Decoherence and noise in an environment of photons and gravitons

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Plan of the talk

- Introduction
- Non-perturbative effects
- Perturbative gravity in cosmology
- Decoherence
- Environment of photons
- Environment of gravitons
Gravitational effects in perturbative QFT are necessarily small because the coupling constant $G$ is small (grav.inter/elect.inter. of electrons $\approx 10^{-42}$).

Non-perturbative effects can be large even if an interaction involves a small coupling. Example: tunneling of wells in QM.

The propagator in QFT: the Green function of

$$M = g^{\mu\nu} \partial_\mu \partial_\nu + A^\mu \partial_\mu$$ (1)

So after averaging over gravity we have

$$\langle M^{-1}(x, y) \rangle$$ (2)

where

$$M^{-1} = \int_0^\infty ds \exp(sM)$$ (3)

The kernel behaves as $s^{-\frac{d}{2}}$.

Claim: if $< g(x)g(y) > \simeq (x - y)^{-2\gamma}$ then

$$\langle M^{-1}(x, y) \rangle \simeq (x - y)^{-d+2+2\gamma}$$ (4)
Rigorously

\[ \int dy \left< \exp(t\mathcal{M})(x, y) \right> |x - y|^4 \sim t^{2(1-\gamma)} \]  

(5)
Consider a perturbation of the homogeneous metric

$$ds^2 = a^2(d\tau^2 - dx^2)$$  \hspace{1cm} (6)

and its perturbation in a special gauge (Newtonian gauge)

$$ds^2 = a^2((1 + 2\phi)d\tau^2 - ((1 - 2\phi)\delta_{jk} + h_{jk})dx^j dx^k)$$  \hspace{1cm} (7)

\(\tau\)-the conformal time \(\tau = \int dt a^{-1} = -H^{-1} \exp(-Ht)\)

where \(a(t) = \exp(Ht)\)
The action for cosmological perturbations is

\[ S = \frac{1}{2} \int dx \left( (\phi')^2 - (\nabla \phi)^2 + z^{-1} z'' \phi^2 \right) \]  

(8)
The Lagrangian equations

\[(\partial_{\tau}^2 - \nabla^2 - z^{-1} z'') \phi = 0\]  

(9)

where \( z = \sqrt{g} a \) and

\[ g = 1 - a^2 (\partial_{\tau} a)^{-2} \partial_{\tau} (a^{-1} \partial_{\tau} a) \]

During inflation in a scalar potential \( V \) in the slow-roll approximation
\[ z^{-1}z'' = (Ha)^2(2 + 5\epsilon - 3\eta) \]  
(10)

where \( H \) is the Hubble variable in the cosmic time, \( Ha \simeq \tau^{-1} \) and

\[
\epsilon = \frac{1}{16\pi G} \left( \frac{V'}{V} \right)^2
\]

\[
\eta = \frac{1}{8\pi G} \frac{V''}{V}
\]
Quantum fields:
The Hamiltonian corresponding to the action reads

\[ \mathcal{H} = \frac{1}{2} \int d\mathbf{x} \left( \Pi^2 + (\nabla \phi)^2 - z^{-1} z'' \phi^2 \right) \]  \hspace{1cm} (11)

From the Hamiltonian we can have the wave function as a solution of the Schrödinger equation.

\[ \psi^g_t = N \exp \left( i \frac{\phi}{2\hbar} \Gamma_\phi (\tau) \phi + i \frac{J_\tau}{\hbar} \phi \right) \]  \hspace{1cm} (12)
\[ \langle \Omega | \phi(k) \phi(k') | \Omega \rangle = A \delta(k + k') |k|^{-3-6\epsilon+2\eta} \]  

where \( A \simeq \rho^{-1} \delta \rho \simeq 10^{-5} \) confirmed by COBE, \( 6\epsilon - 2\eta \simeq 0.02 \) \(|k|^{-3}\) - Harrison-Zeldovich spectrum
Thermal fluctuations

The fluctuations may be described by a mixed state rather than the pure state (vacuum). CMB indicates a thermal state. Calculations give

$$< \phi(\mathbf{k})\phi(\mathbf{k}') > \simeq \delta(\mathbf{k} + \mathbf{k}')|\mathbf{k}|^{-3+2\eta+3\gamma}$$  \hspace{1cm} (14)

$\gamma$-friction constant in the harmonic oscillator equation.
Decoherence in general

When we average over an environment the system is described by a density matrix which instead of the Schrödinger equation satisfies (in the Markovian approximation) the master equation (Lindblad equation). In the photonic environment

\[ \partial_t \rho = -\frac{i}{\hbar} [\mathcal{H}, \rho] - \alpha [P, [P, \rho]] \] (15)

in the graviton environment (non-relativistic particles)

\[ \partial_t \rho = -\frac{i}{\hbar} [\mathcal{H}, \rho] - \alpha [P^2, [P^2, \rho]] \] (16)
Generalization of quantum mechanics beyond the Kopenhagen interpretation

\[ d\psi = -\frac{i}{\hbar} \mathcal{H}\psi\,dt + i\psi\,dW \]  

(17)

\( W(t) \)-Brownian motion.
Such modifications have consequences for the spectrum of cosmological perturbations and can be checked in astrophysical observations.

The unobservable particles of dark matter can constitute an environment They can form the heat bath which leads to the modification of the equations for the density matrix and are changing the Einstein equations.
Classical radiation environment

\( \mathcal{L} = \frac{1}{8\pi} \int d\mathbf{x} \left( (\partial_t \mathbf{A})^2 - \mathbf{B}^2 \right) + \mathbf{A} \frac{d\mathbf{q}}{ds} + \frac{1}{2} \left( \frac{d\mathbf{q}}{ds} \right)^2 - V(q) \).

The equation for the electromagnetic field resulting from the Lagrangian is

\[ \frac{d^2 A^r(k)}{dt^2} + k^2 A^r(k) = 4\pi \Lambda^{rl} \frac{dq^l}{dt} \exp(-i\mathbf{kq}(t)). \quad (18) \]

where \( \Lambda \) is the projection on the transverse part
I write the solution in the form (in the radiation gauge)

\[ A_r(k) = A_r^{th}(k) + A_r^l(k) = e_r^\alpha (a_0^\alpha \cos(kt) + k^{-1}b_0^\alpha \sin(kt)) \]
\[ + \Lambda_r l 4\pi \int_{t_0}^{t} k^{-1} \sin(k(t - t')) \frac{dq}{dt'} \exp(-i\mathbf{kq}(t')dt'), \] (19)
The equation of motion for the coordinate $q^r$ is

$$\frac{d^2 q_r}{dt^2} = -\partial_t (2\pi)^{-3} \int A_r(k) \exp(-i q(t)k)dk.$$  

(20)

I insert $A_t$ from eq.(19), I obtain stochastic Abraham-Lorentz equation
The noise comes from the thermal part of $\mathbf{A}$

$$< N_r(t)N_l(t') >= \int d\mathbf{k} n_r l \cos(k(t-t')) = \frac{2}{3} \partial_t^2 \delta(t-t') \quad (21)$$

The stochastic equation is

$$\frac{d^2 q_r}{dt^2} = \frac{2}{3} \frac{e^2}{mc^3} \frac{d^3 q_r}{dt^3} + N_r(t) \quad (22)$$

For an oscillator $\partial_t^2 q = -\omega^2 q$ hence the friction which is equal to the width of the spectral line is

$$\frac{2e^2 \omega^2}{3mc^3}$$
Quantum environment of oscillators

\[ i\hbar \partial_t \psi_t = \left( -\frac{\hbar^2}{2m} \nabla_x^2 + \frac{m\omega^2 x^2}{2} + V_t(x) \right) \psi_t \equiv H \psi_t. \quad (23) \]

I introduce a solution \( \psi_t^g \) of the Schrödinger equation for the oscillator (\( \omega \) may depend on time)

\[ i\hbar \partial_t \psi_t^g = \left( -\frac{\hbar^2}{2m} \nabla_x^2 + \frac{m\omega^2 x^2}{2} \right) \psi_t^g. \quad (24) \]
I represent the solution of (23) in the form

\[ \psi_t = \psi_t^g \chi_t. \]  

(25)

Then, \( \chi_t \) solves the equation

\[ \partial_t \chi_t = \left( \frac{i\hbar}{2m} \nabla_x^2 + \frac{i\hbar}{m} \nabla_x \ln \psi_t^g \nabla - \frac{i}{\hbar} V_t \right) \chi_t \]  

(26)

Let

\[ dq_s = \frac{i\hbar}{m} \nabla \ln \psi_{t-s}^g(q_s) ds + \sqrt{\frac{i\hbar}{m}} db_s, \]  

(27)

where \( b_t \) is the Brownian motion, i.e., the Gaussian process with the covariance

\[ E[b_t b_s] = \min(t, s). \]
then

$$\chi_t(x) = E \left[ \exp \left( - \frac{i}{\hbar} \int_0^t ds V_{t-s}(q_s(x)) \right) \chi_0(q_t(x)) \right], \quad (28)$$

here $q_s(x)$ is the solution of the Langevin equation (5) with the initial condition $q_0(x) = x$. 
Particle interacting with scalar and tensor perturbations

For a detection of gravitational waves in LIGO/Virgo a system of mirrors is prepared. The interference takes place depending on the distance of the mirrors. I consider a system of two mirrors with masses $m' \gg m$. The action for the particle-metric interaction is

$$S = m \int \sqrt{g_{\mu\nu}} dX^\mu dX^\nu$$

$X^\mu = (\tau, \mathbf{X})$, $\mathbf{X}$ are the Fermi coordinates between the neighboring geodesics of the two mirrors. Calculating the square root in the non-relativistic approximation we obtain

$$S = \frac{m}{2} \int d\tau (\frac{d\mathbf{X}}{d\tau})^2 - \frac{m}{2} \int d\tau R_{0l0r} X^r X^l$$

where $R_{\mu\nu\alpha\beta}$ is the Riemannian tensor. Inserting the expression for the Riemannian tensor with a linear approximation we obtain the action
\[ S_I = \frac{m}{2} \int d\tau \frac{dX^k}{d\tau} \frac{dX^k}{d\tau} - \lambda m \int d\tau \phi(\tau, X(\tau)) \frac{d^2}{d\tau^2} X^2 + \frac{1}{2} m\lambda \int d\tau h_{jk}(\tau, X(\tau)) \frac{d^2}{d\tau^2} (X^k X^j) \]  \hspace{1cm} (29)

where \( \lambda^2 = 8\pi G \).
Thermal perturbations

The Lagrangian in the coordinate space for scalar perturbations is

\[ \mathcal{L} = \frac{1}{2} \int d\mathbf{x} \phi(s, \mathbf{x}) \left( -\partial_s^2 + \triangle \right) \phi(s, \mathbf{x}) + \frac{1}{2} m \frac{dX_r}{ds} \frac{dX_r}{ds} + \int ds \phi \frac{d^2}{ds^2} \mathbf{X}^2 \]

(30)

I calculate expectation values in the Gibbs state \( \exp(-\beta \mathcal{H}) \). I obtain that the evolution of the density matrix is determined by the stochastic equation
for scalar and tensor perturbations

\[ -\frac{d^2 Q_n}{ds^2} + 16 G m Q n \frac{d^5}{ds^5} Q^2 + \frac{8\pi G m}{10\pi} Q^l \frac{d^5}{ds^5} \left( \frac{1}{3} Q_r Q_r \delta_{nl} - Q_n Q_l \right) \]

(31)

\[ = m^{-1} \sqrt{2G \beta^{-\frac{1}{2}}} \left( M_{\frac{1}{2}}^{\frac{1}{2}} \right)_{nr} \partial_{s} b^r_s. \]

\( b^r(s) \) is the Brownian motion
where

\[ M_{rk;ln}(s, s') = \langle \frac{1}{4\pi} \Lambda_{rk;ln} + 8\delta_{rk}\delta_{ln} \rangle \partial_s^2 \partial_{s'}^2 \delta(s - s') \] (32)

and

\[ \langle \Lambda_{ij;mn} \rangle = 4\pi \left( \frac{1}{5} (\delta_{im}\delta_{jn} + \delta_{in}\delta_{jm}) - \frac{2}{15} \delta_{ij}\delta_{nm} \right) \] (33)
Squeezed state of the cosmological perturbations

I consider a general Gaussian function as the environment

\[ \psi^g_{\tau} = \exp\left( \frac{i}{2\hbar} \phi \Gamma(\tau) \phi + \frac{i}{2\hbar} \hbar \Gamma(\tau) h \right) \] (34)
Then for large squeezing $\gamma$

$$
\rho_T(X, X') \simeq \exp \left( - \frac{\lambda^2 m^2}{16\hbar} \int dk k \gamma(k) \left( \int_0^T dsds' (\cos(k\tau))^2 \cos(ks) \cos(ks') \left( \frac{1}{4\pi} < \Lambda_{rl; mn} > + 8\delta_{rl}\delta_{mn} \right) \right) \right)

\left( \frac{d^2}{ds^2} (Q^r y^l + Q^l y^r) \frac{d^2}{ds'^2} (Q^m y^n + Q^n y^m) \right)

+ \left( \frac{d^2}{ds'^2} Q^l y^l \frac{d^2}{ds^2} Q^n y^n \right),
$$

where $X = Q + \frac{\gamma}{2}$, $X' = Q - \frac{\gamma}{2}$

Large $\gamma$ (squeezing)- large noise

Large squeezing during inflation.

Through detection of noise confirmation of quantum nature of gravitational fluctuations(waves)