^{5/2} \left[ \frac{M}{c} \right] \quad \text{here} \quad q = 3/2$$
Black body does not fit

\[ \text{Im}(M\omega_*) \]

\[ x = 1 - 9M/4R \]
New results and conclusions