Hydrodynamic attractors in Heavy Ion Collisions

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Ultrarelativistic Heavy Ion Collisions

- **Semi-hard particle production**
  
  $0 < \tau < 0.3 \text{ fm/c}$

- **Hot Hadron Gas**
  
  $6 < \tau < 10 \text{ fm/c}$

- **Freezeout**
  
  $\tau > 10 \text{ fm/c}$

- **Beam direction**
  
  time

- **Non-equilibrium QGP**
  
  $0.3 < \tau < 2 \text{ fm/c}$

- **Equilibrium QGP**
  
  $2 < \tau < 6 \text{ fm/c}$

Universal dynamics at early times

A far-from-equilibrium attractor is a model of equilibration in which memory of initial conditions is rapidly swept away $\tau \sim 0.5 \text{ fm/c}$

- What mechanism drives the system towards the attractor?
- Can it have experimental imprints?
- How generic it is?
Bjorken flow

• Boost invariant metric $ds^2 = -d\tau^2 + \tau^2 dy^2 + dx_\perp^2$

• Energy momentum tensor is diagonal

$$T_{\mu\nu} = \text{diag}\{\epsilon(\tau), P_L(\tau), P_T(\tau), P_T(\tau)\}$$

• Conditions: $\nabla_\mu T^{\mu\nu} = 0$ and $T^{\mu}_{\mu} = 0$ imply

$$P_L(\tau) = \frac{\epsilon}{3} \left(1 - \frac{2}{3} A\right) \quad P_T(\tau) = \frac{\epsilon}{3} \left(1 + \frac{1}{3} A\right)$$

where $A$ is the pressure anisotropy

• Evolution of the system is captured by a single function $\epsilon(\tau)$

• Strict for an infinite energy collision of infinitely large nuclei

Bjorken flow

• Energy density *defines* local effective temperature

\[ \epsilon(\tau) = \gamma T(\tau)^4 \]

• Dimensionless time variable measured in units of relaxation time \( \tau_R \sim 1/T(\tau) \)

\[ w = \tau T(\tau) \]

• \( w = 1/Kn \) where we use the Knudsen number

\[ Kn = \frac{\lambda_{\text{micro}}}{\lambda_{\text{macro}}} \]


Pressure anisotropy determines the conservation equation

\[ \nabla_\mu T^{\mu\nu} = 0 \quad \iff \quad \tau \partial_\tau \log \epsilon = -\frac{4}{3} + \frac{2}{9} \mathcal{A}(w) \]

or in dimensionless variables

\[ \frac{d \log T}{d \log w} = \frac{\mathcal{A}(w) - 6}{\mathcal{A}(w) + 12} \]

with a solution

\[ T(w) = T(w_0) \exp \left( \int_{w_0}^w \frac{dx}{x} \frac{\mathcal{A}(x) - 6}{\mathcal{A}(x) + 12} \right) \]

Function \( \mathcal{A}(w) \) comes from a microscopic model and encodes initial conditions of the system.
Mueller-Israel-Stewart (MIS) theory

- A simple model of equilibration that possesses exponentially damped, non-hydrodynamic mode
- For the Bjorken flow equations read

\[
C_\tau \Pi \left( 1 + \frac{A}{12} \right) A' + \left( \frac{C_\tau \Pi}{3w} + \frac{C_\lambda}{8C_\eta} \right) A^2 = \frac{3}{2} \left( \frac{8C_\eta}{w} - A \right)
\]

- The attractor is the unique regular solution at \( w = 0 \)

\[
A_*(w = 0) = 6 \sqrt{\frac{C_\eta}{C_\tau \Pi}}
\]

- At late times \( A \) admits a divergent gradient expansion


The MIS attractor

\[ A(w) = \frac{P_T - P_L}{c^3/3} \]

\[ w = \tau T \]


What attracts to attractors?

- **Early time** - system’s expansion rate dominates the dynamics leading to a universal, power-law behaviour determined by the Bjorken symmetry

- **Late times** - decay of the non-hydro mode resulting in a trans-series structure with exponentially decaying modes symmetry independent

What attracts to attractors?

\[ A(w) = \frac{P_T - P_L}{\epsilon / 3} \]

\[ w = \tau T \]

Connecting early and late times

- If an attractor exists then we approximate the time evolution as

\[ T(w) \sim T(w_0) \exp \left( \int_{w_0}^{w} \frac{dx}{x} \frac{A_*(x) - 6}{A_*(x) + 12} \right) \]

- The system follows universal dynamics apart from initial transient

- Almost all memory of initial conditions is lost, only temperature remains

Connecting early and late times

- At late times
  \[\epsilon(\tau) \sim \frac{\Lambda^4}{(\Lambda\tau)^{\frac{4}{3}}}\]

- At early times
  \[\epsilon(\tau) \sim \frac{\mu^4}{(\mu\tau)^{\beta}} \iff A_*(w) \sim 6 \left(1 - \frac{3}{4}\beta\right)\]

- With attractor evolution one can relate the late time energy scale \(\Lambda\) to the initial time energy scale \(\mu\)

- Knowledge of \(\Lambda\) determines final state entropy and whence particle production

Connecting early and late times

- Due to the attractor assumption we can connect particle production with initial energy deposition

\[
\frac{dN}{dy} = h(\beta) \int d^2x_\perp \epsilon(\tau_0, x_\perp) \frac{2}{4-\beta}
\]

- The unknown dependence on the attractor cancels when we consider centrality ratios

\[
Q(c, c') = \frac{\langle dN/dy \rangle_c}{\langle dN/dy \rangle_{c'}}
\]

- Adopting various models of initial energy deposition we get predictions for $\beta$ by fitting to $\sqrt{s} = 2.76$ TeV ALICE data

Models we consider are formulated in terms of a nuclear thickness function $T(x_{\perp})$ obtained by Monte-Carlo sampling.

All considered models use aspects of saturation physics.

The impact parameter $b$ in fm is translated to centrality $c$ in %.

Initial state models

• Dilute-dense

\[ \epsilon^{(I)}(\tau_0, x_\perp) = CT^<(x_\perp) \sqrt{T^>(x_\perp)} \]

with \( T^<(x_\perp) = \min(T(x_\perp + b/2), T(x_\perp - b/2)) \)

• Trento \( p = -1 \)

\[ \epsilon^{(II)}(\tau_0, x_\perp) = C \frac{T(x_\perp + b/2) T(x_\perp - b/2)}{T(x_\perp + b/2) + T(x_\perp - b/2)} \]

• Dense-dense

\[ \epsilon^{(III)}(\tau_0, x_\perp) = CT(x_\perp + b/2) T(x_\perp - b/2) \]
Connecting early and late times

\[ Q(c_{20}^\prime) \]

\[ \text{centrality } \% \]

\[ \beta = 1.12 \quad \beta = 1.96 \quad \beta = 0.44 \]

- Pb + Pb@\sqrt{s} = 2.76 \text{ TeV}
- Xe + Xe@\sqrt{s} = 5.44 \text{ TeV}
- U + U@\sqrt{s} = 193 \text{ GeV}
- Au + Au@\sqrt{s} = 130 \text{ GeV}
- Cu + Cu@\sqrt{s} = 62.4 \text{ GeV}

Conecting early and late times

- The three adopted models of initial energy deposition predict
  \[ \beta^{(I)} = 1.12 \quad \beta^{(II)} = 1.96 \quad \beta^{(III)} = 0.44 \]

- In contrast free-streaming, ubiquitous in kinetic theory, implies
  \[ \beta = 1 \]

- Pre-hydrodynamic flow is tightly connected to the initial state model
- Interactions are relevant at the earliest stages of the evolution

Conclusions and questions

- The attractor provides a simple conceptual link between early and late times

- How to define an attractor in a conceptually transparent fashion?

- To answer that one can try to use insights from dynamical system theory ...

- Relax the restrictive symmetry assumptions (conformal, Bjorken ...) and perform the Bayesian analysis